Due 16 September at 6:00 pm

We want to consider an RG58U lossless transmission line used in different applications. We will also be using only the lossy transmission line model from PSpice but will input only lossless parameters. That is we will choose the resistance and conductance per unit length to be zero. Note that their default value is 1. The time delay of the line is *66.67ns* and the excitation frequency is *f = 100kHz*.

1. Transmission Line Characteristics

Since you will need them later, write down the following characteristics of the line and waves on the line.

The inductance per unit length *l =0.25µH/m*

The capacitance per unit length *c =100pF/m*

The characteristic impedance $Z_o=50\Omega$

The propagation velocity $u=2x10^8 m/s$

The length of the line *L=approximately 80m*

The angular frequency $\omega = 2\pi f$

The propagation constant $\beta = \omega\sqrt{lc}$

2. PSpice Simulation of the Transmission Line with Sinusoidal Input Voltage

First we will consider how the line behaves with the sinusoidal input voltage $V(t) = 10\cos\omega t$ where the frequency *f = 100kHz*. The voltage source and the load are both matched to the line. Set up the following representation. Note that none of the parameters are specified in this diagram. Also, this diagram shows the lossless model. You should use the lossy line, but you might want to do both to check your results.
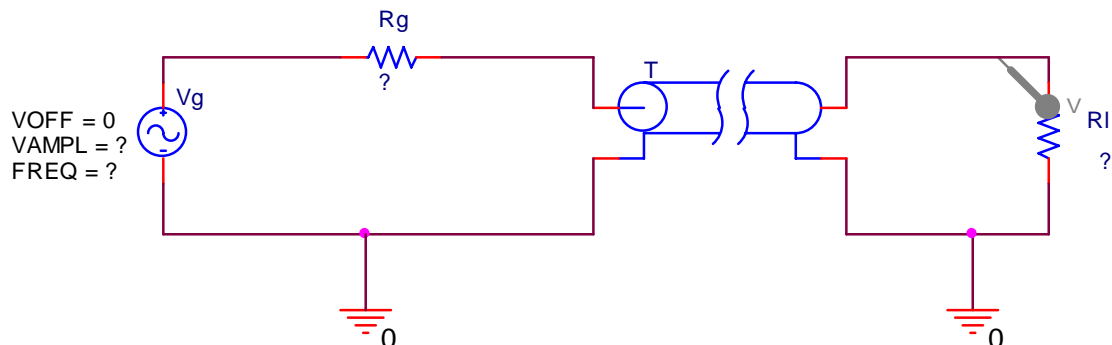
When setting up your simulation profile, use the following AC Sweep. Note that the range of frequencies goes from 100kHz to 15MHz (the latter is the maximum frequency that can be produced by the function generators in JEC 4107). Place voltage probes at both the input and output ends of the transmission line. Note that to make the AC Sweep work, you have to specify an AC value for the voltage source.
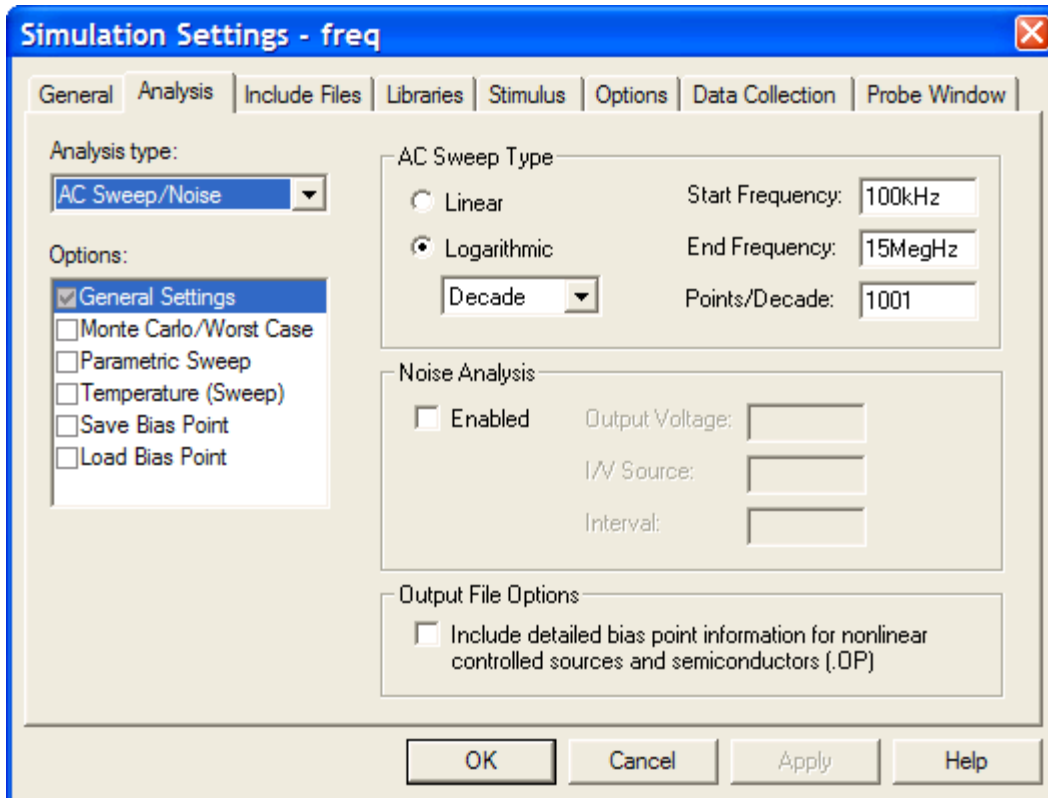
Generate and save the plots of the input and output voltages for the line. Repeat your analysis for the following three cases (the first case has been included in the table for completeness). Z_o is the characteristic impedance of the line.

