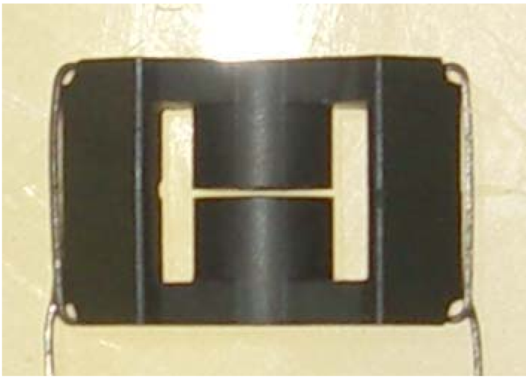
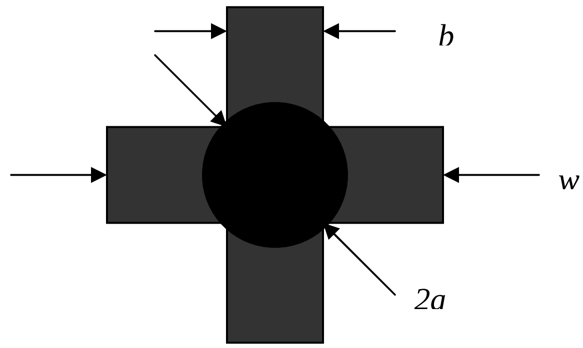
Homework 6

Due 4 November at 6:00 pm

1. **Magnetic Circuits** Core with and without air gap.

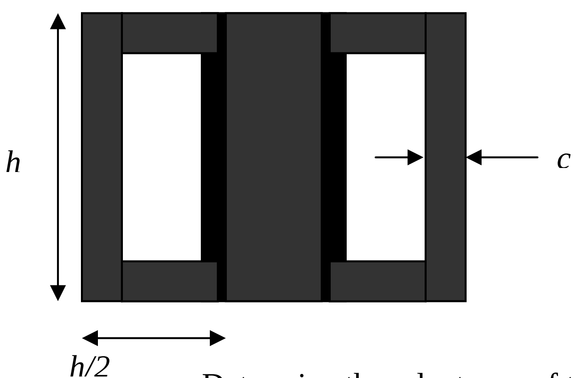


Commercial Core



Top View

The core of interest has a circular center leg and four rectangular outer legs. The cross sectional area of the round center leg is equal to the sum of areas of the four rectangular legs. The radius of the center core is *a* and the dimensions of the cross section of the outer legs are *b x c*.



Assume that there are *N* turns of wire carrying a current *I* wrapped around the center leg. Analyze two cases, one with no gap and one with a gap l_g in the central leg. You are to find the reluctances and inductance for both cases and then use that information to determine the maximum possible energy stored for each case give that the B field cannot exceed B_{max} or the core will saturate. Also simplify your solution for the gap case by assume the reluctance of the gap is much larger than that of the core. Core $\mu >> \mu_o$.

a. Determine the reluctance of the configuration with and without the gap.
b. Determine the inductance of both configurations.
c. Determine the energy stored in terms of B_{max}.

Note for this problem, you might find the following will make interesting reading.

- A discussion of air-gapped magnetic cores from the University of Surrey in the UK: http://info.ee.surrey.ac.uk/Workshop/advice/coils/gap/index.html
- The 2001 Magnetics Design Handbook from TI, especially the chapter on Inductor and Flyback Transformer Design. http://focus.ti.com/docs/training/catalog/events/event.jhtml?sku=SEM401014
- Electrical Power Transformer and Inductor from Lazar's Power Electronics Guide http://www.smps.us/magnetics

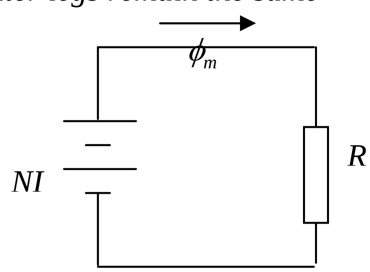
a. *Reluctance of configuration, no air gap.*

$$\mathcal{R} = \frac{l}{\mu A} = \mathcal{R}_{center} + \mathcal{R}_{legs} \text{ and note that } \pi a^2 = 4bc = A$$

$$\mathcal{R}_{center} = \frac{h}{\mu \pi a^2} \text{ and } \mathcal{R}_{legs} = \frac{2h}{\mu bc}$$ *Since the areas are the same for all legs, the total reluctance is* $\mathcal{R} = \frac{l}{\mu A} = \mathcal{R}_{center} + \mathcal{R}_{legs} = \frac{h}{\mu A} + \frac{2h}{\mu A} = \frac{3h}{\mu A}$

