



1. Coaxial Cable Electric Field

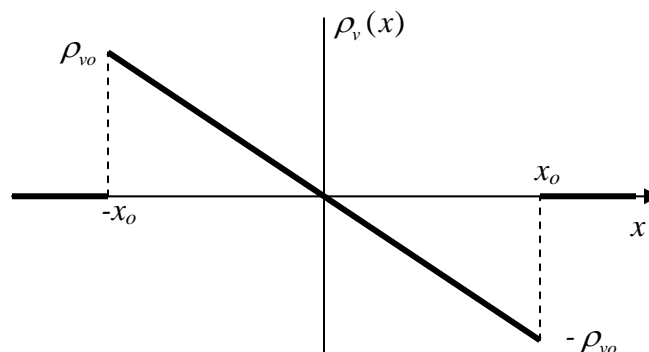
- a. Using the solution to problem 2 of HW3, write the electric field $\vec{E}(r)$ of the CATV coaxial cable in full vector form (with unit vector) making sure you have provided an expression for all values of r . Also, write down the values of the inner and outer radii of the cable and its dielectric constant ϵ .
- b. Assuming that outer conductor is grounded, determine the voltage (electric potential) $V(r)$ for all values of r . Plot your solution as a function of radius.
- c. Show that your solution satisfies Laplace's Equation.

2. The Earth-Ionosphere Capacitor

- a. From the solution to problem 3 of HW2, write down the expression for the electric field vector $\vec{E}(r)$ as a function of radius in the region between the surface of the earth and the ionosphere. Be sure it is in full vector form.
- b. Assuming the surface of the earth is grounded, determine the voltage (electric potential) $V(r)$ for all values of r . Plot your solution as a function of radius.
- c. Show that your solution satisfies Laplace's Equation.

3. The Electric Field and Potential of a Charge Distribution

- a. The volume charge distribution in the planar structure of problem 4, HW3 is reproduced below. From the solution to this problem, write down the electric field as a function of x $\vec{E}(x)$ in full vector form.



- b. Assuming that the voltage is equal to zero at $x=0$, determine the voltage as a function of x $V(x)$ for all values of x . Plot your solution. *Hint: to use Matlab for the plot, you will have to pick numerical values for the unspecified constants.*
- c. Show that your solution for $V(x)$ satisfies Poisson's Equation.



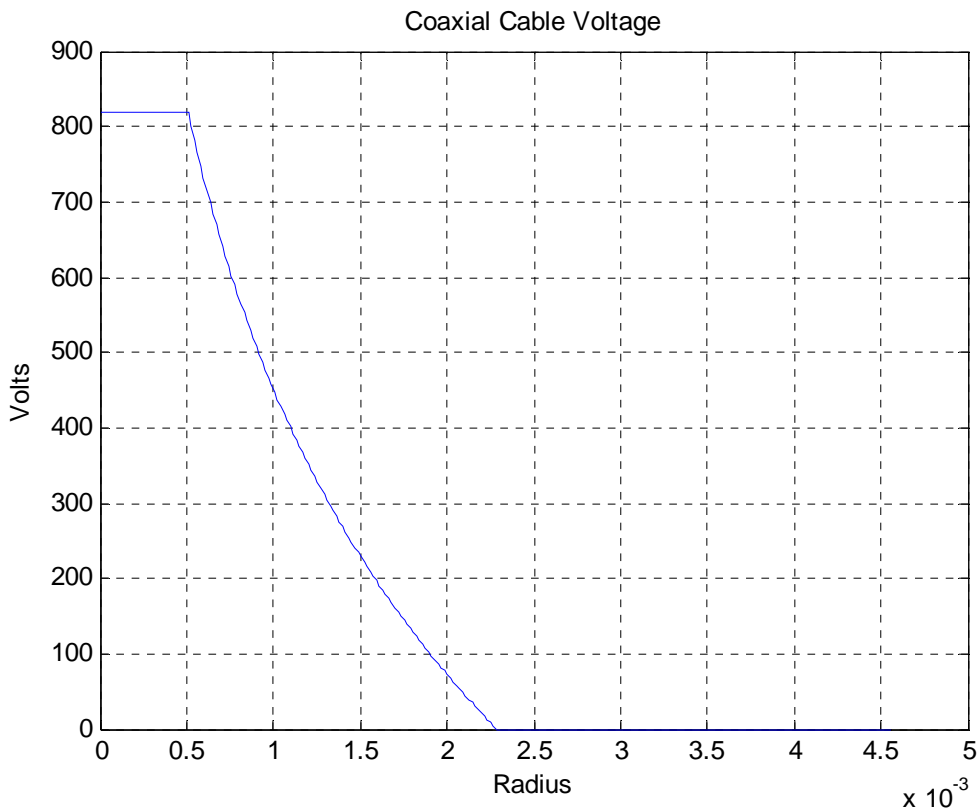
Most of the solution for problem 1 is found on the handwritten pages that follow. The plotting can be done by Matlab.

% Problem 1

```
a=.00051;b=.002285;  
epsr=1.38;epso=8.95e-12;  
Vo=546*log(b/a);%Voltage on Inner Conductor  
r0=[a:(b-a)/100:b];V0=546.*log(b./r0);%Space between conductors  
r1=[0:(b-a)/100:a];for i=1:29;V1(i)=Vo;end;%Inside Inner Conductor  
r2=[b:(b-a)/100:2*b];for j=1:129;V2(j)=0;end%Outside Conductor and Outside  
r=[r1 r0 r2];V=[V1 V0 V2];
```

%Plotting

```
plot(r,V);grid;title('Coaxial Cable Voltage');xlabel('Radius');ylabel('Volts');
```



Note that the voltage is a constant for both conducting regions and varies as $\ln(b/r)$ in between the conductors (in the insulator region).