	Source Resistance	Load Resistance
Case 1	Z_o	Z_o
Case 2	$0.5Z_o$	Z_o
Case 3	Z_o	$0.5Z_o$
Case 4	$0.5Z_o$	$0.5Z_o$

You should have four plots. Note that you could also analyze all four cases at once and produce a single, complicated plot.
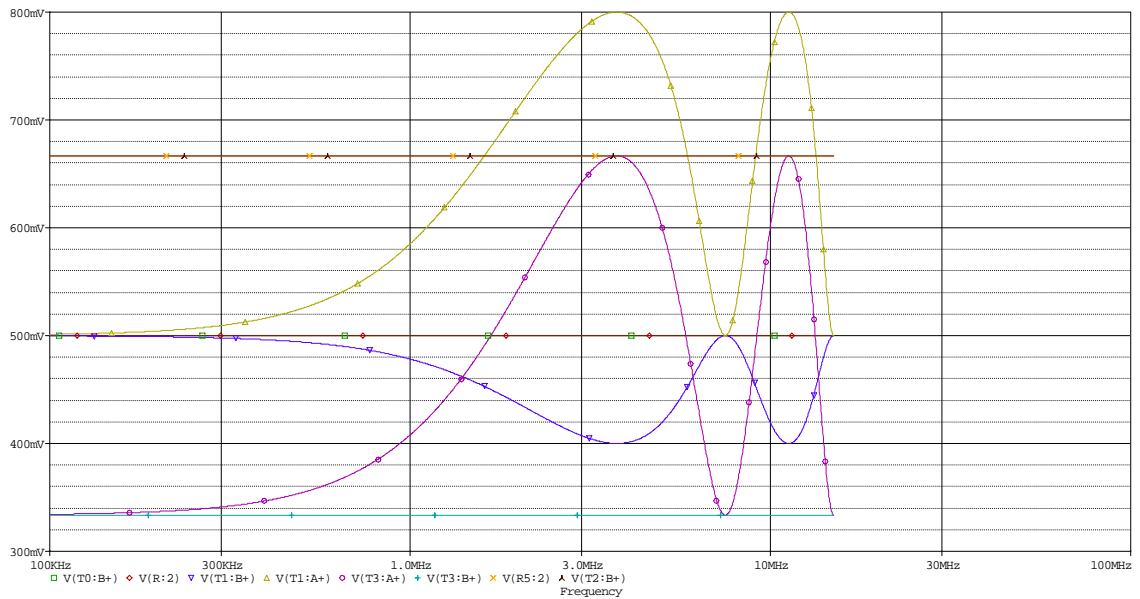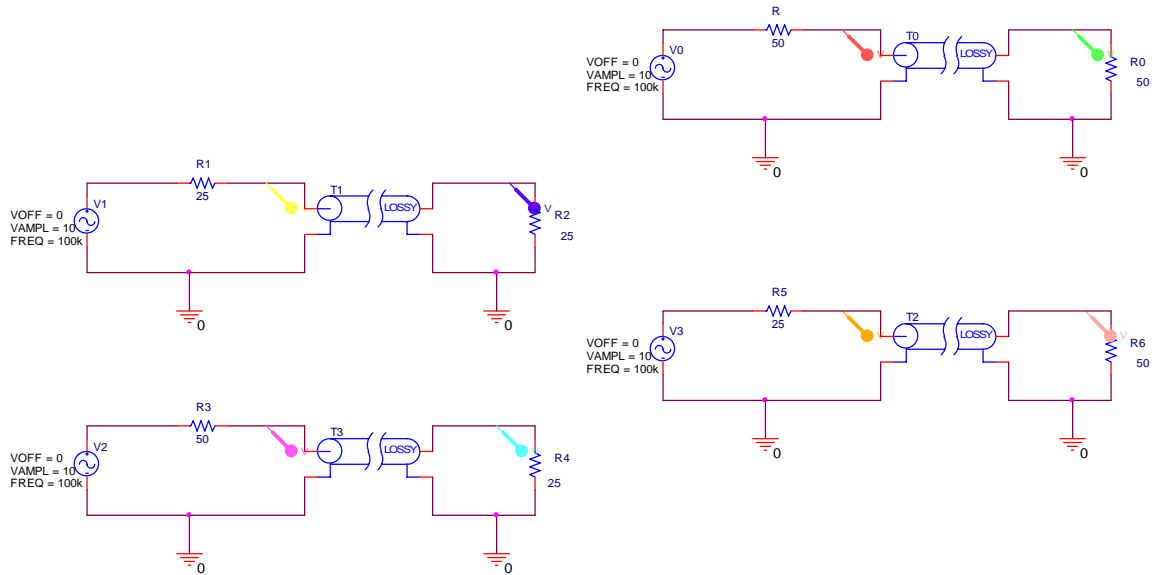
For the frequency scan, using the lossless transmission line models.
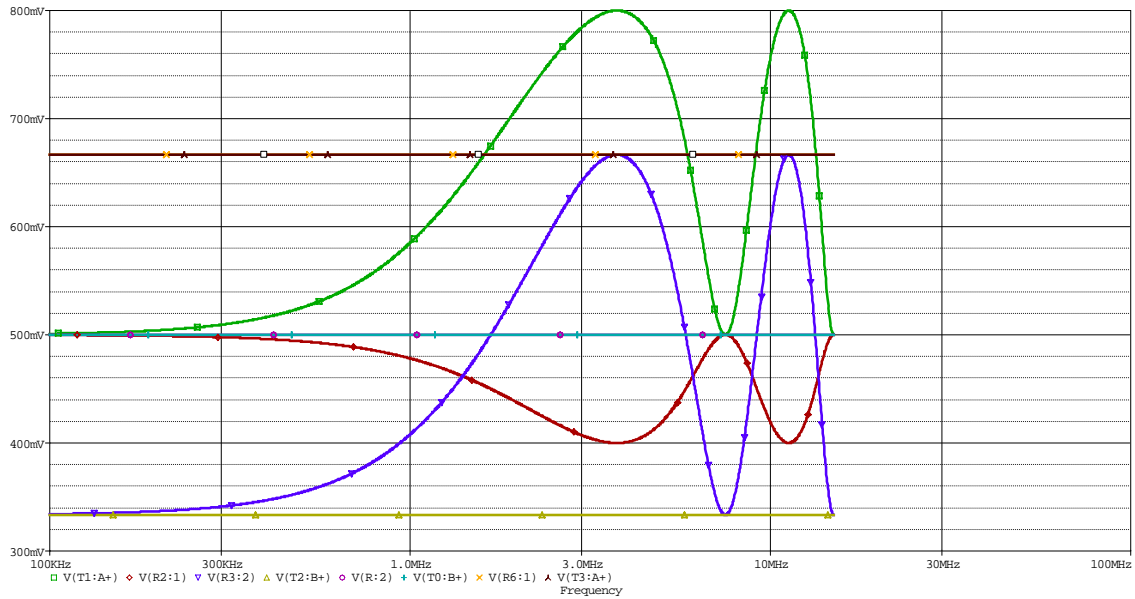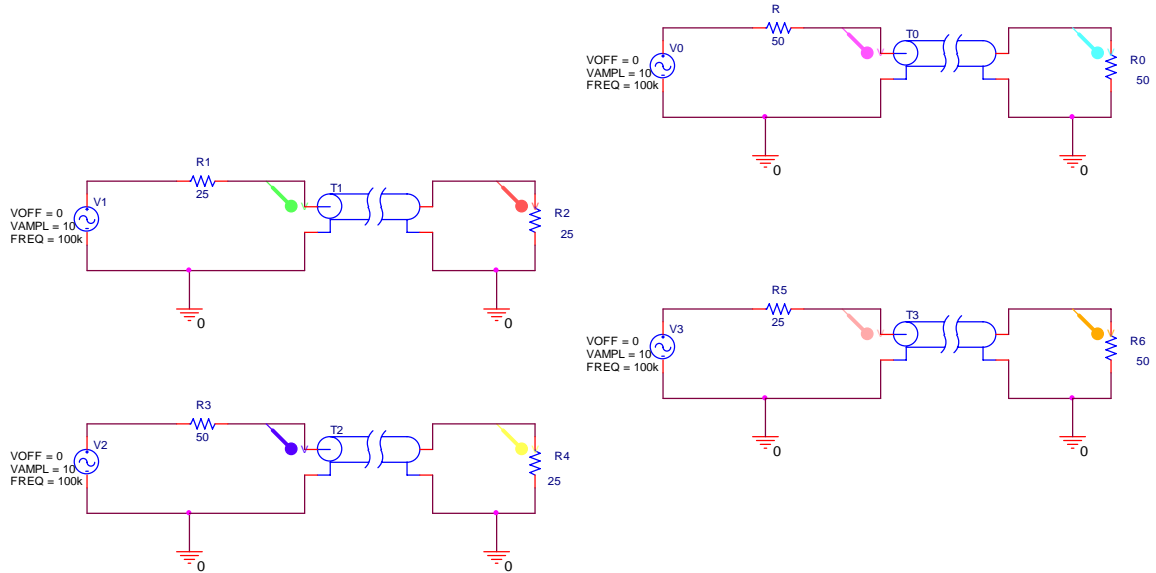
Using the lossy line model, produces the same result.

