Due 9 September 2009 at 6:00 pm

## 1. Waves and Phasor Notation

Be sure that you read the following questions carefully. Also, when you convert an expression from phasor to time domain form or vice versa, convert it back, carefully following the rules, to check your answer.
a. Write the following voltage phasor expression in time domain form $\tilde{V}=100 e^{+\frac{j 7 \pi}{4}}$. Note that this is just a voltage, not a voltage wave. Assume $f=30 \mathrm{MHz}$.
b. Write the following time domain current wave expression in phasor

$$
\text { formi }(z, t)=I_{o} \sin \left(\omega t+\frac{\omega z}{a}\right) .
$$

c. Is the expression in part b a standing or traveling wave? If it is a traveling wave, what direction does it travel and what is its velocity?

## 2. Plane Wave Representations

The numbers given in this problem are realistic but not necessarily real. That is, your answers should come out in a reasonable range, but the numbers are not based on a real, commercially available transmission line.
The current on a transmission line is given by $i(z, t)=\cos \left(7 \pi 10^{6} t+0.03 \pi z\right)$.
a. Is this a standing wave or a traveling wave? If it is a traveling wave, what direction does it travel and what is its velocity?
b. What is the period of this wave $T$ ? What is the wavelength $\lambda$ ?
c. Plot this expression as a function of space at $t=0, t=T / 3, t=2 T / 3$ using Maple or Matlab or some similar program. Note that the wave propagates to the left (order is blue, red, green)


## Homework 1

d. Write this current expression in phasor form.
e. Assume that the transmission line has a capacitance per unit length of $43 \mathrm{pF} / \mathrm{m}$.

Find the characteristic impedance of the line $Z_{o}$ and then the voltage on the line in phasor form.

## 3. PSpice Simulated Experiments

PSpice can show us a lot about how the voltages look at the input and output ends of transmission lines. The simulation can, in effect, replace an experiment by giving us essentially the same results.
a. To get some practice using PSpice, set up the following simulation using the parameters of your transmission line from problem 2. That is the source, line and load impedances should all be equal to the characteristic impedance of the line you determined in problem 2. The source voltage and frequency should be selected to obtain the current given in problem 2 on the line. Set the offset voltage to zero. Use the lossless line model and assume the length of the line is 40 meters. Be careful to use the zero ground, since it is the only one that works with PSpice.


For a line that is 40 m long, the time delay (needed for T1) will be
$T D=$ length $/$ velocity $=40 / 2.33 \times 10^{8}=1.7143 \times 10^{-7}$
Run the simulation shown below and display your results. The numbers given are approximate. You may want to change them some to get a better display. To see the results better, the time range should be reduced. However, this is not a critical issue.


b. Explain why your result makes sense. Be sure to thoroughly label your plot.

The green sine wave is the input and the red sine wave is the output. The line is lossless so the input and output should have the same amplitude. The 40 meter line produces a time delay of 171 ns , which is exactly what is shown. Note that to obtain the specified voltage on the line of 100 V , the voltage source had to be 200V, as shown below.


## 4. Lumped Transmission Lines

A transmission line can be replaced by a series of lumped circuit elements, under certain conditions. In this exercise, you will compare the response of such a lumped model line with the coil of coax. Obtain a coil of coax, a 50 Ohm terminator and one of the lumped component transmission lines from a TA. Note: we will be analyzing the lumped configuration using PSpice in lecture. Thus, your measurements should be guided by what we obtain there. Provide the number of your coax coil and lumped model below: Coax \#: $\qquad$ Lumped Model \#: $\qquad$
a. First, we will repeat an experiment from the first studio session at a different frequency. Put the 50 Ohm terminator across the output of the coaxial cable and simultaneously measure input and output signals on the oscilloscope. Set the input voltage at $1 \mathrm{~V}_{\text {P-p }}$ with a frequency of 3 MHz . Measure the time delay between the signals. What else is different about the input and output voltages?
b. Replace the coaxial cable with the lumped version and repeat the experiment.
c. You should have observed that the lumped line behaves in a qualitatively similar manner to the spool of coax. Thus, it must represent a similar length of line. We want to determine the actual length of line. Each $L-C$ combination represents some equivalent length of line. Since there are 20 such combinations, we only need to figure out what length each combo represents and multiply it by 20. To understand better how the lumped circuit is configured, look at the diagram below, done with PSpice. (This diagram has a load impedance R2 of 93 Ohms, which is not what we are using here.) Note that the inductance and capacitance for each section is indicated. Given your knowledge of the actual capacitance and inductance per unit length for the RG58/U coaxial cable, approximately what length does each section represent?

Node 11


## Homework 1

d. Remove the load resistor and replace it with a short circuit. Measure the voltage at the $11^{\text {th }}$ node. Adjust the frequency somewhere between 2 MHz and 4 MHz until the voltage at the $11^{\text {th }}$ node is a minimum. It should be somewhere around 100 mV . Put the terminating resistor back on as a load. Measure the voltage at each of the nodes for the case where the lumped line is terminated with 50 Ohms. Remove the resistor and repeat for the short circuit. Where are the minimas and maximas located? (One minimum should be at node 11.) Plot your results for the measured voltages as a function of distance (using the distance between nodes that you determined above). Plot your results again but now in terms of wavelength rather than meters. (You can use the same plot, if you wish, and provide two sets of labels.)
e. Show that the maximas and minimas are located where they should be in terms of wavelength.
f. Run the corresponding case using the simulation at BesserNet, mentioned in lecture.

