Rensselaer | EE EPE CSE

Department of Electrical, Computer, & Systems Engineering

Homework 8

Due 3 December at 5:00 pm

Solution

1. Plane Wave In Lossless Media

In my home office, my Wi-Fi signal level (measured using Inssider, which is free from Metageek.net) is -33dBm (channel 6). My router (Linksys WRT54G2) transmits total power of 18dbm. My laptop is located approximately 1m from the router.

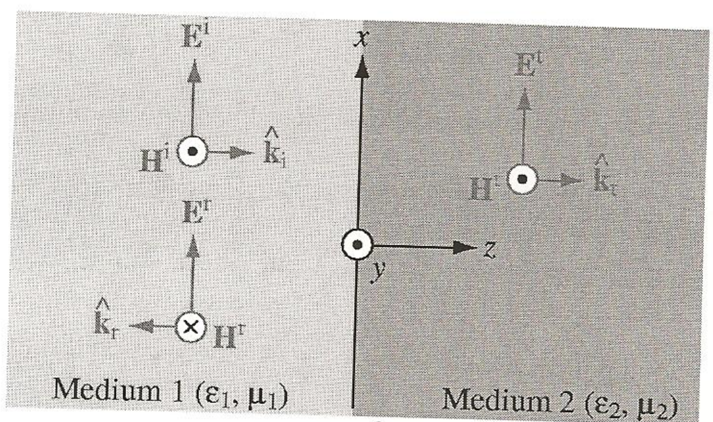
a. Determine the power density this represents in W/m^2 and the equivalent cross-sectional area of my laptop antenna.

b. Determine the electric and magnetic field intensities of the Wi-Fi signal. Also, what is η?

c. What is the center frequency of channel 6? (There is a range of frequencies used by this channel, but we will assume everything occurs at the center frequency.) Determine ω, β, and λ for this wave. (Assume this *f* for all of this assignment.)

d. Write the electric and magnetic fields in phasor form, assuming propagation in the z-direction.

2. Reflection and Transmission at Normal Incidence



$z = 0$

The wave of problem 1 is incident on a wall made of sheetrock (Medium 2) which, according to the Microwaves101 website, has a dielectric constant of *2.4*. In this problem, we will assume normal incidence for simplicity. The normal direction will be defined as the z-direction, as shown above.

a. For a wave propagating in air (Medium 1), incident normally on an infinitely thick region of sheetrock (we first consider only a single boundary), determine the reflection and transmission coefficients, Γ and τ.

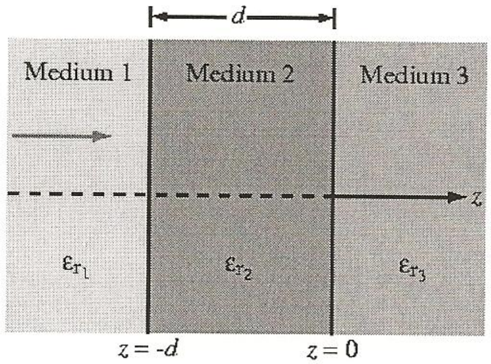
b. Write the reflected and transmitted electric and magnetic fields in phasor form.

c. Determine the reflected and transmitted average power densities.

d. Plot the standing wave pattern for the electric field in air.

(All figures from Ulaby)