For the next question, note that the amplitudes of all 8 voltages take on only 3 values at the lowest frequency: 3.33V, 5V and 6.67V

3. Input Impedance and Power

First, repeat the analysis for each of the four cases above in the time domain at the specified frequency of *100kHz*. The simulation profile is shown below. Note that the data is saved only after a specified time to avoid having to deal with start up transients. This is probably not a problem here, but it is a good idea to get used to this approach.
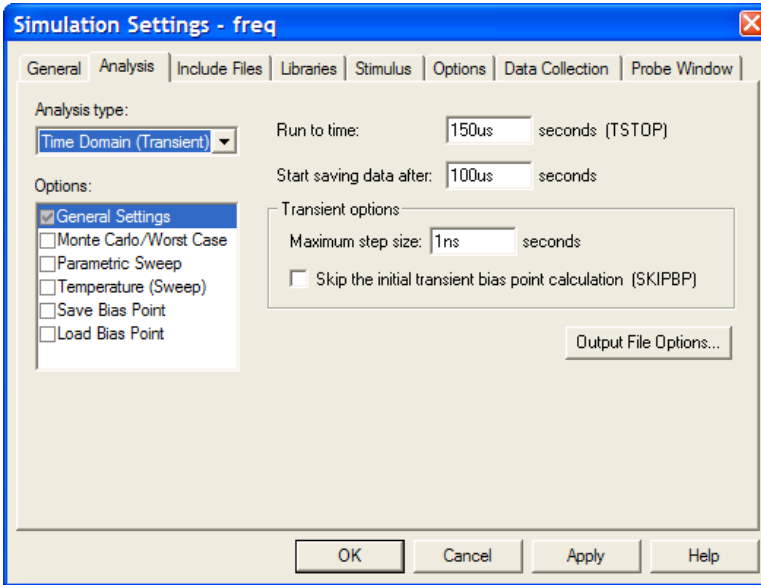
Plotting all signals at once might look too confusing, but note that there are only three amplitudes shown: 3.33V, 5V and 6.67V, which are exactly the values shown in the previous plot at *100kHz* (the lowest frequency)
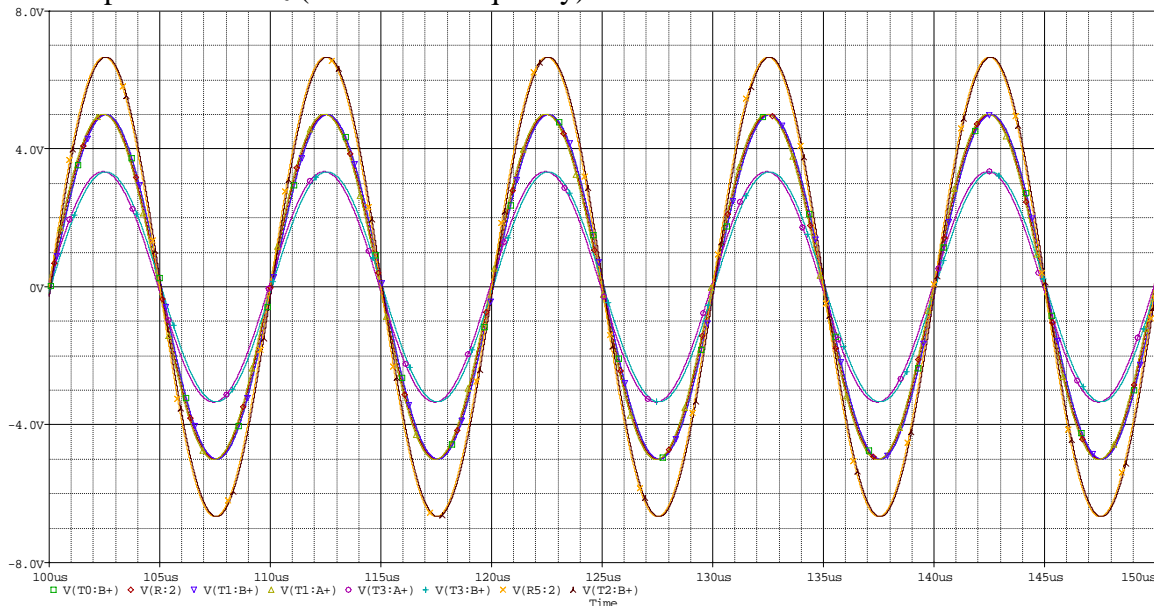
Check to be sure that your plots obtained at *100kHz* agree with the AC Sweep scan results. *The agreement is noted above.*

Calculate (obtain a mathematical expression for) the input impedance Z_{in} for the transmission lines for each of the four cases. First obtain the expression for any frequency between *100kHz* and *15MHz* and then specialize the expression for *100kHz*. For which of the four cases is the input impedance frequency dependent? Which are frequency independent? Explain your answers.