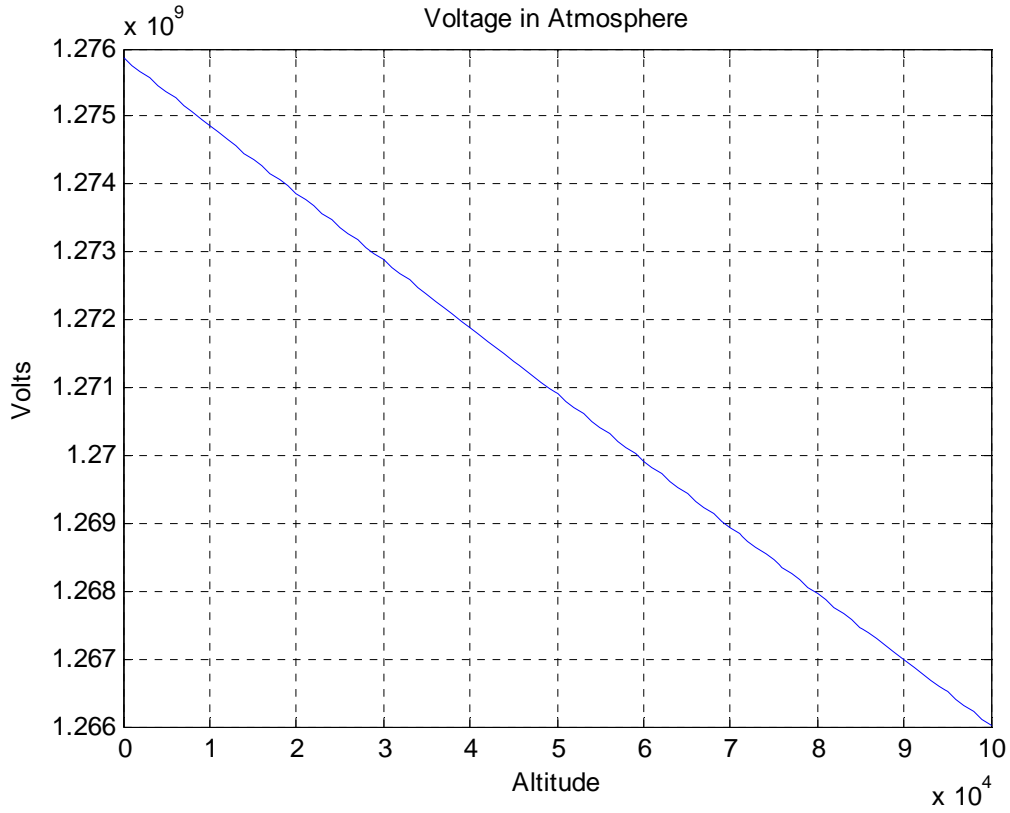


% Problem 2

```
re=6.38e6;ri=re+1e5;rs=[re:1e3:ri];rp=[0:1e3:1e5];
```

```
Vei=4.07e15.*((rs).^(-1)+(re).^(-1));
```

```
plot(rp,Vei);grid;title('Voltage in Atmosphere');xlabel('Altitude');ylabel('Volts');
```





% Problem 3

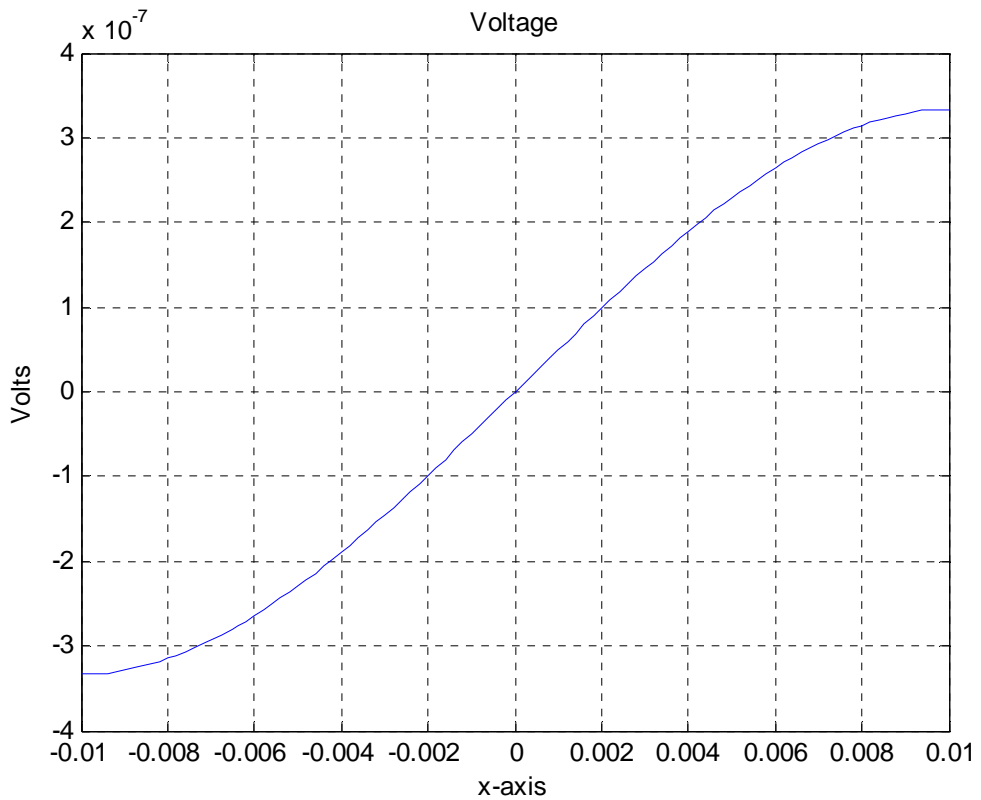
% Choose a value for xo. Also let the charge density divided by eps0 = 1

xo=.01;

x=[-xo:xo/50:xo];

Vx=0.5.*((xo.^2).*x-(x.^3)./3);

plot(x,Vx);grid;title('Voltage');xlabel('x-axis');ylabel('Volts');



For this plot, note the values used for the constants.

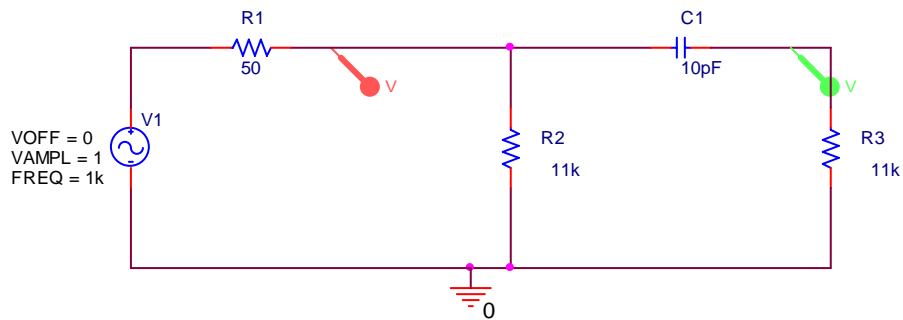


Homework 4

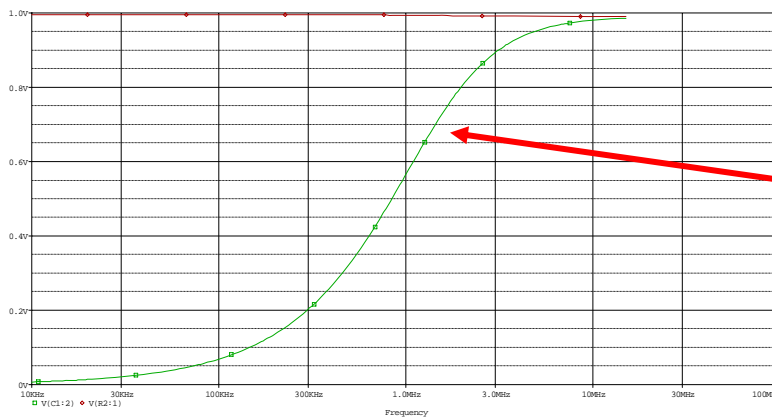
Part Two

Due 21 October at 6:00 pm

4. **Capacitance Measurements** Repeat the experiment done the first day of class where you made a small capacitor from two twisted wires on a protoboard. (Use a length of 20-30cm.) Calculate the capacitance of your configuration (you will have to carefully measure the dimensions of the wires and their separation distance and record the numbers in your solution) and compare with the measurement using the impedance bridge. Remember to take into account the capacitance of the cable you use to connect to your wires. Set up the same circuit you used previously and scan the frequency from 10kHz to 15MHz. Take enough data points to show the general form of the frequency dependence of the input and output voltages. Before coming to class, setup the following simulation using PSpice and show it to a TA or instructor at the beginning of your assigned section. Then modify the simulation based on your measured capacitance and run it again. Compare your experimental results with your simulation. Ideally, plot your measured data on the PSpice plot.