Note that the lumped lines work slightly differently if different capacitors are used. Thus, please note the color of the capacitors on the board you are using. The correct answer to part $d$ will depend on what capacitors are used on your board. Before running the experiment, we can first simulate it using PSpice.


## Homework 1

For this case, the signals that should be observed at the last several nodes are plotted below. The thick green dashed line is for node 11 and is indeed a null. The only other null observed is at the load end where there is a short circuit.


We also should look at the BesserNet applet, which shows the standing wave pattern with a null at the half wavelength distance from the shorted load. The wavelength for this case should be given by the ratio of the velocity to the frequency or about 72 meters for the wavelength, so the null should be located about 36 meters from the load or 9 nodes away at node 11, were we do indeed find it.

Reflectometer Calculator


Copyright 2004-5 BAI All Rights Reserved. Free Applets at www.bessernet.com

## Homework 1

Taking data from the artificial line, we see the following pattern in the voltages at each node.


The standing wave pattern is quite clear. It appears that the next minimum is between nodes 1 and 2 and thus maybe the width of the half wavelength is a bit more than 36 meters. These data were taken with a case where the minimum was achieved at 2.6 MHz (the blue capacitors). We can compare with an idealized PSpice calc too.


We probably cannot do much better than this, but there is a systematic tilt to the data that may be due to a small resistance in the inductors and maybe even a little real conductance in the capacitors. The impact of the scope is also not included.

Homework 1

HUI

1. a. $\hat{V}=100 e^{j \frac{7 \pi}{4}} \quad f=30 \mathrm{MHz}$

$$
\begin{aligned}
& \omega=2 \pi f=2 \pi 3 \times 10^{7}=6 \pi \times 10^{7} \\
& V(z, t)
\end{aligned}=\operatorname{Re}\left\{100 e^{j \frac{7 \pi}{4}} e^{j 6 \pi \times 10^{7 t}}\right\}
$$

could also be written with phase $\frac{\pi}{4}$
b.

$$
\begin{aligned}
i(z, t) & =I_{0} \sin \left(\omega t+\frac{\omega z}{a}\right) \\
& =I_{0} \cos \left(\omega t+\frac{\omega z}{a}-\frac{\pi}{2}\right) \\
\tilde{I} & =I_{0} e^{j \omega \frac{z}{a}-j \frac{\pi}{2}}
\end{aligned}
$$

$$
=\underbrace{I_{0} e^{-j \frac{\pi}{2}} e^{+j \omega \frac{z}{a}}}_{\text {complex amplitude }}
$$

C. Traveluy wave

$$
\begin{aligned}
& \text { Velocity }=\frac{\omega}{\beta}=\frac{w}{w / a}=a \\
& \text { Velocity }=a \text { in }-z \text { direction }
\end{aligned}
$$

When $w \sum_{1}^{k} \beta$ terms he me some sign, the ware piupagutes in the neg $z$ direction
2. $\quad i(z, t)=\cos \left(7 \pi \times 10^{6} t+0.03 \pi z\right)$
a. Traveling ware, prop in $-z$ direction

$$
\text { velocity }=\frac{\omega}{\beta}=\frac{7 \pi \times 10^{6}}{0.03 \pi}=2.333 \times 10^{80} \frac{\mathrm{~m}}{\mathrm{~s}}
$$

b. Period $T=\frac{1}{f} \quad f=3.5 \times 10^{6}$
$=2.86 \times 10^{-7} \mathrm{~s}$

$$
\lambda=\frac{2 \pi}{\beta}=\frac{2 \pi}{.03 \pi}=66.7 \mathrm{~m}
$$

c. The 3 time are shown abas in the problem statement.

$$
\text { d. } \frac{\tilde{I}}{I}=e^{* ; 0.03 z}
$$

$$
\text { e. } \quad z_{0}=\sqrt{\frac{l}{c}} \quad c=43 \mathrm{pF} / \mathrm{m}
$$

$$
v=\frac{1}{\sqrt{l c}}
$$

$$
Z_{0}=\frac{1}{c v}=\frac{1}{\left(43 \times 10^{-12}\right)\left(2.33 \times 10^{8}\right)}
$$

$$
=99.67 \approx 100 \Omega
$$

$$
l=z_{0}^{2} c=4.27 \times 10^{-7} \mathrm{H} / \mathrm{m} \simeq .43 \mathrm{\mu H} / \mathrm{m}
$$

The current on the line is $i(z, t)=\cos \left(7 \pi 10^{6} t+0.03 \pi z\right)$, which is phasor form is $\tilde{I}=e^{+j 0.03 z}$. Thus, the voltage on the line in phasor form is $\tilde{V}=Z_{o} \tilde{I}=100 e^{+j 0.03 z}$