Reluctance with air gap.

$$\mathcal{R} = \frac{l}{\mu A} = \mathcal{R}_{center} + \mathcal{R}_{gap} + \mathcal{R}_{legs}$$ *where we will assume that the gap is sufficiently small that the reluctance of the center leg remains the same. Clearly the outer legs remain the same so that the only new term is* $\mathcal{R}_{gap} = \frac{l_g}{\mu_o A}$. *For the case where The gap dominates,* $\mathcal{R} = \mathcal{R}_{gap}$



b. *To determine the inductance first draw the circuit model. Then we have the flux as* $\phi_m = \frac{NI}{\mathcal{R}}$ *The total flux linking the circuit will be* N *times this so that the inductance is*

$$L = \frac{\Lambda}{I} = \frac{N\phi_m}{I} = \frac{N^2}{\mathcal{R}}$$ *thus all we need to know is the reluctance. For the case with no gap, we have then that* $L = \frac{N^2}{\mathcal{R}} = \frac{\mu A N^2}{3h}$ *while for the gap,* $L = \frac{N^2}{\mathcal{R}} = \frac{\mu_o A N^2}{l_g}$

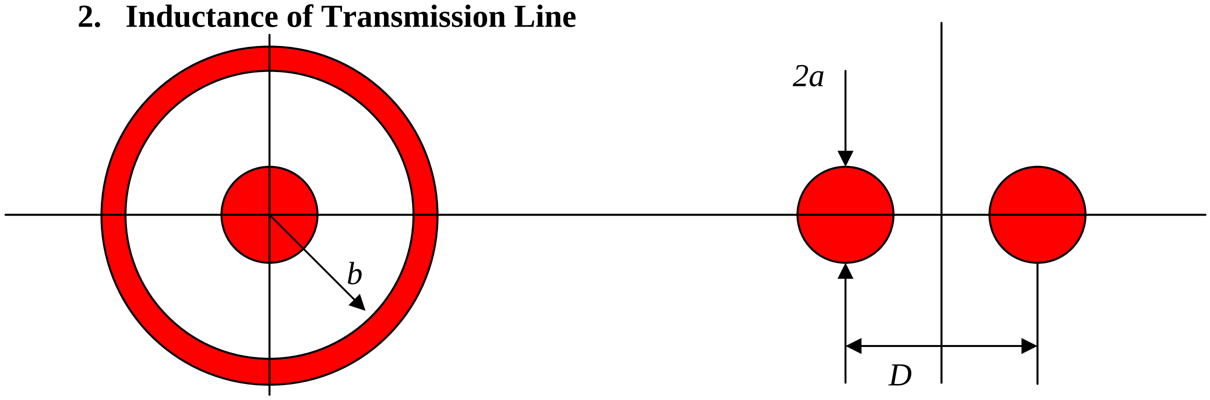
c. *The magnetic field in a solenoid is given by* $B = \frac{\mu NI}{l} = \frac{\mu NI}{3h}$ *for the case with no gap.*

If the magnetic field is B_{max}, *then* $I = \frac{3hB_{\max}}{\mu N}$ *and the energy stored is*

$$W_m = \frac{LI^2}{2} = \frac{\mu A N^2}{6h}\left(\frac{3hB_{\max}}{\mu N}\right)^2 = \frac{1}{2}\frac{B_{\max}{}^2}{\mu}\frac{3hA}{1}$$ *For the gap case,*

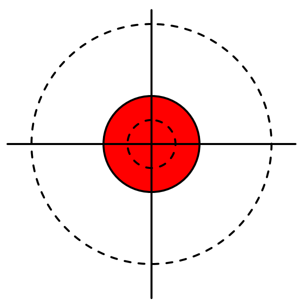
$$W_m = \frac{LI^2}{2} = \frac{1}{2}\frac{\mu_o A N^2}{l_g}\left(\frac{l_g B_{\max}}{\mu_o N}\right)^2 = \frac{1}{2}\frac{B_{\max}{}^2}{\mu_o} l_g A$$. *Note that since* $\frac{l_g}{\mu_o} >> \frac{3h}{\mu}$ *the energy stored in the gap case is larger, even though most of the energy is stored in the gap.*

2. Inductance of Transmission Line



Two common transmission lines we considered previously are shown above. Note that the analysis of these two lines can be found in many places in the reference materials provided for this course. However, we will go through all of the steps so that they are both addressed thoroughly.

a. Begin by analyzing the magnetic field $\vec{B}$ produced by a long, straight wire carrying a current *I*. Assume that the wire has a radius = *a*. (This is the radius of all wires except for the outer shield of the coax.)