See lecture 12 slides, examples 2 and 3 for the general method. Each line will be slightly different, depending on its length and the type of wire used. The capacitance obtained should be similar. Typical frequency response from PSpice is shown below.

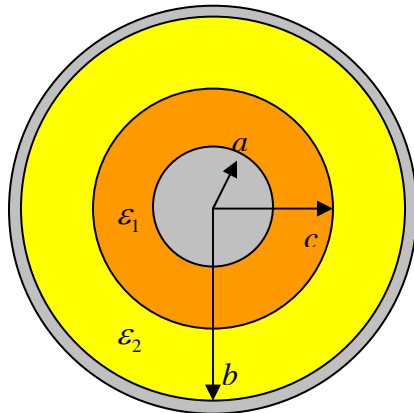


Note that the corner freq. for this circuit is $f = (2\pi RC)^{-1} = 1.45\text{MHz}$ and one would expect.



5. **Capacitor Design Choices** Assume that we have a limited amount of dielectric material and wish to investigate some choices for how we could use it in a simple capacitor configuration.

- a. Begin by determining the capacitance of the spherical structure below. Note that there are two different dielectric regions. Assume that the inner conductor voltage is V_o and the outer conductor is grounded. You can use either the charge or energy method.



- b. Now assume that the inner region is filled with an insulating material with $\epsilon_1 = 6\epsilon_o$ and the outer region is air $\epsilon_2 = \epsilon_o$; $a=1cm$ and $b=10cm$. Also assume the voltage on the inner conductor is $1000V$ and the outer conductor is grounded. Determine the capacitance and energy stored in this configuration. Also, determine the maximum value for the electric field magnitude. Assume that half of the volume between the two conductors is filled with insulator.
- c. Now assume that the outer region is filled with the insulating material so that $\epsilon_1 = \epsilon_o$ and $\epsilon_2 = 6\epsilon_o$; $a=1cm$ and $b=10cm$. Also assume the voltage on the inner conductor is $1000V$ and the outer conductor is grounded. Determine the capacitance and energy stored in this configuration. Also, determine the maximum value for the electric field magnitude. Assume that half of the volume between the two conductors is filled with insulator.
- d. Discuss the tradeoffs between the three possible configurations, including the choice to not use the insulating material at all.



%Spherical Cap Problem
%HW 4 Fall 2009
% K. A. Connor

%Parameters

```
a=.01;b=.1;
c=((a^3+b^3)/2)^(1/3);%Half the volume is filled with insulator
r1=[a:(c-a)/100:c];r2=[c:(c-a)/100:b];r=[r1 r2];
epso=8.85e-12;Vo=1000;%Voltage on Inner Conductor. Outer is grounded.
epsr=6;
```

%Inner half filled with insulator

```
C1=4*pi*epso/(1/(epsr*a)-1/(epsr*c)+1/c-1/b);Q1=C1*Vo;We1=0.5*C1*Vo^2;
%E Field
E11=Q1./(4.*pi.*epsr.*epso.*r1.^2);E12=Q1./(4.*pi.*epso.*r2.^2);
E1=[E11 E12];E1max=max(E1);
plot(r,E1);grid;xlabel('Radius');ylabel('E Field');title('Insulator in Inner Region');
```

%Outer half filled with insulator

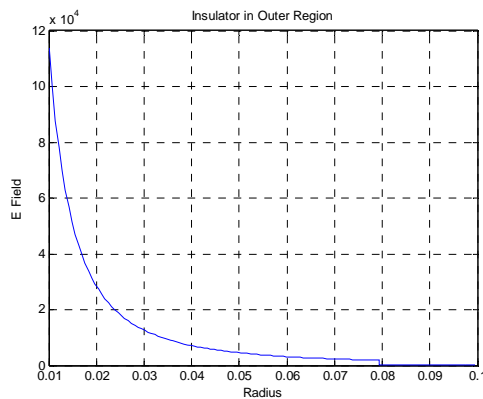
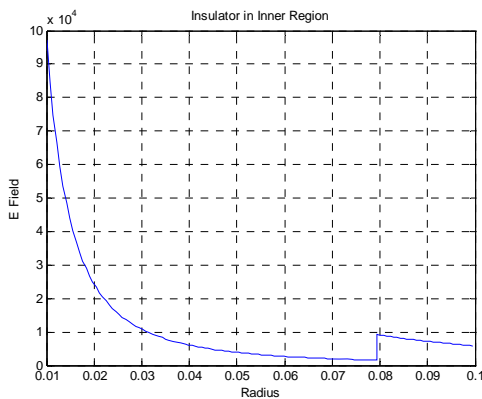
```
C2=4*pi*epso/(1/(epsr*c)-1/(epsr*b)+1/a-1/c);Q2=C2*Vo;We2=0.5*C2*Vo^2;
%E Field
E21=Q2./(4.*pi.*epso.*r1.^2);E22=Q2./(4.*pi.*epsr.*epso.*r2.^2);
E2=[E21 E22];E2max=max(E2);
figure;plot(r,E2);grid;xlabel('Radius');ylabel('E Field');title('Insulator in Outer Region');
```

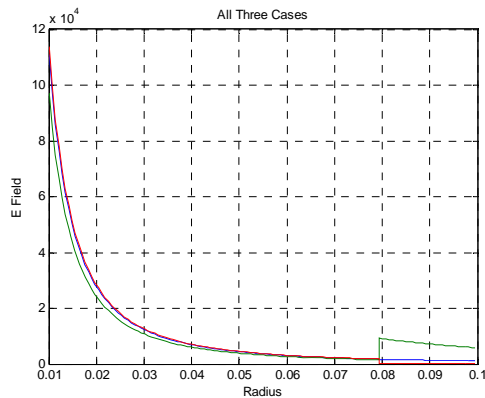
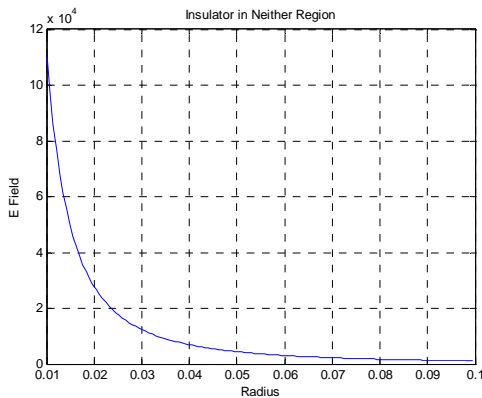
%Neither half filled with insulator

```
C=4*pi*epso/(1/a-1/b);Q=C*Vo;We=0.5*C*Vo^2;
%E Field
E=Q./(4.*pi.*epso.*r.^2);Emax=max(E);
figure;plot(r,E);grid;xlabel('Radius');ylabel('E Field');title('Insulator in Neither Region');
```