Using the formula from the 'Comments on Lossless Transmission Lines,'

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d}$$ *we can specialize for each of the four cases.*

Case 1: $Z_{in} = Z_o \dfrac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} = Z_o \dfrac{Z_o + jZ_o \tan \beta d}{Z_o + jZ_o \tan \beta d} = Z_o$

Case 2: $Z_{in} = Z_o \dfrac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} = Z_o \dfrac{Z_o + jZ_o \tan \beta d}{Z_o + jZ_o \tan \beta d} = Z_o$

Case 3:

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} = Z_o \frac{0.5Z_o + jZ_o \tan \beta d}{Z_o + j0.5Z_o \tan \beta d} = Z_o \frac{0.5 + j \tan(2\pi f (66.67 ns))}{1 + j0.5 \tan(2\pi f (66.67 ns))}$$

$$Z_{in} = Z_o \frac{0.5 + j \tan(2\pi 10^5 (66.67)10^{-9})}{1 + j0.5 \tan(2\pi 10^5 (66.67)10^{-9})} = Z_o \frac{0.5 + j0.0419}{1 + j0.5(0.0419)} \approx 0.5 Z_o$$

Case 4:

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} = Z_o \frac{0.5Z_o + jZ_o \tan \beta d}{Z_o + j0.5Z_o \tan \beta d} = Z_o \frac{0.5 + j \tan(2\pi f (66.67 ns))}{1 + j0.5 \tan(2\pi f (66.67 ns))}$$

$$Z_{in} = Z_o \frac{0.5 + j \tan(2\pi 10^5 (66.67)10^{-9})}{1 + j0.5 \tan(2\pi 10^5 (66.67)10^{-9})} = Z_o \frac{0.5 + j0.0419}{1 + j0.5(0.0419)} \approx 0.5 Z_o$$

Cases 1 and 2 are constants, so frequency independent, because the load matches the line. The other two cases are frequency dependent because the load does not match the line.

Determine the power delivered to the load in each case.

First, determine the input voltage, $V_{in} = V_s \dfrac{Z_{in}}{Z_s + Z_{in}}$.

Case 1: $V_{in} = V_s \dfrac{Z_0}{Z_o + Z_o} = \dfrac{V_s}{2}$

Case 2: $V_{in} = V_s \dfrac{Z_0}{0.5Z_o + Z_o} = \dfrac{2V_s}{3}$

Case 3: $V_{in} = V_s \dfrac{0.5Z_o}{Z_o + 0.5Z_o} = \dfrac{V_s}{3}$

Case 4: $V_{in} = V_s \dfrac{0.5Z_o}{0.5Z_o + 0.5Z_o} = \dfrac{V_s}{2}$

Now, the average input power is given by $P_{ave} = \text{Re}\left(\dfrac{V_{in}V_{in}^{*}}{2Z_{in}^{*}}\right) = \dfrac{V_{in}^{2}}{2Z_{in}}$ *for all four cases*

because both the input voltage and impedance are real.

Case 1: $P_{ave} = \dfrac{V_{in}^{2}}{2Z_{in}} = \dfrac{V_s^{2}}{8Z_o} = \dfrac{100}{400} = 0.25W$

Case 2: $P_{ave} = \dfrac{V_{in}^{2}}{2Z_{in}} = \dfrac{4V_s^{2}}{18Z_o} = \dfrac{400}{900} = 0.44W$

Case 3: $P_{ave} = \dfrac{V_{in}^{2}}{2Z_{in}} = \dfrac{V_s^{2}}{9Z_o} = \dfrac{100}{450} = 0.22W$

Case 4: $P_{ave} = \dfrac{V_{in}^{2}}{2Z_{in}} = \dfrac{V_s^{2}}{8Z_o} = \dfrac{100}{200} = 0.5W$

We can also do the power calc at the load end, which we will do here to check our

answers. $P_{ave} = \text{Re}\left(\dfrac{V_L V_L^{*}}{2Z_L^{*}}\right) = \dfrac{V_L^{2}}{2Z_L}$

Case 1: $P_{ave} = \dfrac{V_L^{2}}{2Z_L} = \dfrac{25}{100} = 0.25W$

Case 2: $P_{ave} = \dfrac{V_L^{2}}{2Z_L} = \dfrac{44.5}{100} = 0.44W$

Case 3: $P_{ave} = \dfrac{V_L^{2}}{2Z_L} = \dfrac{11}{50} = 0.22W$

Case 4: $P_{ave} = \dfrac{V_L^{2}}{2Z_L} = \dfrac{25}{50} = 0.5W$

Note that the cases we have addressed include matched load and source, matched load and unmatched source, unmatched source and matched load, and unmatched load and source so we can see the importance of matching. *However, in the last case, the source and the input are matched again, because the input impedance has changed. The maximum power transfer for each source impedance is achieved when the load is equal to the source. The transmission line is really too short in this case to have a significant effect.*

4. Experiment and Analysis

On Wednesday, you will consider essentially the same problems as in questions 2 and 3, but for a different configuration. You will find the input impedance and power delivered to the load for the same four cases, but using a series combination of one of the reels of coax and a lumped line box as your total transmission line. You will connect to the reel first, then the lumped line box and then the load. You will not know ahead of time exactly what reel or box you will work with, so you will not know the length of the reel or the time delay for either the reel or the box. However, you should set everything up so that you only need to plug in the numbers for the line parameters once you determine them. If you have set up your analysis for the previous problems in a sufficiently general form, you will be ready, but you should probably guess the information you will use (taken from your previous work in the lab) and see how your approach works.

Repeat the analysis you did in problems 2 and 3. In the parts that require a specific frequency, do your analysis at both $f = 100kHz$ and $f = 1MHz$. The results at these two frequencies can be very different, but that depends on the specific hardware you have.

For the experiment, each group is set up an experiment with their cables. The first measurement you each need to make is the transit time to go from one end of the coax reel to the other and then repeat this with the lumped box line. You can do this any way you wish, but you should do it both with a sinusoidal and a pulsed source (the latter generally gives better results, but it is always good to use more than one method to make a measurement). From the transit time and your knowledge of the propagation velocity, determine the actual length of your coax reel. As an additional confirmation of the length, use the LCZ Bridge to measure the capacitance of the cable with no load (open circuit). Using the known capacitance per unit length of the cable, determine its length.

