Magnetic Fields

MAXWELL'S EQUATIONS

Differential Form

$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{H} = \vec{J}$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Integral Form

$$\oint \vec{B} \cdot d\vec{S} = 0$$
$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S}$$
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{S}$$
or $\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \oint \vec{u} \times \vec{B} \cdot d\vec{l}$

VECTOR POTENTIAL

$\nabla \times \vec{A} = \vec{B}$ or $\oint \vec{A} \cdot d\vec{l} = \int \vec{B} \cdot d\vec{S}$

$$\mu_o = 4\pi x 10^{-7} \; H/m$$

MATERIALS

$$\vec{B} = \mu\vec{H} = \mu_o\left(\vec{H} + \vec{M}\right)$$
$$\vec{M} = \chi_m \vec{H}$$
$$\mu = \mu_o\left(1 + \chi_m\right)$$

BOUNDARY CONDITIONS

General

$$\hat{n} \cdot \left(\vec{B}_1 - \vec{B}_2\right) = 0$$
$$\hat{n} \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_S$$

Magnetic-Magnetic

$$B_{1n} = B_{2n}$$
$$H_{1t} = H_{2t}$$

Magnetic-Conductor

See General

FORCE

$\vec{F} = q\left(\vec{u} \times \vec{B}\right)$ *Newtons per particle*

$\vec{F} = -\nabla W_m$ *Newtons*

$\vec{f} = \vec{J} \times \vec{B}$ *Newtons/m^2 force per volume*

ENERGY

$$W_m = \frac{1}{2} L I^2 = \frac{1}{2}\int \vec{B} \cdot \vec{H} dv = \int \frac{B^2}{2\mu} dv$$

INDUCTANCE

$$L = \frac{\Lambda}{I}$$

$$L = \frac{N\Phi_m}{I}$$ *if all turns enclose the same flux*

Magnetic Flux

$$\Phi_m = \int \vec{B} \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{l}$$

Mutual Inductance

$$L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2 \Phi_{12}}{I_1}$$

$$\Phi_{12} = \int \vec{B}_1 \cdot d\vec{S}_2$$

BIOT-SAVART LAW

$$\vec{B} = \frac{\mu_o I}{4\pi} \oint_{C'} d\vec{l}' \times \frac{\hat{R}}{R^2}$$

$$\vec{A} = \frac{\mu_o I}{4\pi} \oint_{C'} \frac{d\vec{l}'}{R}$$

OHM'S LAW – RESISTIVITY

$$\vec{J} = \sigma\vec{E} \text{ and } R = \frac{l}{\sigma A}$$

MAGNETIC CIRCUITS

Magnetomotive Force (mmf)

$$V_m = NI$$

Magnetic Flux

$$\Phi_m = \int \vec{B} \cdot d\vec{S} \text{ (analog of current)}$$

Reluctance

$$\mathcal{R} = \frac{l}{\mu A}$$