Rensselaer | EE EPE CSE

Department of Electrical, Computer, & Systems Engineering

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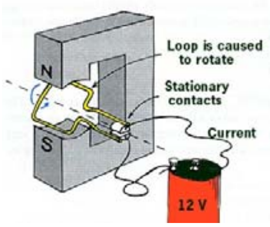
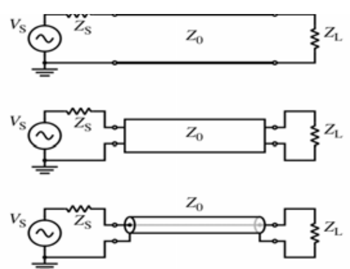
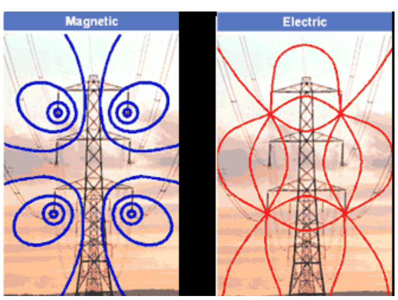
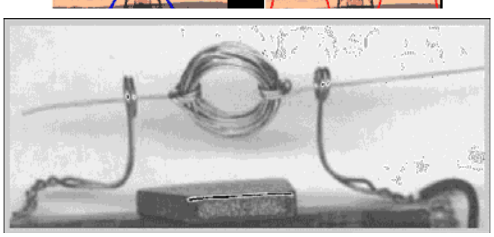
Homework 9
Due 9 December at 6pm









1. Short Questions (8 Points)

a. (1 Point) ____ {Trans Lines}
b. (1 Point) ____ {Inductance}
c. (1 Point) ____ {Waves}
d. (1 Point) ____ {Magnetic Ckts & Inductance}
e. (1 Point) ____ {Trans Lines}
f. (1 Point) ____ {Resistance & Inductance}
g. (1 Point) ____ {Faraday's Law}
h. (1 Point) ____ {Boundary Conditions}
j. (1 Point) ____ {Waves}
k. (1 Point) ____ {Lossy Materials}
l. (1 Point) ____ {Misc}
m. (1 Point) ____ {Vocabulary}

Total ______

2. Electric Fields (3 Points) ______

3. Magnetic Fields (3 Points) ______

4. Transmission Lines (3 Points) ______

5. Capacitance & Laplace's Equation (3 Points) ______

Total (20 Points) ______

Some Comments and Helpful Info:
In this course, we use two types of notation for unit vectors. Keep in mind that

$$\hat{a}_x = \hat{x} \qquad \hat{a}_y = \hat{y} \qquad \hat{a}_z = \hat{z}$$

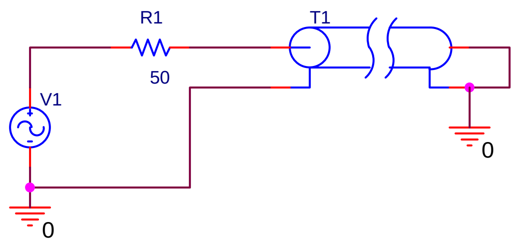
$$\hat{a}_r = \hat{r} \qquad \hat{a}_\phi = \hat{\phi} \qquad \hat{a}_\theta = \hat{\theta}$$

Also, sometimes R is used for spherical radius instead of r, so R is another term that gets used for more than one purpose. Pay attention to the context of the questions to minimize problems.

Notes:
1. **Please read over all questions before you begin your work. There may be some information in a later question that helps you with an earlier question.**
2. For short answer questions, you should add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees & use 4 significant digits.

1. Short Questions (8 Points)

a. (1 Point) Transmission Lines



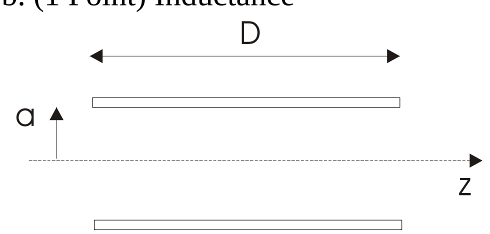
A 200 meter long air insulated transmission line has a short circuit at its load end. At low frequencies (near ω = 0), does the line look like an inductor, a capacitor or a resistor? (Circle the correct answer.) Which of the following frequencies are low? (Again, circle the correct answers.)