In your report, you should include the following:
   a. The number on your cable spool and the letter of your lumped line box.
   b. The measured delay time and resulting length of the coax cable and the delay time for the box. *A typical total is 900ns for the combination.*
   c. The measured capacitance and resulting length of the coax cable.
One of the cables measured 84m both using both methods. Such good agreement should not be expected.

Once you have this information, you will be able to re-do the analysis you set up above for the exact length of the cable reel and the combined delay time of the reel and lumped box.

We will use the formulas developed above, but for longer lines.
$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d}$$ *we can specialize for each of the four cases.*

Case 1: $Z_{in} = Z_o \dfrac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} = Z_o \dfrac{Z_o + jZ_o \tan \beta d}{Z_o + jZ_o \tan \beta d} = Z_o$

Case 2: $Z_{in} = Z_o \dfrac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} = Z_o \dfrac{Z_o + jZ_o \tan \beta d}{Z_o + jZ_o \tan \beta d} = Z_o$

Case 3: $Z_{in} = Z_o \dfrac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} = Z_o \dfrac{0.5Z_o + jZ_o \tan \beta d}{Z_o + j0.5Z_o \tan \beta d} = Z_o \dfrac{0.5 + j \tan \left(2\pi f (900ns) \right)}{1 + j0.5 \tan \left(2\pi f (900ns) \right)}$

At 100kHz

$$Z_{in} = Z_o \dfrac{0.5 + j \tan \left(2\pi 10^5 (900)10^{-9} \right)}{1 + j0.5 \tan \left(2\pi 10^5 (900)10^{-9} \right)} = Z_o \dfrac{0.5 + j \tan(.18\pi)}{1 + j0.5 \tan(.18\pi)} \approx 25.6 + 6.77j$$

At 1MHz

$$Z_{in} = Z_o \dfrac{0.5 + j \tan \left(2\pi 10^6 (900)10^{-9} \right)}{1 + j0.5 \tan \left(2\pi 10^6 (900)10^{-9} \right)} = Z_o \dfrac{0.5 + j \tan(1.8\pi)}{1 + j0.5 \tan(1.8\pi)} \approx 86.5904 - 28.7385j$$

Case 4: $Z_{in} = Z_o \dfrac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} = Z_o \dfrac{0.5Z_o + jZ_o \tan \beta d}{Z_o + j0.5Z_o \tan \beta d} = Z_o \dfrac{0.5 + j \tan \left(2\pi f (900ns) \right)}{1 + j0.5 \tan \left(2\pi f (900ns) \right)}$

At 100kHz

$$Z_{in} = Z_o \dfrac{0.5 + j \tan \left(2\pi 10^5 (900)10^{-9} \right)}{1 + j0.5 \tan \left(2\pi 10^5 (900)10^{-9} \right)} = Z_o \dfrac{0.5 + j \tan(.18\pi)}{1 + j0.5 \tan(.18\pi)} \approx 25.6 + 6.77j$$

At 1MHz

$$Z_{in} = Z_o \dfrac{0.5 + j \tan \left(2\pi 10^6 (900)10^{-9} \right)}{1 + j0.5 \tan \left(2\pi 10^6 (900)10^{-9} \right)} = Z_o \dfrac{0.5 + j \tan(1.8\pi)}{1 + j0.5 \tan(1.8\pi)} \approx 86.5904 - 28.7385j$$

Cases 1 and 2 are constants, so frequency independent, because the load matches the line. The other two cases are frequency dependent because the load does not match the line.

Determine the power delivered to the load in each case.

First, determine the input voltage, $V_{in} = V_s \dfrac{Z_{in}}{Z_s + Z_{in}}$.

Case 1: $V_{in} = V_s \dfrac{Z_0}{Z_o + Z_o} = \dfrac{V_s}{2}$

Case 2: $V_{in} = V_s \dfrac{Z_0}{0.5Z_o + Z_o} = \dfrac{2V_s}{3}$

Case 3: $V_{in} = V_s \dfrac{25.6 + j6.77}{50 + 25.6 + j6.77} = \left(4.2904 + 1.5080j\right)$ *at 100kHz*

$V_{in} = V_s \dfrac{84.6 - j28.7}{50 + 84.6 - j28.7} = \left(4.4850 - 1.5851j\right)$ *at 1MHz*

Case 4: $V_{in} = V_s \dfrac{25.6 + j6.77}{25 + 25.6 + j6.77} = \left(6.1587 + 1.4607j\right)$ *at 100kHz*

$V_{in} = V_s \dfrac{84.6 - j28.7}{25 + 84.6 - j28.7} = \left(6.3560 - 1.4931j\right)$ *at 1MHz*

Now, the average input power is given by $P_{ave} = \mathrm{Re}\left(\dfrac{V_{in}V_{in}^{*}}{2Z_{in}^{*}}\right) = \dfrac{V_{in}^{2}}{2Z_{in}}$ *for the first two*

cases because both the input voltage and impedance are real. The last two are nearly real, so we may be able to approximate the answer, but we will use the full form this time.

Case 1: $P_{ave} = \mathrm{Re}\left(\dfrac{V_{in}V_{in}^{*}}{2Z_{in}^{*}}\right) = 0.22W$

Case 2: $P_{ave} = \mathrm{Re}\left(\dfrac{V_{in}V_{in}^{*}}{2Z_{in}^{*}}\right) = 0.24W$

Case 3: $P_{ave} = \mathrm{Re}\left(\dfrac{V_{in}V_{in}^{*}}{2Z_{in}^{*}}\right) = 0.39W$

Case 4: $P_{ave} = \mathrm{Re}\left(\dfrac{V_{in}V_{in}^{*}}{2Z_{in}^{*}}\right) = 0.42W$

The Matlab File used to find the expressions above

```
% HW2 Calcs for Fall 2009
% Input Voltage
% Input Impedance
% Input Power


% Char Imp
zo=50;


% Source Voltage
Vs=10;


% Line Length (delay)
td=900e-9;
f1=1e5;D1=tan(2*pi*f1*td);
f2=1e6;D2=tan(2*pi*f2*td);


% Load Impedance
zl=25;


% Input Impedance
zin1=zo*(zl+j*zo*D1)/(zo+j*zl*D1);
zin2=zo*(zl+j*zo*D2)/(zo+j*zl*D2);


% Input Voltage for zs1=50 and zs2=25
zs1=50;zs2=25;
Vin11=Vs*zin1/(zs1+zin1);
Vin12=Vs*zin2/(zs1+zin2);
Vin21=Vs*zin1/(zs2+zin1);
Vin22=Vs*zin2/(zs2+zin2);


% Input Power
Pin11=real(Vin11*conj(Vin11)/(2*zin1));
Pin12=real(Vin12*conj(Vin12)/(2*zin1));
Pin21=real(Vin21*conj(Vin21)/(2*zin2));
Pin22=real(Vin22*conj(Vin22)/(2*zin2));
```