Begin by determining the current enclosed by a closed circle of radius *r* for the two cases shown. That is, for *r*<*a* and *r*>*a*. Clearly label your results. Then use your results to find the expressions for the magnetic field $\vec{B}$ for all values of *r*.

For the inner circle, the current enclosed will be determined by the area it encloses, which is πr^2 *so that the current will be* $\frac{\pi r^2}{\pi a^2} I$ *. For the outer circle, the current enclosed is all of the current or I. Then we have for the magnetic field* $\oint \vec{B} \cdot d\vec{l} = B_\phi 2\pi r = \mu_o I_{encl}$ *and* $\vec{B} = \hat{\phi}\frac{\mu_o I}{2\pi r}$ *for the region outside the current and* $\vec{B} = \hat{\phi}\frac{\mu_o I r}{2\pi a^2}$ *inside the wire.*

b. Before considering the external inductance for the coax or two-wire lines, first evaluate the internal inductance per unit length of the long, straight wire using the energy method. The flux method is also possible, but somewhat more difficult in this case.

The magnetic field energy inside the wire is given by

$$\iiint \frac{1}{2\mu_o}\vec{B}\cdot\vec{B}dv = \frac{2\pi}{2\mu_o}\int_0^a \frac{\mu_o Ir}{2\pi a^2}\frac{\mu_o Ir}{2\pi a^2}rdr = \frac{\mu_o I^2}{4\pi a^4}\int_0^a r^3 dr = \frac{I^2}{2}\frac{\mu_o}{2\pi}\frac{a^4}{4a^4}$$

So that the inductance is $L_i = \frac{\mu_o}{2\pi}\frac{a1}{4} = \frac{\mu_o}{8\pi}$ *per unit length*

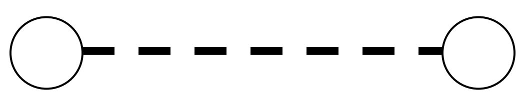
c. Find the external inductance per unit length for the coaxial cable using either the flux or energy method. They are equally easy for this case. We will not determine the internal inductance of the coax shield because that is generally not large enough to matter.

The energy method involves similar integrations.

$$\iiint \frac{1}{2\mu_o}\vec{B}\cdot\vec{B}dv = \frac{2\pi}{2\mu_o}\int_0^a \frac{\mu_o I}{2\pi r}\frac{\mu_o I}{2\pi r}rdr = \frac{\mu_o I^2}{4\pi}\int_0^a \frac{1}{r}dr = \frac{I^2}{2}\frac{\mu_o}{2\pi}\ln\frac{b}{a}$$

So that the inductance is $L_e = \frac{\mu_o}{2\pi}\ln\frac{b}{a}$ *per unit length*

d. Assuming that each of the two long, straight wires in the two-wire line produce the same $\vec{B}$ field, although with different locations, determine the external inductance per unit length for the line. The flux method is generally easier for this configuration.