f = 100 Hz   f = 1 kHz.   f = 10 kHz   f = 100 kHz   f = 1 MHz   f=10 MHz

Answer: The line looks like an inductor at low frequencies. A low frequency is one for which the line is electrically short. That is $d = 200 << \lambda = c/f$. *Since*

$f = \frac{3x10^8}{200} = 1.5x10^6$, *all frequencies well under 1MHz are low.*

b. (1 Point) Inductance



A solenoidal inductor can be constructed using any of the following combinations of materials. Which will result in the largest inductance? Assume that the length of the solenoid is equal to *D* and the radius is equal to *a*.

i) The solenoid has 250 turns of wire, length *D = 10cm*, radius *a = 5.64mm* and is wound on mild steel ($\mu_r = 2000$).

ii) The solenoid has 150 turns of wire, length *D = 20cm*, radius *a = 5.64mm* and is wound on iron ($\mu_r = 5000$).

iii) The solenoid has 50 turns of wire, length D =*30cm*, radius a = *5.64mm* and is wound on mild steel ($\mu_r = 2000$).

Answer: For an ideal solenoid, $L = \dfrac{\mu N^2 \pi a^2}{D}$. *Since the number of turns N dominates the expression, the first answer is the largest (nothing else changed by more than a factor of 3). Plugging in numbers confirms this.*

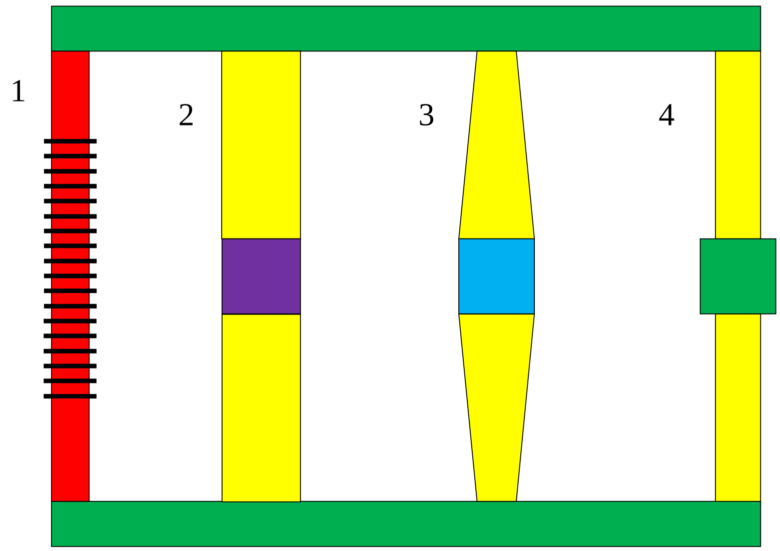
c. (1 Point) Electromagnetic Waves (Circle the correct answers.)

i. The closest distance between a minimum and a maximum in a voltage standing wave pattern is (λ/4, λ/2, λ, 2λ).

ii. A uniform plane wave is propagating in water (ε_r = 1.5 at optical frequencies) and is incident obliquely at 30^o on the water/air surface at z=0. The angle of incidence is (equal to, greater than, less than) the angle of reflection. Which of the following is correct? (There is no transmitted wave into air, the wave transmitted into air propagates with an angle greater than 30^o, the wave transmitted into air propagates at an angle less than 30^o).

Answer: The critical angle is about 55^o so there is a transmitted angle, which has to be larger.

d. (1 Point) Magnetic Circuits and Inductance



The magnetic circuit at the left consists of four legs. The left-most leg is wrapped with N turns of wire carrying a current I. Each of the legs looks the same from each side. The square sections in the middle of the second, third, and fourth legs are identical cubes. The permeability of the cubes is μ_{cube} and the permeability of all legs, including the horizontal legs at the top and bottom is μ_{legs}. Also, $\mu_{cube} >> \mu_{legs}$. The dimensions of the four legs are not given, but you can assume their relative sizes correspond to what is shown in the diagram.

i. Put the reluctances of the four vertical legs in order from largest to smallest.