%Plot all Three Cases

```
figure;plot(r,E,r,E1,r,E2);grid;xlabel('Radius');ylabel('E Field');title('All Three Cases');
```





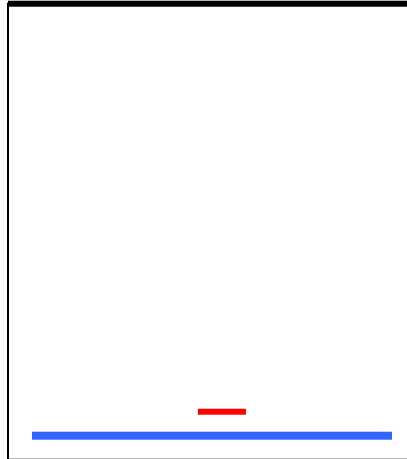
6. **Capacitance Using a Spreadsheet** The device we are going to simulate is a variation of a capaciflector, which was invented by a NASA engineer. This uses a capacitor to determine the distance to a surface, usually as part of a robot obstacle sensor. For our purposes, you should use a 100x100 array (use rows 1 to 101 and columns A to CW). The outer boundary cells should be grounded ($V=0$). The sensor electrode is relatively small (cells AT89 to BD89). There is also a reflector to force the field lines toward the surface to be measured (cells E97 to CS97). Both the sensor and reflector are at the same voltage (choose anything you like, but 10V works). However, we are only to be concerned with the capacitance of the sensor. [In application, the reflector is driven by a voltage follower connected to the sensor.] For your analysis, you can determine the charge on the sensor using the boundary condition method or the flux integral method. The latter should be more accurate, but the differences will not be large. All space is filled with air in this problem.
- Determine the charge on the sensor electrode for your choice of voltage. Use this to determine the capacitance of the configuration. Plot the equipotentials for this configuration, adding a few E field lines for clarity. Why are your results reasonable?

The object whose position is to be measured is the top of the shield. Now we will move the top plate (add a grounded surface) at other locations to see how the capacitance will change.

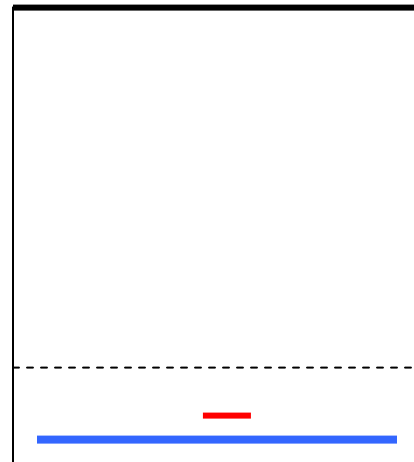
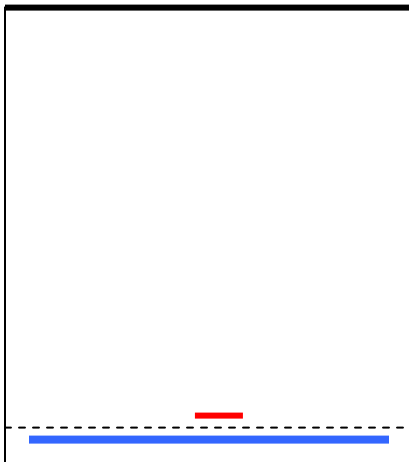
- Repeat the calculations of part a for three additional cases: a grounded plane at row 26, 51 and 76, respectively. For each case, plot the equipotentials, adding a few E field lines for clarity. Why are your results reasonable?



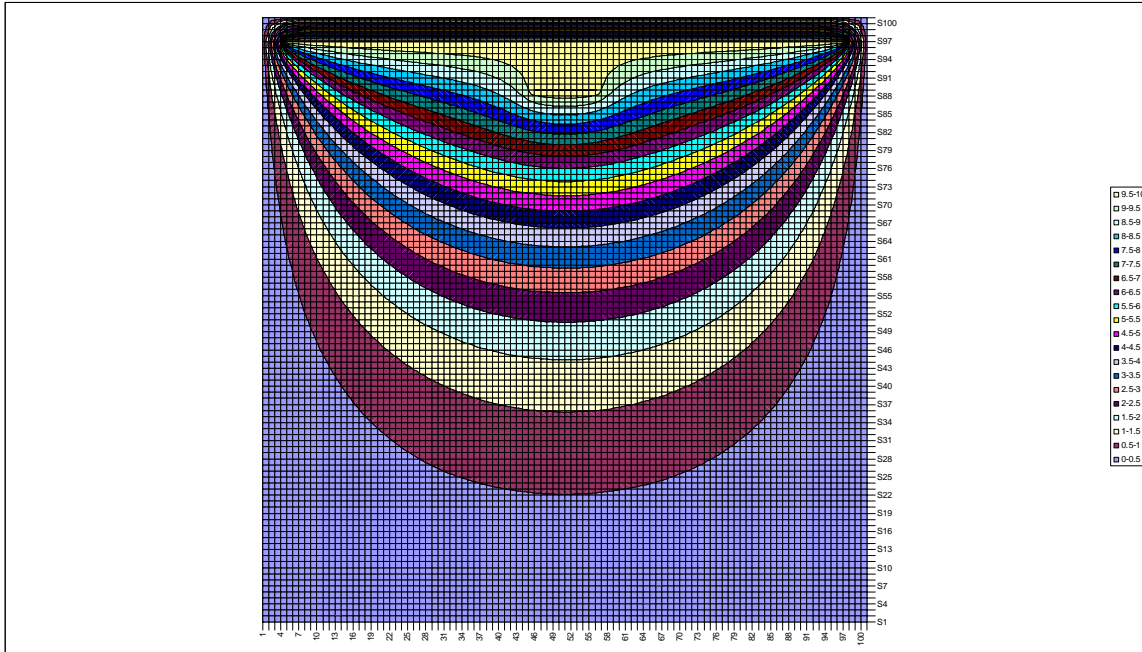
For your analysis, please explain each step, especially if you use the spreadsheet to do most of the work.



Extra Credit: Assume that the purpose of the moveable top plate is to separate a region filled with water from the air filled space just analyzed. Now also assume that the water leaks into the chamber (somehow). Consider two cases: one where the water fills the chamber up to row 95 and one where it fills the chamber up to row 76. Determine the capacitance for both cases. Use a relative permittivity of 81 for water. Only consider the situation where the top of the chamber is in its original position (top row).



Discuss your results.