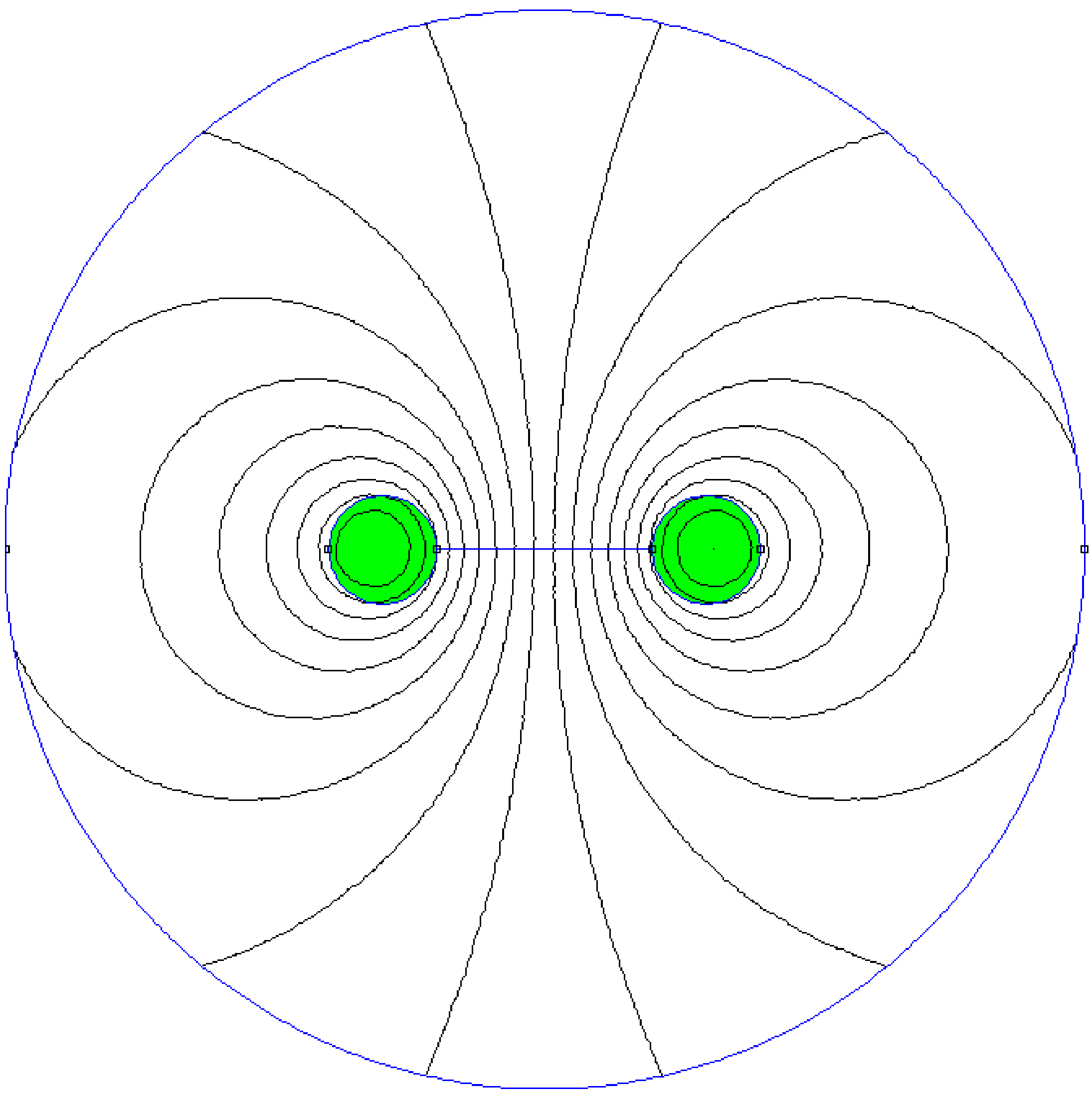


The integral is done from the edge of one wire to the edge of the other, as shown in the dashed line above. $\phi_m = 2\int_a^{D-2a}\frac{\mu_o I}{2\pi r}dr = 2\frac{\mu_o I}{2\pi}\ln\frac{D-2a}{a} = \Lambda = LI$ *so that*

