Electric Fields

MAXWELL'S EQUATIONS

Differential Form

$$\nabla \cdot \vec{D} = \rho_v$$
$$\nabla \times \vec{E} = 0$$

Integral Form

$$\oint \vec{D} \cdot d\vec{S} = \int \rho_v dv = Q_{encl}$$
$$\oint \vec{E} \cdot d\vec{l} = 0$$
$$Q_{encl} = \int \rho_v dv = \int \rho_S dS = \int \rho_l dl$$

VOLTAGE EQUATIONS

Electric Scalar Potential

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \text{ (Poisson's Eqn)}$$
$$V_2 - V_1 = -\int_{r_1}^{p_2} \vec{E} \cdot d\vec{l}$$
$$\vec{E} = -\nabla V$$

Finite Difference Approximation to Laplace's Equation

$$V_{i,j} = \frac{\left(V_{i+1,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1}\right)}{4}$$

DEFINITIONS & DIELECTRIC PROPERTIES

Electric Flux Density

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_o \vec{E} + \vec{P}$$

Polarization

$$\vec{P} = \varepsilon_o \chi_e \vec{E}$$

Permittivity

$$\varepsilon = \varepsilon_o\left(1 + \chi_e\right)$$
$$\varepsilon_o \approx \frac{1}{36\pi} x10^{-9} \equiv 8.854x10^{-12} \; F/m$$

Capacitance

$C=Q/V_{12}$
Q = charge on either conductor
V_{12} = voltage difference between conductors

Force

$$\vec{F} = q\vec{E}$$
$$\vec{F} = \hat{a}_{12} \frac{q_1 q_2}{4\pi\varepsilon_o R^2}$$
$$\vec{F} = -\nabla W_e$$

BOUNDARY CONDITIONS

General

$$\hat{n} \times \left(\vec{E}_1 - \vec{E}_2 \right) = 0$$
$$\hat{n} \cdot \left(\vec{D}_1 - \vec{D}_2 \right) = \rho_S$$

Dielectric-Dielectric

$$E_{t1} = E_{t2}$$
$$D_{1n} = D_{2n}$$

Conductor-Dielectric

$\vec{E}$ *is zero in conductor*

$D_n = \rho_S$ *in dielectric*

$$\hat{n} \times \left(\vec{E}_1 - \vec{E}_2 \right) = 0$$

$E_t = 0$ *in dielectric*

COULOMB'S LAW

$$\vec{E} = \frac{1}{4\pi\varepsilon_o} \sum_{k=1}^{n} \frac{q_k \left(\vec{R} - \vec{R}_k \right)}{\left| \vec{R} - \vec{R}_k \right|^3} = \frac{1}{4\pi\varepsilon_o} \int_{V'} \frac{\rho_v \left(\vec{R} - \vec{R}' \right)}{\left| \vec{R} - \vec{R}' \right|^3} dV'$$

$$V = \frac{1}{4\pi\varepsilon_o} \sum_{k=1}^{n} \frac{q_k}{\left| \vec{R} - \vec{R}_k \right|} = \frac{1}{4\pi\varepsilon_o} \int_{V'} \frac{\rho_v}{\left| \vec{R} - \vec{R}' \right|} dV' = \frac{1}{4\pi\varepsilon_o} \int_{V'} \frac{\rho_S}{\left| \vec{R} - \vec{R}' \right|} dS'$$

ENERGY

$$W_e = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \int \vec{D} \cdot \vec{E} dv = \int \frac{\varepsilon E^2}{2} dv$$

COORDINATE SYSTEMS

Rectangular

$d\vec{l} = \hat{x}dx + \hat{y}dy + \hat{z}dz$
$dS_x = dydz$
$dS_y = dxdz$
$dS_z = dxdy$
$dv = dxdydz$

$d\vec{l} = \hat{r}dr + \hat{\phi}rd\phi + \hat{z}dz$
$dS_r = rd\phi dz$
$dS_\phi = drdz$
$dS_z = rdrd\phi$
$dv = rdrd\phi dz$