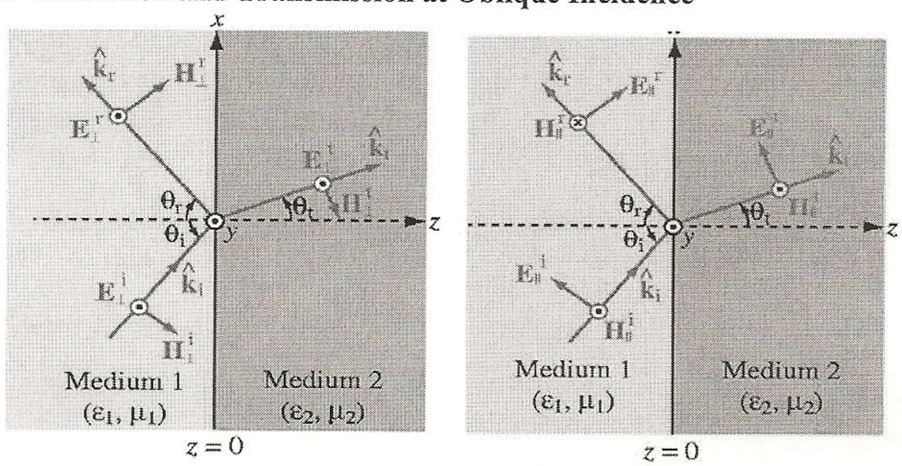
3. Multiple Boundaries



Now assume, as is more realistic, that the sheetrock has finite thickness (*d=5/8"*). Convert the thickness to SI units.

a. Determine the values of η, β, and λ for this wave in the sheetrock.
b. Determine the input impedance $Z_{in}=Z(-d)$ for this slab of material.
c. Write the reflected and transmitted electric and magnetic fields in air on either side of the slab of sheetrock (regions 1 and 3).
d. Determine the reflected and transmitted average power densities.

4. Reflection and Transmission at Oblique Incidence



(a) Perpendicular polarization (b) Parallel polarization

Assume the wave is now incident on the sheetrock (single boundary as in problem 2) at an angle of *30°*.

a. What is the transmitted angle?
b. Assuming that the wave is randomly polarized so that there is equal power in perpendicular and parallel polarization, determine the reflected and transmitted electric field in each polarization.
c. Determine the reflected and transmitted average power density in each polarization.

1. Plane Wave in Lossless Media

a. $-33\,dBm = .5\,\mu W$ (There are hundreds of calculators online for this)

$18\,dBm = 63\,mW$

$$S_{ave} = \frac{63\,mW}{4\pi(1)^2} = \frac{5\,mW}{m^2} = 5\times10^{-3}\,\frac{W}{m^2}$$

at a distance of 1 m.

The equivalent area is $\frac{5\times10^{-7}}{5\times10^{-3}} = 10^{-4}\,m^2 = (1\,cm)^2$

b. $5\times10^{-3} = \frac{1}{2}\frac{E^2}{\eta_0}$ $\quad \eta_0 = 120\pi = 377\,\Omega$

$E_m = |E| = 1.94\,\frac{V}{m}$ $\quad H_m = |H| = \frac{1.94}{377} = 5\times10^{-3}\,\frac{A}{m}$

c. $f = 2.437$ GHz for channel 6

$\omega = 2\pi f = 15.3\times10^9$

$\beta_0 = \frac{\omega}{c} = 51$

$\lambda_0 = 12.3\,cm$

Checking $\lambda_0 = \frac{c}{f}$
$= \frac{3\times10^8}{2.437\times10^9}$
$= .123\,m$

d. $\vec{E} = \hat{x}\,E_m e^{-j\beta_0 z}$

$\vec{H} = \hat{y}\,H_m e^{-j\beta_0 z}$

2 Reflection & Transmission at Normal Incidence

$\varepsilon_{r2} = 2.4\,\varepsilon_0 \qquad \sqrt{2.4} = 1.55$

a. $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \dfrac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} = \dfrac{1 - 1.55}{1 + 1.55} = -.216$

$\tau = \dfrac{2}{2.55} = .784$

b. $E_r = \Gamma E_m \qquad \vec{E}_r = \hat{x}\,\Gamma E_m e^{+j\beta_0 z}$

$E_t = \tau E_m \qquad \vec{E}_t = \hat{x}\,\tau E_m e^{-j\beta z}$

$\beta = \beta_0 \sqrt{\varepsilon_r} = 79.1 \qquad \lambda = \dfrac{2\pi}{\beta} = 8\text{ cm}$