Measure the input and output voltages for the combined reel plus lumped box at the two frequencies given above. Compare your results to the analysis you have done. If you find the results are quite different, provide an explanation for the differences.
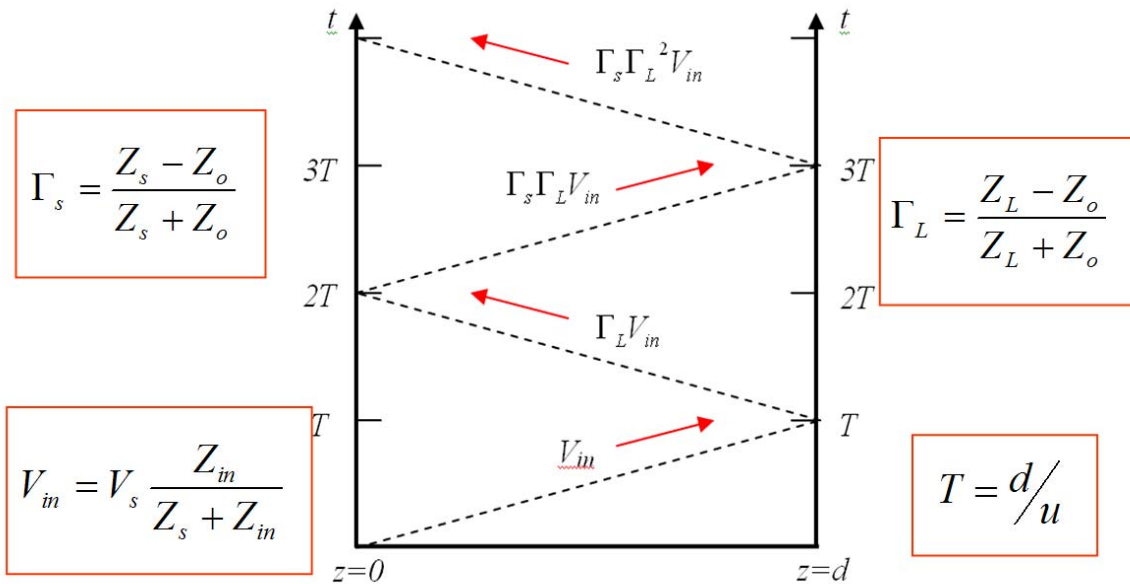
5. Pulsed Excitation

Now we wish to replace the sinusoidal source with a pulsed source. For the PSpice simulation, the four cases will look like
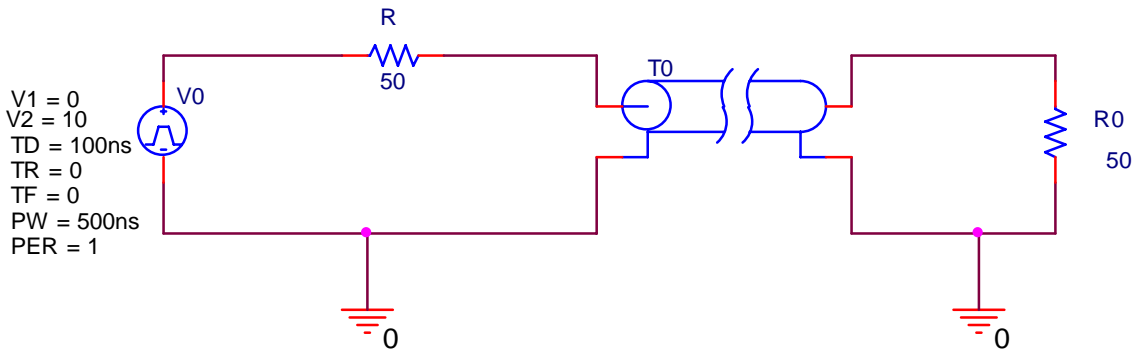
with the appropriate resistance chosen for each case. Note that there is a small delay added so that the pulses can be fully seen in the plots. Do all four cases again, using the information from the coax reel and lumped box. Here you will only need to find the voltages at the input and output ends of the cable.

Set up the experiment and make the actual measurements at the input and output. You will need to use the amplitude, frequency, square wave, burst, duty cycle and offset features of the function generator to reproduce this in hardware.

Finally, do the lattice or bounce diagram analysis of the four cases.

$$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o}$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$V_{in} = V_s \frac{Z_{in}}{Z_s + Z_{in}}$$

$$T = \frac{d}{u}$$

The time delay T is the measured number.

The reflection coefficients are:

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}, \; \Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o}, \; \text{and the input voltage is } V_{in} = V_s \frac{Z_{in}}{Z_s + Z_{in}} = V_s \frac{Z_o}{Z_s + Z_o}$$

Case 1: $\Gamma_L = \dfrac{Z_L - Z_o}{Z_L + Z_o} = 0, \; \Gamma_s = \dfrac{Z_s - Z_o}{Z_s + Z_o} = 0, \; V_{in} = V_s \dfrac{Z_o}{Z_s + Z_o} = 10\dfrac{1}{2} V = 5V$

Case 2: $\Gamma_L = \dfrac{Z_L - Z_o}{Z_L + Z_o} = 0, \; \Gamma_s = \dfrac{0.5Z_o - Z_o}{0.5Z_0 + Z_o} = -\dfrac{1}{3}, \; V_{in} = V_s \dfrac{Z_o}{Z_s + Z_o} = 10\dfrac{2}{3} V = 6.67V$