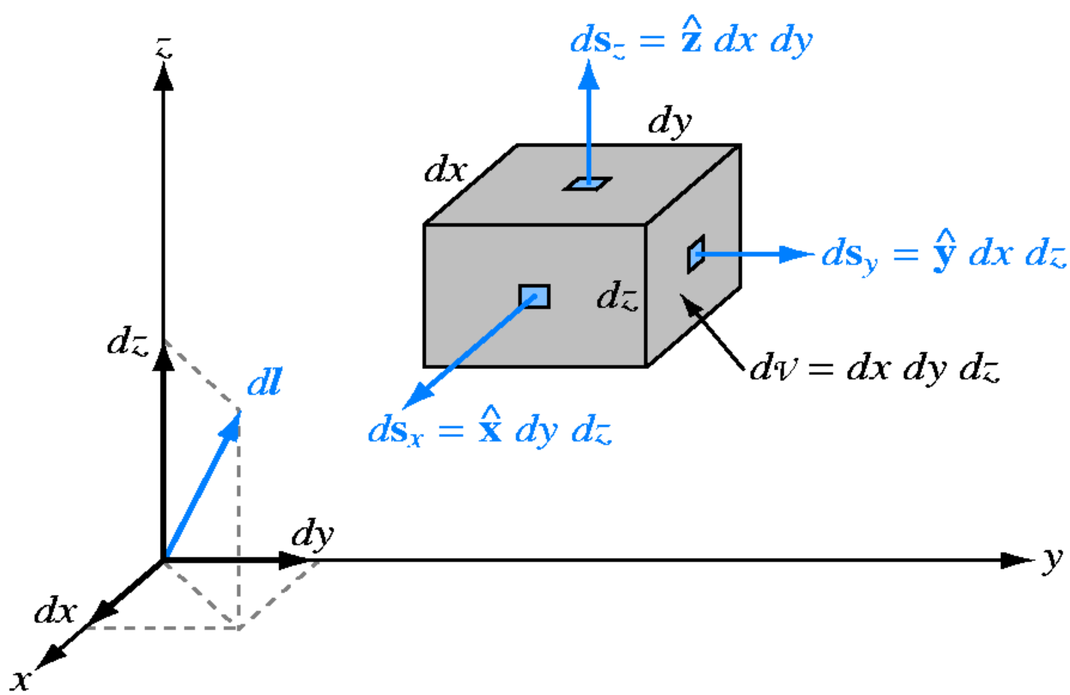
$d\vec{l} = \hat{r}dr + \hat{\theta}rd\theta + \hat{\phi}r\sin\theta d\phi$
$dS_r = r^2\sin\theta d\theta d\phi$
$dS_\theta = r\sin\theta drd\phi$
$dS_\phi = rdrd\theta$
$dv = r^2\sin\theta drd\theta d\phi$

Spherical

Cylindrical

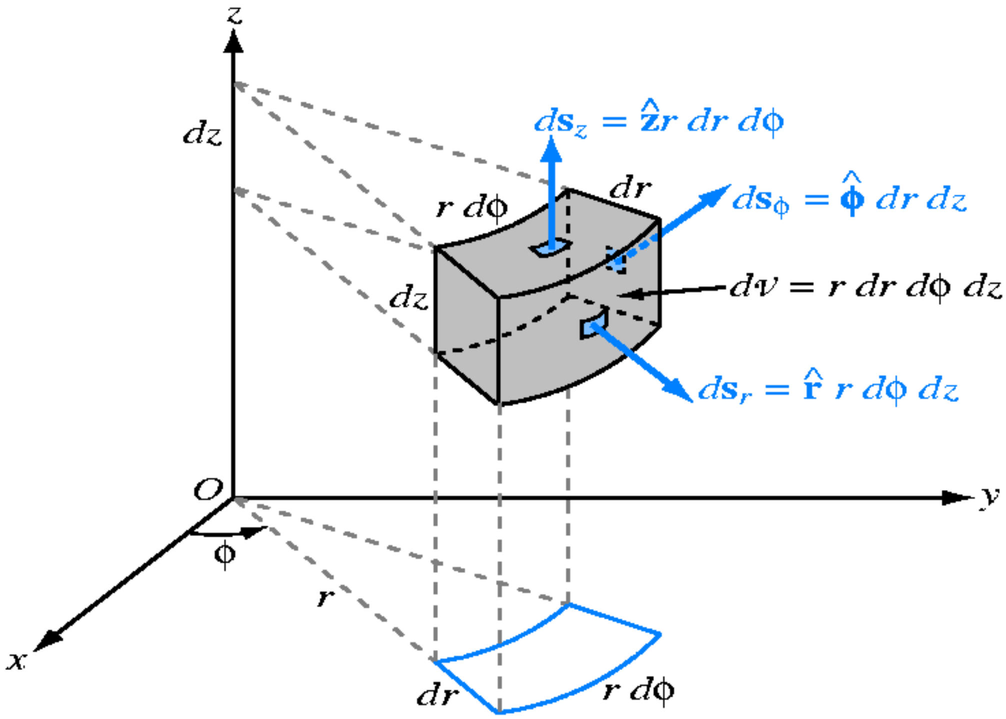
Note: Most Vector Formula Info is Found on the Inside Covers of the Text

Rectangular



Note: In Ulaby (where these figures are from), the spherical radial direction is indicated by upper case R while here we use lover case r. It is almost always simple to determine whether we are using cylindrical or spherical r and we have decided to use R for resistance. The diagrams on the next page show the cylindrical and spherical cases.

Cylindrical



Spherical