$\vec{H}_r = \hat{y}\,\dfrac{-\Gamma E_m}{\eta_0} e^{+j\beta_0 z} \qquad \vec{H}_t = \hat{y}\,\dfrac{\tau E_m}{\eta} e^{-j\beta z}$

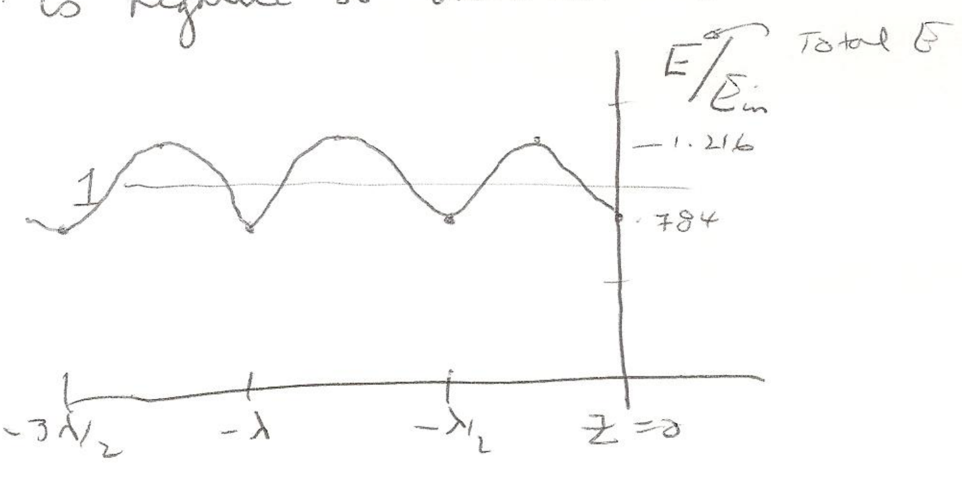
c. $\dfrac{E_{refl}^2}{2\eta_0} = \Gamma^2 S_{in} = (.216)^2 S_{in} = 0.047\, S_{in}$

$\dfrac{S_{refl}}{S_{in}} = 0.047 \qquad S_{trans} = 1 - .047 = .953$

d. $1 - |\Gamma| = 1 - .216 = .784$

$1 + |\Gamma| = 1.216$

Γ is negative so min at $z = 0$



3. $d = 5/8'' = 1.5875$ cm

a. η, β, λ are the same as noted above $\beta d = 1.26$

$\tan \beta d = 3.08$

$\beta d = \pi/5$

b. $Z_{in} = Z(-d) = \eta \dfrac{\eta_0 + j\eta \tan\beta d}{\eta + j\eta_0 \tan\beta d}$

$= \eta \dfrac{1.55 + j \tan\beta d}{1 + j(1.55)(\tan\beta d)} = \eta \dfrac{1.55 + j\,3.08}{1 + j(1.55)(3.08)}$

$\eta = \dfrac{\eta_0}{\sqrt{2.4}} = 243\,\Omega$

$Z_{in} = 166 - j44$

c. $\Gamma = \dfrac{Z_{in} - \eta_0}{Z_{in} + \eta_0} = \dfrac{166 - j44 - 377}{166 - j44 + 377}$

$= -.38 - j.11$

$\tau = 1 + \Gamma = .62 - j.11$

$E_r = E_m \Gamma e^{+j\beta_0 z}$ $\qquad E_t = E_m \tau e^{-j\beta_0 z}$

$H_r = \dfrac{E_m \Gamma}{\eta_0} e^{+j\beta_0 z}$ $\qquad \underbrace{H_t = \dfrac{E_m \tau}{\eta_0} e^{-j\beta_0 z}}$

phase term missing
due to slab

d. $S_{ave\,refl} = \underbrace{\Gamma^2}_{\Gamma\Gamma^*} \dfrac{E_m^2}{2\eta_0}$ $\qquad S_{ave\,Trans} = \underbrace{\tau^2}_{\tau\tau^*} \dfrac{E_m^2}{2\eta}$