Shield Voltage 0
 Electrode Voltage 10

Capacitance Calc Using Flux to Find Charge

Average Inner Voltage 9.1139591
 Average Outer Voltage 8.9786606
 Step Size 0.01
 Area Per Unit Length 0.56
 Average E Field 13.529849
 Average D Field 1.20E-10
 Flux 6.71E-11
 Charge Per Unit Length 6.71E-11
 Capacitance Per Unit Length 6.71E-12

The calculations done in Excel to determine the charge on the sensor for the given voltage are shown above. Note that two voltage averages are taken and the difference between them gives the average E field normal to the surface. From E we get D, which, when multiplied by the area, gives the flux. The flux is equal to the charge.



The following table shows the region around the sensor electrode. The electric flux was determined through a surface that surrounds the sensor. This is more accurate than the surface charge method, but here the difference is not great.

6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	7	7
6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7	7	7	7	7	7	8	8	8	8
7	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8	8
7	7	7	7	7	8	8	8	8	8	8	8	8	8	8	8	8
7	7	7	8	8	8	8	8	8	8	8	8	8	9	9	9	9
8	8	8	8	8	8	8	8	8	8	9	9	9	9	9	9	9
8	8	8	8	8	8	8	8	9	9	9	9	9	9	9	9	9
8	8	8	8	8	8	9	9	9	9	9	9	10	10	10	10	10
8	8	8	8	9	9	9	9	9	9	10	10	10	10	10	10	10
8	9	9	9	9	9	9	9	9	9	10	10	10	10	10	10	10
9	9	9	9	9	9	9	9	9	9	10	10	10	10	10	10	10
9	9	9	9	9	9	9	9	9	9	10	10	10	10	10	10	10
9	9	9	9	9	9	9	9	9	9	10	10	10	10	10	10	10
9	9	9	9	9	10	10	10	10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8	8
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

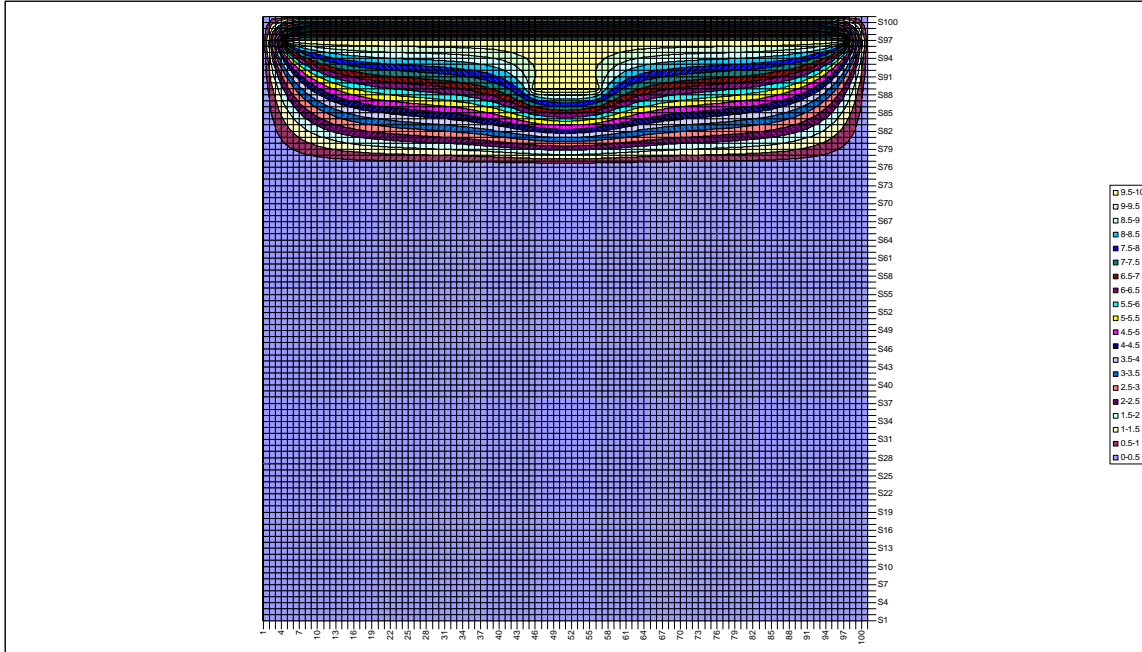
The surface charge method is also used.

Capactiance Using Surface Charge

Average Voltage	9.7164018
Average E Field	28.35982
Average D Field	2.51E-10
Area Per Unit Length	0.24
Charge Per Unit Length	6.03E-11
Capactiance Per Unit Length	6.03E-12



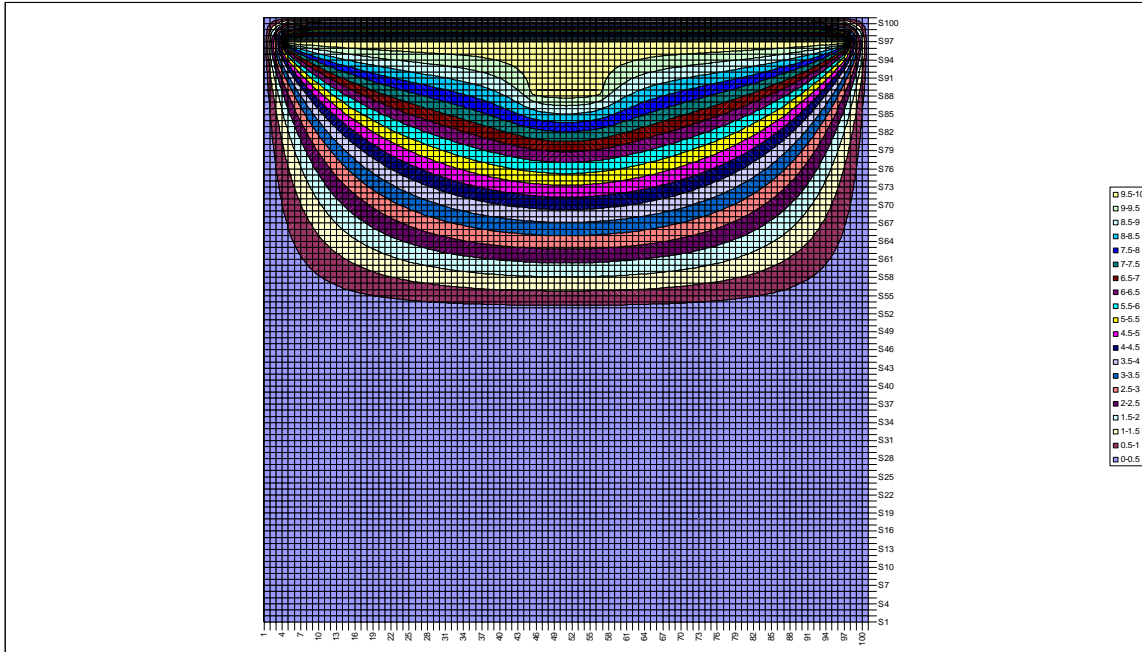
The following info is for the other three locations for the grounded top of the chamber.