Case 3: $\Gamma_L = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{0.5Z_o - Z_o}{0.5Z_o + Z_o} = -\dfrac{1}{3}, \; \Gamma_s = \dfrac{Z_s - Z_o}{Z_s + Z_o} = 0,$
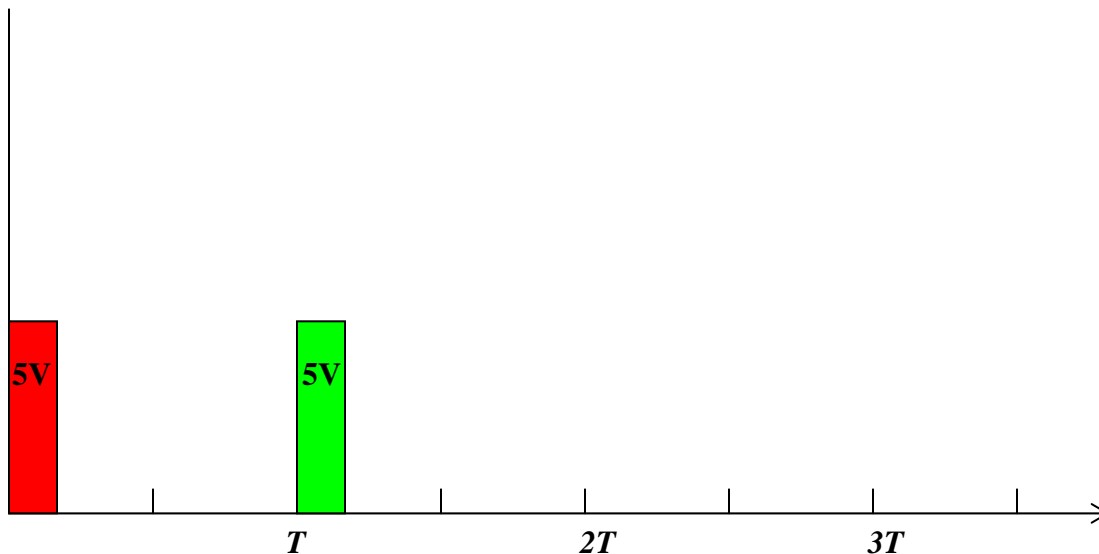
$$V_{in} = V_s \frac{Z_o}{Z_s + Z_o} = 10\frac{1}{2} V = 5V$$

Case 4: $\Gamma_L = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{0.5Z_o - Z_o}{0.5Z_o + Z_o} = -\dfrac{1}{3}, \; \Gamma_s = \dfrac{0.5Z_o - Z_o}{0.5Z_0 + Z_o} = -\dfrac{1}{3},$

$$V_{in} = V_s \frac{Z_o}{Z_s + Z_o} = 10\frac{2}{3} V = 6.67V$$

Time delay, t=900ns (sample case). We don't have to know T to do the plotting since we can leave the horizontal scale as generic.
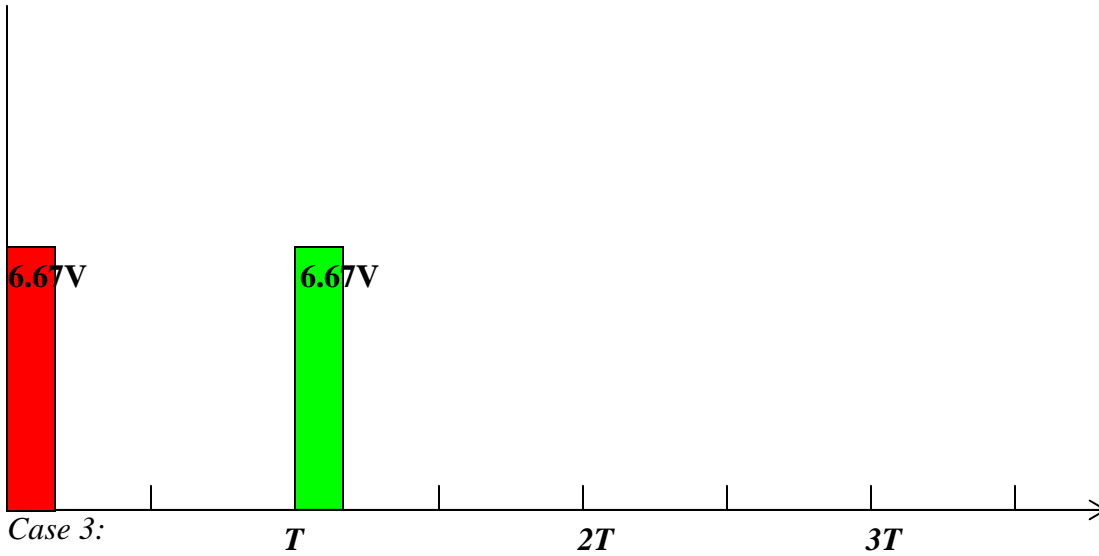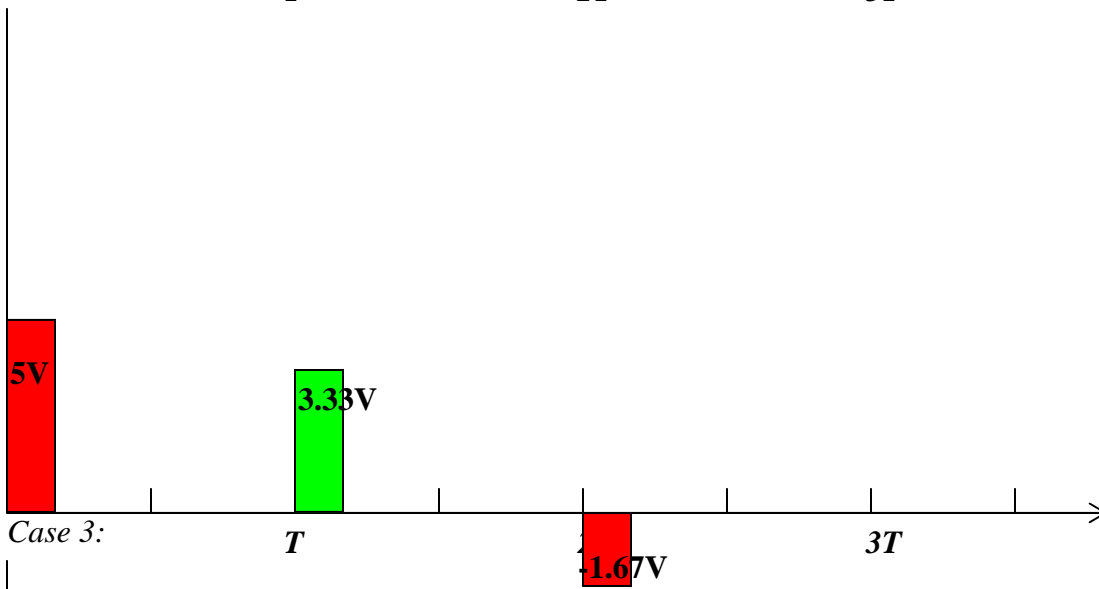*Case 1:*