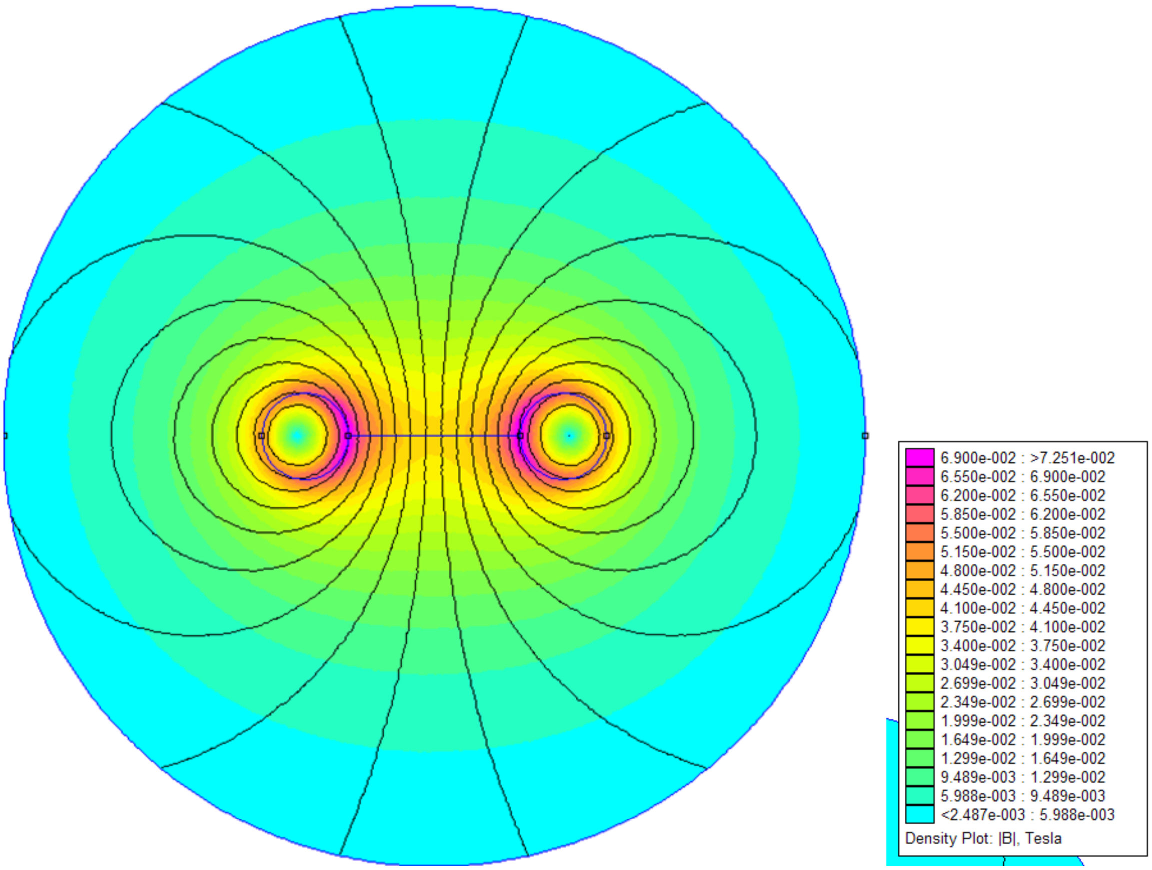
$$L = 2\frac{\mu_o}{2\pi}\ln\frac{D-2a}{a} \approx \frac{\mu_o}{\pi}\ln\frac{D}{a}$$ *per unit length.*

3. **Numerical Methods** Assessing results.

We can quite easily analyze the two-wire line using FEMM. Shown below is the field structure determine for two parallel lines, each with a radius of *1cm*, whose centers are separated by a distance of *6cm.* The conductors are shown in green. No direction is shown for the field but one current goes into the page and one out. The outer circle (radius = *10cm*) is just the limit of the analysis region, since numerical methods can only consider a region of finite extent. There is no boundary or any other object there.



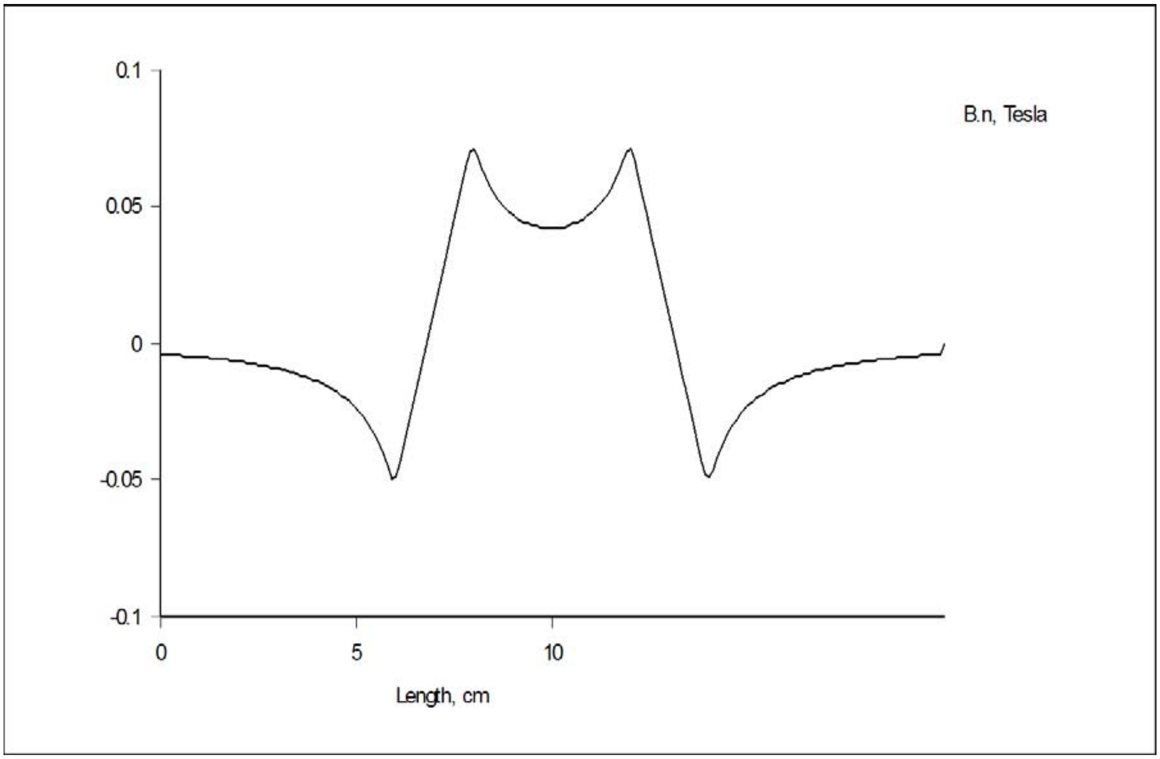
The magnetic field intensity is shown below. Note that there is field throughout this region, inside and outside the conductors.