Capacitance Calc Using Flux to Find Charge

Average Inner Voltage	7.7450702
Average Outer Voltage	7.4070474
Step Size	0.01
Area Per Unit Length	0.56
Average E Field	33.802282
Average D Field	2.99E-10
Flux	1.68E-10
Charge Per Unit Length	1.68E-10
Capacitance Per Unit Length	1.68E-11
Capacitance Using Surface Charge	

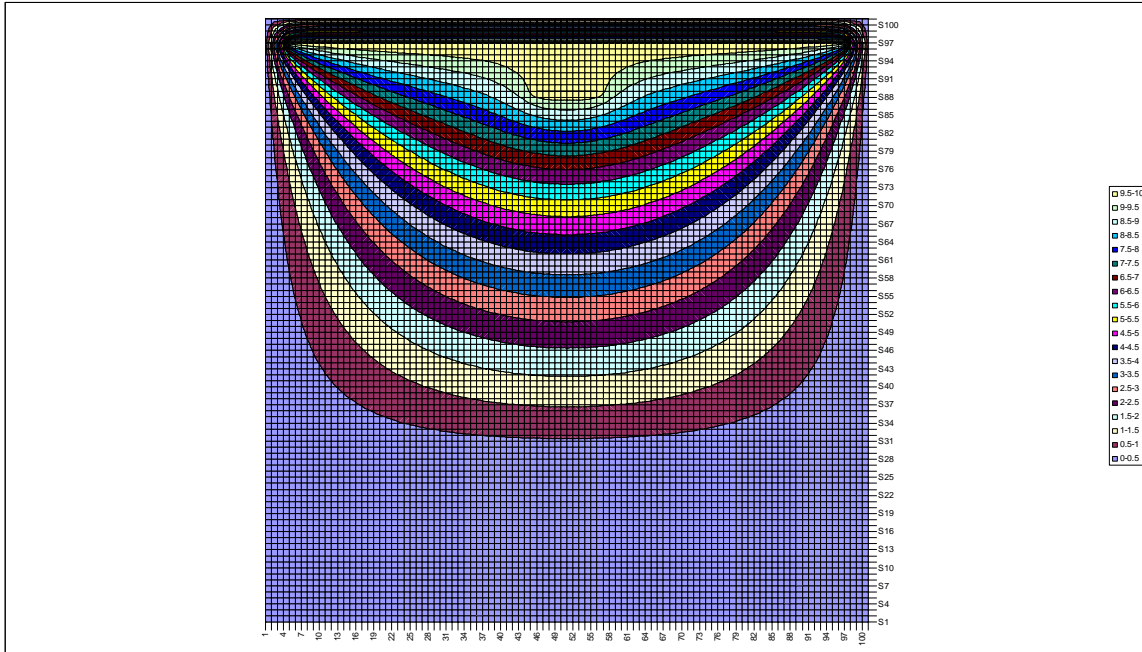
Average Voltage	9.2705775
Average E Field	72.942249
Average D Field	6.46E-10
Area Per Unit Length	0.24
Charge Per Unit Length	1.55E-10
Capacitance Per Unit Length	1.55E-11



Capacitance Calc Using Flux to Find Charge

Average Inner Voltage 8.9425443
 Average Outer Voltage 8.781826
 Step Size 0.01
 Area Per Unit Length 0.56
 Average E Field 16.07183
 Average D Field 1.42E-10
 Flux 7.97E-11
 Charge Per Unit Length 7.97E-11
 Capacitance Per Unit Length 7.97E-12
 Capacitance Using Surface Charge

Average Voltage 9.6606121
 Average E Field 33.938787
 Average D Field 3.01E-10
 Area Per Unit Length 0.24
 Charge Per Unit Length 7.21E-11
 Capacitance Per Unit Length 7.21E-12



Capacitance Calc Using Flux to Find Charge

Average Inner Voltage	9.1100977
Average Outer Voltage	8.9742452
Step Size	0.01
Area Per Unit Length	0.56
Average E Field	13.585248
Average D Field	1.20E-10
Flux	6.74E-11
Charge Per Unit Length	6.74E-11
Capacitance Per Unit Length	6.74E-12
Capacitance Using Surface Charge	

Average Voltage	9.715142
Average E Field	28.485798
Average D Field	2.52E-10
Area Per Unit Length	0.24
Charge Per Unit Length	6.05E-11
Capacitance Per Unit Length	6.05E-12



Redoing the first case, just to check to see if the structure is really sensitive to the change in top plate location:

Capacitance Calc Using Flux to Find Charge

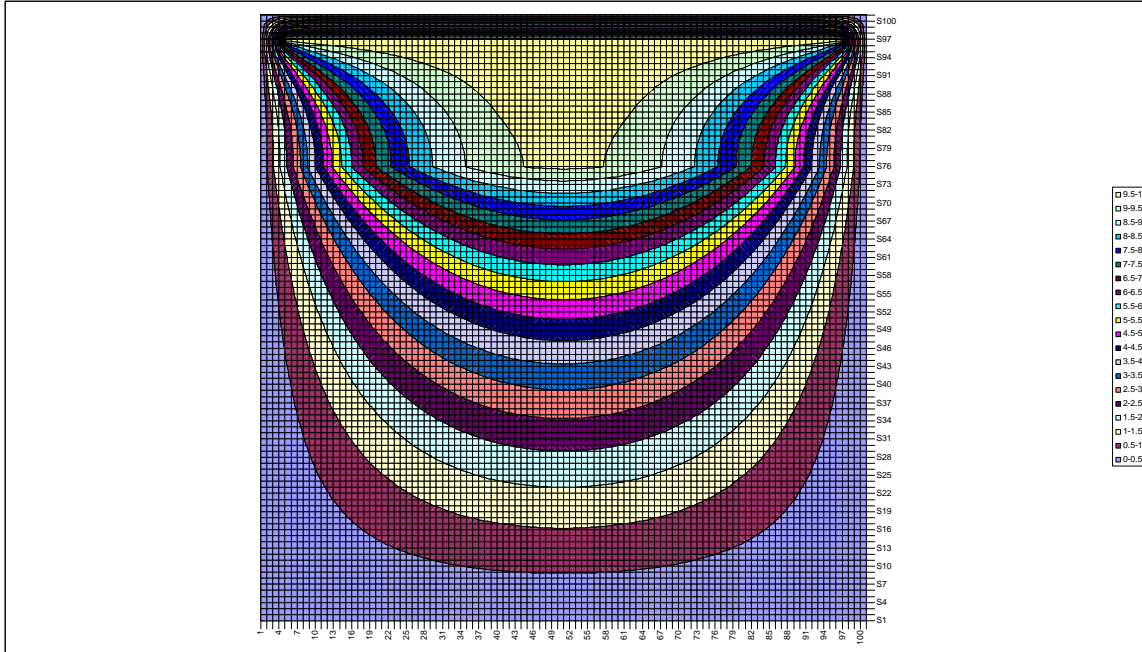
Average Inner Voltage	9.1108326
Average Outer Voltage	8.9750855
Step Size	0.01
Area Per Unit Length	0.56
Average E Field	13.574705
Average D Field	1.20E-10
Flux	6.73E-11
Charge Per Unit Length	6.73E-11
Capacitance Per Unit Length	6.73E-12
Capacitance Using Surface Charge	

Average Voltage	9.7153818
Average E Field	28.461822
Average D Field	2.52E-10
Area Per Unit Length	0.24
Charge Per Unit Length	6.05E-11
Capacitance Per Unit Length	6.05E-12

Thus, we see no change at all so this is probably not in the range where one can measure any real change.