In these plots, the red pulses are at the source end and the green pulses are at the load end. The only case that requires any real analysis is #4. First pulse to the right (6.67V), first pulse reflected from the load (-2.22V), first pulse reflected from the source (.74V), second pulse reflected from the load (-.25V).
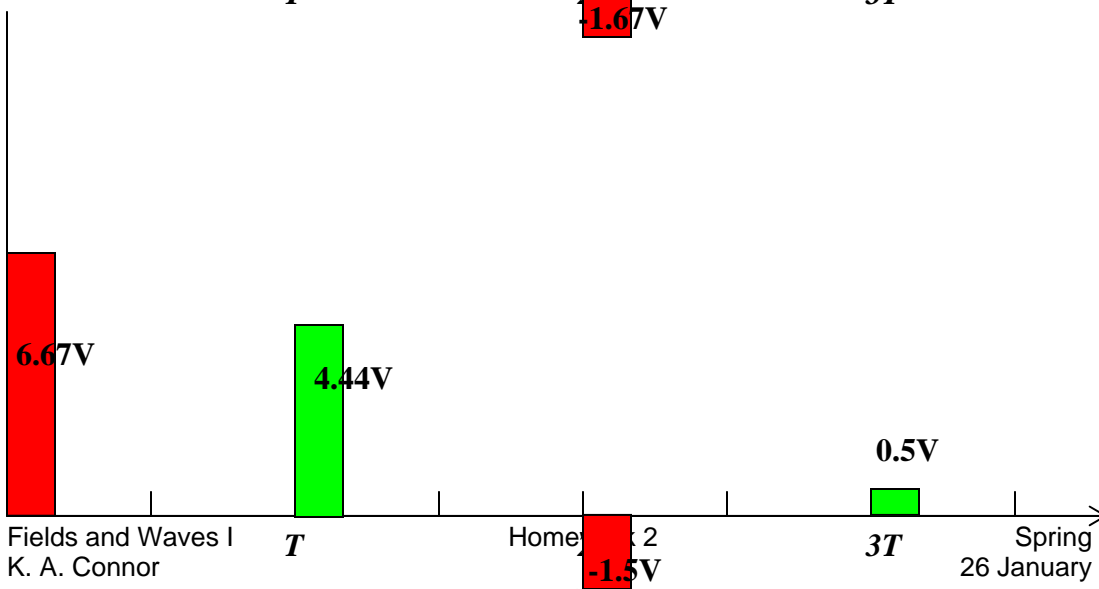
Case 2:

6.67V   6.67V

T   *2T*   *3T*

Case 3:

5V

3.33V

-1.67V

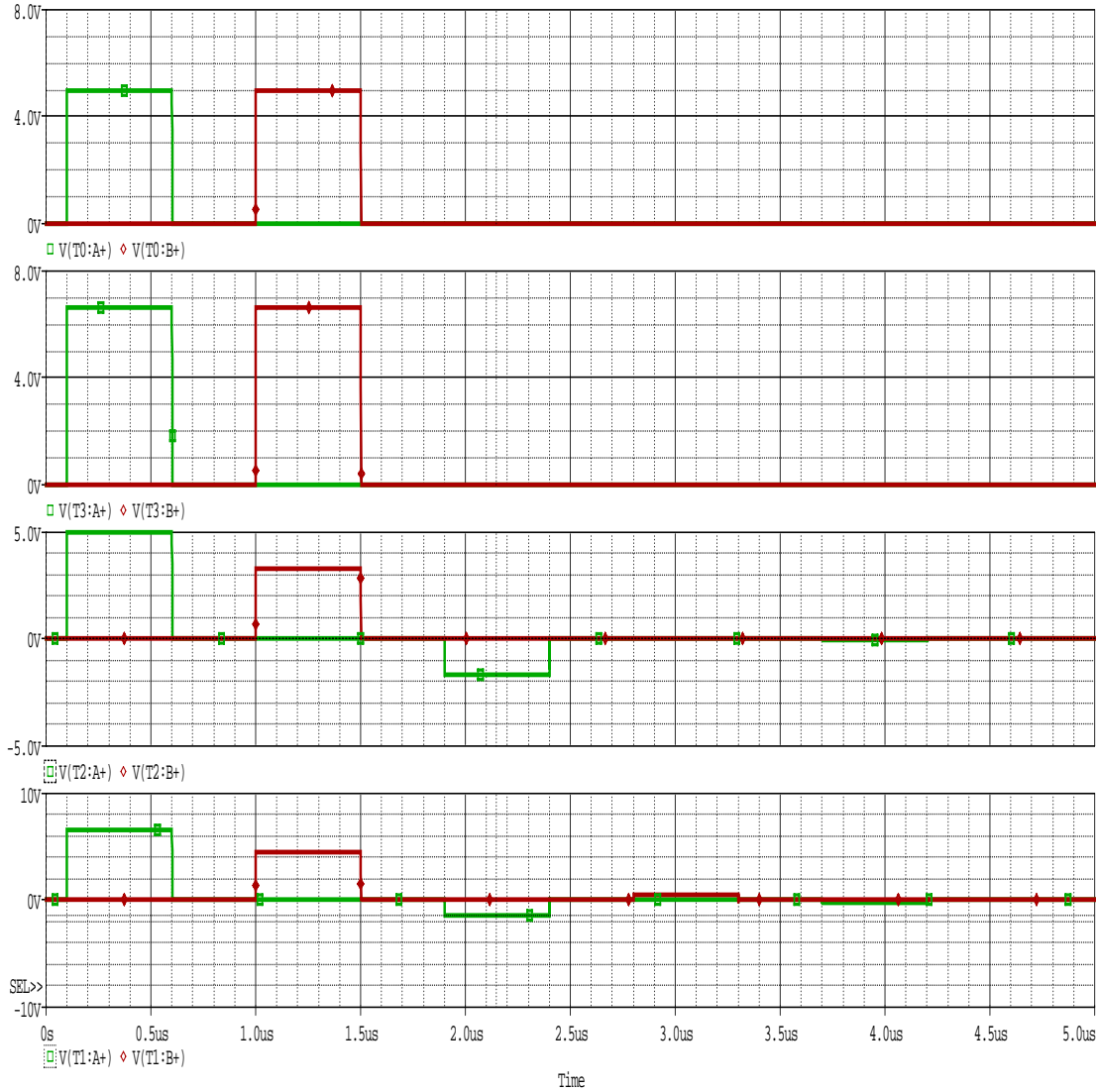*T*   *3T*

Case 3:

6.67V

4.44V

0.5V

-1.5V

T   *3T*

Using PSpice, we obtain the same results.(The colors are backwards from above.)

Compare your results from the three types of analysis for the four cases.

Note that you will be generating several plots of data from PSpice and from experimental measurements. You should always annotate your plots so that other people (the grader in this case) will know what information is contained and why your answers make sense. Use the ideas we discussed in lecture for this purpose.

Measurements to be added later.