4. a. Transmitted angle

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{\sin 30^\circ}{\sqrt{2.4}} = \frac{.5}{\sqrt{2.4}} = .3227$$

$$\theta_2 = 18.8^\circ$$

b. $\Gamma_\perp = -.2574$ See formula sheet for formulas

$\tau_\perp = 1 + \Gamma_\perp$

$\Gamma_\parallel = -.1727$

Note! → $\tau_\parallel = (1+\Gamma_\parallel)\frac{\cos\theta_i}{\cos\theta_2}$

Perp Pol $\hat{a}_E = \hat{y}$

Par Pol $\hat{a}_H = \pm\hat{y}$

$\vec{k}_i = \hat{x}\,\cancel{\sin}\,\sin\theta_I + \hat{z}\cos\theta_i$

$\vec{k}_r = \hat{x}\sin\theta_1 - \hat{z}\cos\theta_1$

$\vec{k}_t = \hat{x}\sin\theta_2 + \hat{z}\cos\theta_2$

Par Pol

$\hat{a}_{E_i} = +\hat{x}\cos\theta_1 - \hat{z}\sin\theta_1$

$\hat{a}_{E_r} = \hat{x}\cos\theta_1 + \hat{z}\sin\theta_1$

$\hat{a}_{E_t} = \hat{x}\cos\theta_2 \cancel{+} \hat{z}\sin\theta_2$

Perp Pol

$\vec{a}_{H_i} = -\hat{x}\cos\theta_1 + \hat{z}\sin\theta_1$

$\hat{a}_{H_r} = \hat{x}\cos\theta_1 + \hat{z}\sin\theta_1$

$\hat{a}_{H_t} = -\hat{x}\cos\theta_2 + \hat{z}\sin\theta_2$

$\vec{E} = \hat{a}_E E_m e^{-j\bar{k}\cdot\bar{r}}$ $\vec{H} = \hat{a}_H \frac{E_m}{\eta} e^{-j\bar{k}\cdot\bar{r}}$

When $E_m = \frac{E_i}{\sqrt{2}}$ or $\Gamma\frac{E_i}{\sqrt{2}}$ or $\tau\frac{E_i}{\sqrt{2}}$

$\sqrt{2}$ for half of power in each pol.

C. Reflected & Transmitted Power Density

Reflectivity

$$R_\perp = \Gamma_\perp^2 = .07 \qquad T_\perp = \tau_\perp^2 \frac{\eta_1}{\eta_2} \frac{\cos\theta_2}{\cos\theta_1} = .93 \checkmark$$

Transmissivity

$$R_\parallel = \Gamma_\parallel^2 = .03 \qquad T_\parallel = \tau_\parallel^2 \frac{\eta_1}{\eta_2} \frac{\cos\theta_2}{\cos\theta_1} = .97 \checkmark$$

calculate 1st to be able to check for conservation of power. Then convert to S ratios

see discussion in Ulaby

For inc wave

$$A_i = A\cos\theta_1 \qquad A_r = A_i = A\cos\theta_1$$

$$A_t = A\cos\theta_2$$

$$S_i A_i = P_i \qquad S_r A_r = P_r \qquad S_t A_t = P_t$$

$$\frac{S_r}{S_i} = \frac{P_r}{P_i} = R \qquad \frac{S_t}{S_i} = \frac{P_t}{P_i}\frac{A_i}{A_t} = \frac{P_t}{P_i}\frac{\cos\theta_1}{\cos\theta_2} = T\frac{\cos\theta_1}{\cos\theta_2}$$

$$S_r = S_i \frac{P_r}{P_i} = S_i R \qquad S_t = S_i \frac{P_t}{P_i}\frac{\cos\theta_1}{\cos\theta_2} = S_i T \frac{\cos\theta_1}{\cos\theta_2}$$

Now can plug in the numbers.