On the next page is plotted the normal component of the magnetic field $\vec{B}\cdot\hat{n}$ on the horizontal axis (the line that passes through the centers of both wires). The data used in this plot are also in the Excel spreadsheet HW6_f09.xls. The first column is the horizontal position, with the origin located between the wires. The second column is the vertical (normal) component of $\vec{B}$.

a. The task in this problem is to use the results from problem 2 part a to confirm that the FEMM analysis is reasonable. Note that perfect agreement is not possible, but in this case, the agreement should be very good. In column 3, evaluate the magnetic field for the locations of column 1, for the wire on the left. In column 4, evaluate the magnetic field for the wire on the right. Be sure to use the appropriate expressions for inside and outside the wires. You are not given the currents in the two wires, so you will have to assume some value based on the magnitudes shown for the field and then correct it later. In column 5, add the contributions from the two wires. Finally, plot the field values vs. position. Now, adjust the value of the current in the wires until you obtain the best possible agreement.

What value of current produces the best possible agreement? Plot your results. Discuss any differences you observe between the FEMM results and those obtained using the ideal fields from two long, straight wires.



b. FEMM does not directly evaluate the inductance. However, it can give us the following: Total energy stored in the magnetic field 0.0384018 Joules or the volume integral of $\vec{A}\cdot\vec{J}$ 0.0803573 Henry Amp^2. The problem solved is 2D but FEMM does not consider the per unit length values. It requires the actual depth to be specified. From your knowledge of the inductance per unit length for this line, determine the depth used by FEMM in this problem. Note, FEMM used an integer value for this dimension, so your result should also be an integer. Discuss your answer.

The external inductance per unit length for this line is given by the expression in the previous problem $L = 2\frac{\mu_o}{2\pi}\ln\frac{D-2a}{a} \approx \frac{\mu_o}{\pi}\ln\frac{D}{a} = \frac{\mu_o}{\pi}\ln\frac{6}{1} = 7.16x10^{-7}$ *The calculations in the gray cells in the spreadsheet show that the closest integer is 1cm. The two results do not agree exactly, but they are close. Also, the inductance used here is just the external inductance, so the values from FEMM should be a little bit larger, which they are. The internal inductance is* $L_i = 2\frac{\mu_o}{8\pi} = 1x10^{-7}$ *which brings the results closer together. The differences are not large enough to be concerned about.*

c. FEMM also calculates the force on conductors. For this case, the force given is 0.328163 N, which also uses the depth. From your knowledge of the forces between current-carrying conductors, what is the direction of the force experienced by each conductor and is this result reasonable? Explain your answer.

Forces between currents traveling in opposite directions are in a direction that would increase the distance between the conductors. The force per unit length for a wire carrying the given current due to the field from the other is ILB or a number which is almost exactly equal to the one from FEMM. This calculation is done in the spreadsheet in the yellow area.

Note: The point of this problem is to assess the quality of the solution for the field. Be sure your discussion includes more than just brief comments.