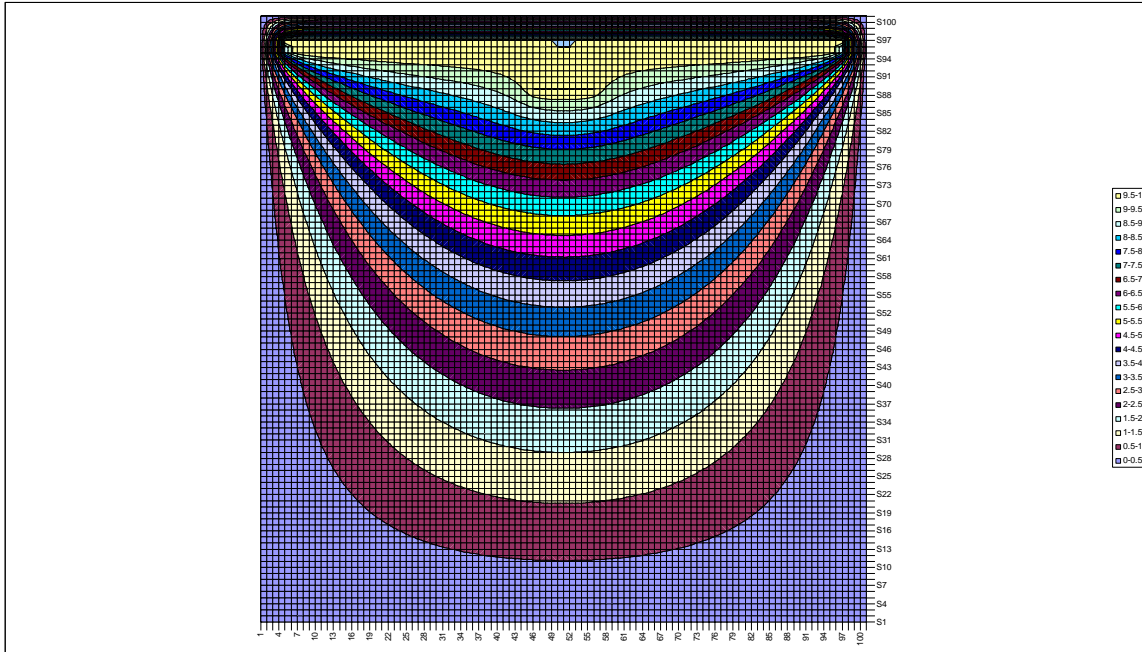
Extra Credit:



Capacitance Calc Using Flux to Find Charge

Average Inner Voltage	9.8080206
Average Outer Voltage	9.7767558
Step Size	0.01
Area Per Unit Length	0.56
Average E Field	3.1264785
Average D Field	2.24E-09
Flux	1.26E-09
Charge Per Unit Length	1.26E-09
Capacitance Per Unit Length	1.26E-10
Capactiance Using Surface Charge	

Average Voltage	9.9411275
Average E Field	5.8872505
Average D Field	4.22E-09
Area Per Unit Length	0.24
Charge Per Unit Length	1.01E-09
Capacitance Per Unit Length	1.01E-10



Capacitance Calc Using Flux to Find Charge

Average Inner Voltage	9.268195
Average Outer Voltage	9.1588874
Step Size	0.01
Area Per Unit Length	0.56
Average E Field	10.930759
Average D Field	9.68E-11
Flux	5.42E-11
Charge Per Unit Length	5.42E-11
Capacitance Per Unit Length	5.42E-12
Capactiance Using Surface Charge	

Average Voltage	9.7634174
Average E Field	23.658259
Average D Field	2.09E-10
Area Per Unit Length	0.24
Charge Per Unit Length	5.03E-11
Capacitance Per Unit Length	5.03E-12

1. From HW 3

$$a. \quad \vec{E} = \begin{cases} E_r(r) \hat{a}_r = \frac{546}{r} \hat{a}_r & \text{for } a \leq r \leq b \\ 0 & \text{elsewhere.} \end{cases}$$

$$a = 0.51 \text{ mm}, \quad b = 2.285 \text{ mm.} \quad (\text{should be near these numbers,})$$
$$\epsilon = 1.38 \epsilon_0$$

$$b. \quad V = 0 \text{ for all } r > b$$

$$V(r) - \underset{0}{V(b)} = - \int_b^r \frac{546}{r} dr = 546 \ln \frac{b}{r}$$

$$V(a) = 546 \ln \frac{b}{a} = 818.9$$

$$c. \quad \nabla^2 V = 0 \text{ is Laplace's Eqn}$$

$$\nabla^2 546 \ln \frac{b}{r} = \nabla^2 (546 \ln b - 546 \ln r)$$

$$= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right)$$

$$= \frac{1}{r} \frac{\partial}{\partial r} \left(r \left(0 - 546 \frac{1}{r} \right) \right)$$

$$= \frac{1}{r} \frac{\partial}{\partial r} (-546) = 0 \quad \checkmark$$

$$2. \quad \vec{E} = \hat{a}_r E_r(r) = \hat{a}_r \frac{4.07 \times 10^{15}}{r^2}$$

in the atmosphere, zero elsewhere.

$$V(r) = - \int_{r_0}^r \frac{4.07 \times 10^{15}}{r^2} dr$$

↙ 6.38×10^6

$$V(r) = - \frac{4.07 \times 10^{15}}{r} + \frac{4.07 \times 10^{15}}{r_0} \quad \text{for pos voltage}$$

(Sign is not specified in this problem because the direction of \vec{E} was not.)

$$3. a. \quad \vec{E} = \hat{a}_x \frac{\rho_{v0}}{\epsilon_0 x_0} \frac{x_0^2 - x^2}{2} \quad -x_0 \leq x \leq x_0$$

$= 0$ elsewhere.

$$b. \quad V(x) - \underbrace{V(0)}_0 = - \int_0^x \frac{\rho_{v0}}{x_0 \epsilon_0} \frac{x_0^2 - x^2}{2}$$

$$V(x) = \frac{\rho_{v0}}{x_0 \epsilon_0 2} \left[\int_0^x x_0^2 dx - \int_0^x x^2 dx \right]$$

$$= \frac{\rho_{v0}}{2 x_0 \epsilon_0} \left[x_0^2 x - \frac{x^3}{3} \right]$$

since this is odd, it works for pos & neg x .

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} \frac{\rho_{v0}}{2x_0 \epsilon_0} \left[x_0^2 x - \frac{V^3}{3} \right]$$

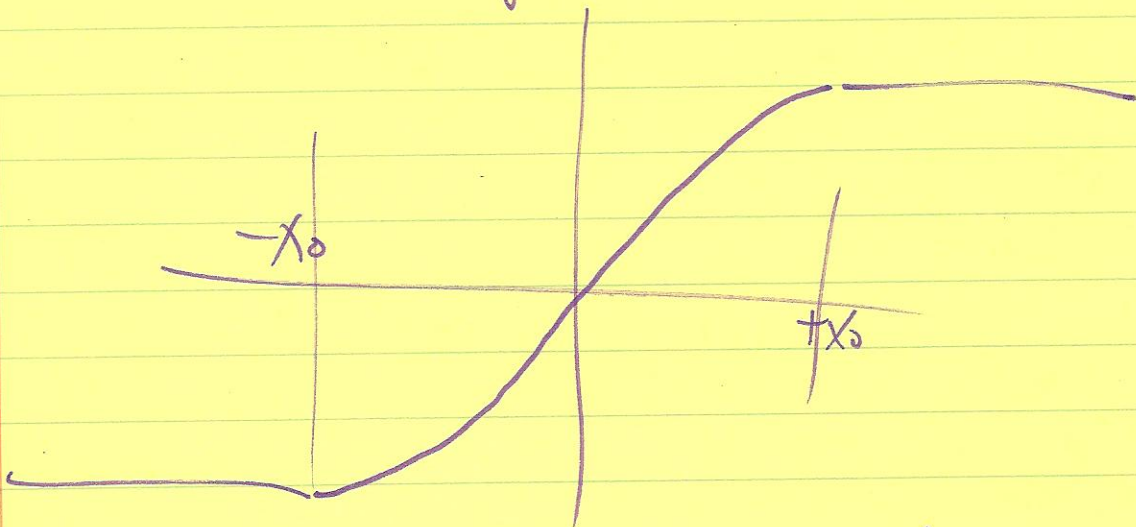
$$= \frac{\partial}{\partial x} \frac{\rho_{v0}}{2x_0 \epsilon_0} \left[x_0^2 - x^2 \right]$$

$$= \frac{\rho_{v0}}{2x_0 \epsilon_0} \left[0 - 2x \right]$$

$$= - \frac{\rho_{v0}}{x_0 \epsilon_0} x \quad \text{which is what we started with}$$



Note that for the Madlob plot

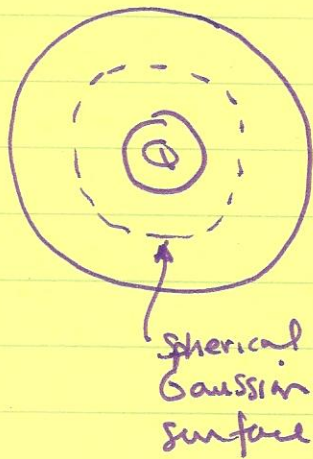


The outer parts are missing,

4. Solution in problem text

5. Capacitance

a. Assume a charge Q on the inner conductor



$$\oint \vec{D} \cdot d\vec{S} = Q_{\text{enc}} = Q$$

$$d\vec{S} = \hat{a}_r r^2 d\phi \sin\theta d\theta$$

$$\vec{D} = \hat{a}_r D_r(r) \text{ from symmetry}$$

$$\begin{aligned} \oint \vec{D} \cdot d\vec{S} &= D_r(r) r^2 \int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta \\ &= D_r(r) 4\pi r^2 \end{aligned}$$

$$\vec{D} = \hat{a}_r \frac{Q}{4\pi r^2}$$

$$\vec{E} = \begin{cases} \hat{a}_r \frac{Q}{4\pi\epsilon_1 r^2} & a \leq r \leq c \\ \hat{a}_r \frac{Q}{4\pi\epsilon_2 r^2} & c \leq r \leq b \end{cases}$$

$$V(a) - V(b) = - \int_b^c \frac{Q}{4\pi\epsilon_1 r^2} dr - \int_c^a \frac{Q}{4\pi\epsilon_2 r^2} dr$$

\downarrow
0 reference

$$V(a) = \frac{Q}{4\pi\epsilon_1} \left(\frac{1}{c} - \frac{1}{b} \right) + \frac{Q}{4\pi\epsilon_2} \left(\frac{1}{a} - \frac{1}{c} \right)$$

$$\begin{aligned} \frac{1}{C} &= \frac{V(a)}{Q} = \frac{1}{4\pi\epsilon_1} \left(\frac{1}{c} - \frac{1}{b} \right) + \frac{1}{4\pi\epsilon_2} \left(\frac{1}{a} - \frac{1}{c} \right) \\ &= \frac{1}{C_1} + \frac{1}{C_2} \quad \text{Caps in series} \end{aligned}$$

$$b. \frac{1}{C} = \frac{1}{4\pi\epsilon_1} \left(\frac{1}{a} - \frac{1}{c} \right) + \frac{1}{4\pi\epsilon_2} \left(\frac{1}{c} - \frac{1}{b} \right)$$

$$\frac{4}{3}\pi (c^3 - a^3) = \frac{4}{3}\pi (b^3 - c^3) \quad \text{for equal volumes}$$

$$2c^3 = b^3 + a^3$$

$$c^3 = \frac{b^3 + a^3}{2}$$

$$\frac{1}{C} = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{b} \left(\frac{1}{a} - \frac{1}{c} \right) + \left(\frac{1}{c} - \frac{1}{b} \right) \right]$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{1}{ba} - \frac{1}{bc} + \frac{1}{c} - \frac{1}{b} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{1}{ba} + \frac{5}{6c} - \frac{1}{b} \right]$$

$$C = 4\pi\epsilon_0 \left/ \left(\frac{1}{6a} - \frac{1}{b} + \frac{5}{6\sqrt[3]{\frac{a^3+b^3}{2}}} \right) \right.$$

$$a = .01, b = .10 \quad C = .08 \quad (.0794)$$

Using Matlab (See files attached)

$$C = 6.48 \text{ pF}$$

$$W_E = \frac{1}{2} C V_0^2 = 3.24 \times 10^6 \text{ J.}$$

$$E_{\max} = 9.7 \times 10^4 \text{ V/m}$$

c. Same approach as b.

$$C = 1.27 \text{ pF}$$

$$W_E = 6.33 \times 10^{-7} \text{ J.}$$

$$E_{\text{max}} = 1.14 \times 10^5 \text{ V/m}$$

d. For comparison, the empty case:

$$C = 1.24 \text{ pF}$$

$$W_E = 6.18 \times 10^{-7} \text{ J.}$$

$$E_{\text{max}} = 1.1 \times 10^5 \text{ V/m}$$

By placing some insulator in the wire
loop, the C goes up & E_{max} goes down
so that it seems worth it to add some
insulator, as long as it stays attached to
the wire conductor.