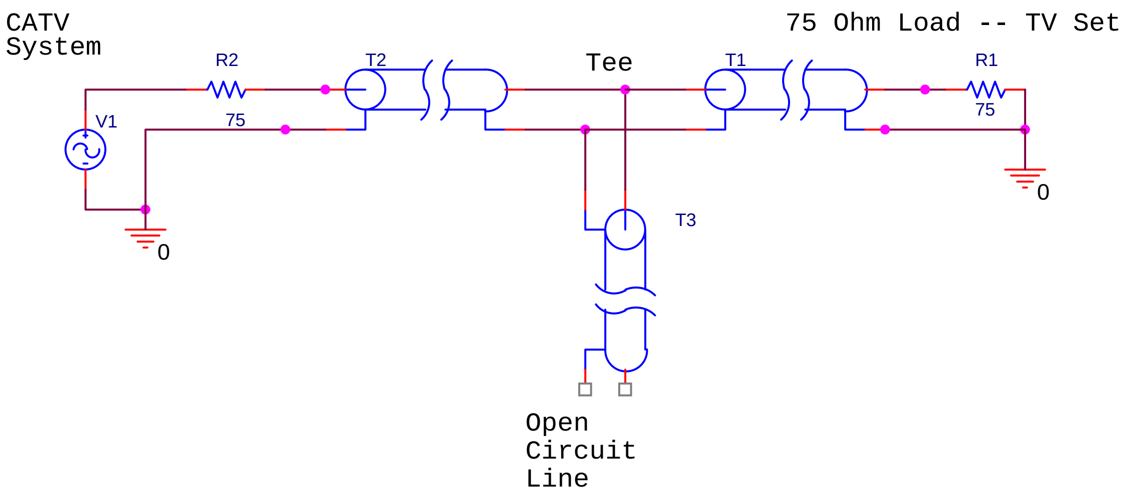
ii. Put the flux carried by the four vertical legs in order from largest to smallest

iii. You wish to maximize the inductance of this configuration by removing all, some or none of the cubes. What will produce the largest inductance?

Answer: Reluctance is given by $R = \dfrac{l}{\mu A}$ *so that* R_1 *is the largest. The average area for 2 is larger than that for 3 which is larger than 4. Thus,* $R_1 > R_4 > R_3 > R_2$*. All flux goes through leg 1 and flux is inversely proportional to reluctance so that*

$\Phi_1 > \Phi_2 > \Phi_3 > \Phi_4$. $L = \dfrac{N\Phi}{I} = \dfrac{NNI}{RI} = \dfrac{N^2}{R}$ *so the reluctance should be maximized, which can be done by removing all cubes.*

e. (1 Point) Transmission Lines



An open-circuited, 0.20 meter long transmission line with $Z_o = 75\Omega$ and $v_{ph} = 2.5 \times 10^8$ m/s connected to a CATV line with a Tee, as shown. Which of the following channels will be blocked by this line? (The frequencies are given in MHz.)

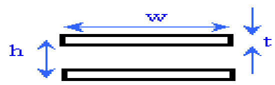
Channel	2	11	19	37	70	97
Frequency	55.25	199.25	151.25	301.25	499.25	103.25

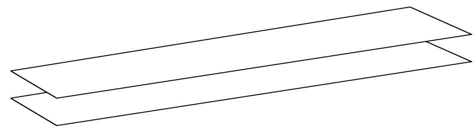
Answer: An open circuit line will look like a short circuit, if it is an odd multiple of $\lambda/4$.

There is only one frequency that matches this condition. $f = \dfrac{u}{\lambda} = \dfrac{2.5x10^8}{4{*}.2} n_{odd}$

f. (1 point) Resistance and Inductance

A parallel plate transmission line is constructed as shown in the figure. Assume that the length of the line is given by *l*.





i) Determine the resistance of this line. Circle the correct answer.

$R = \frac{2l}{\sigma wt}$ $\quad R = \frac{2l}{\sigma hw}$ $\quad R = \frac{l}{\sigma wt}$ $\quad R = \frac{l}{\sigma hw}$

ii) Determine the inductance of this line. Circle the correct answer.

$L = \frac{\mu_o t2l}{w}$ $\quad L = \frac{\mu_o h2l}{w}$ $\quad L = \frac{\mu_o tl}{w}$ $\quad L = \frac{\mu_o hl}{w}$

Answer: The current flows down one plate and back the other so that the total length is 2l. The cross-sectional area is wt. For the inductance, the magnetic flux is found in the area between the plates which is lh. The depth in this case is the distance along the magneticfield direction, w.

g. (1 Point) Faraday's Law

A small permanent magnet is dropped through a coil like the one we used in the Beakman's motor project (with 300 turns of wire and a radius of 1 cm). The voltage measured across the terminals of the coil looks like the following figure as a function of time. A good approximation to this voltage is given by the expression $V(t) = A_o t e^{-\left(\frac{t}{\tau}\right)^2}$ volts, where $A_o=2300$ and $\tau = 6ms$ (this is from real data taken in 4107)

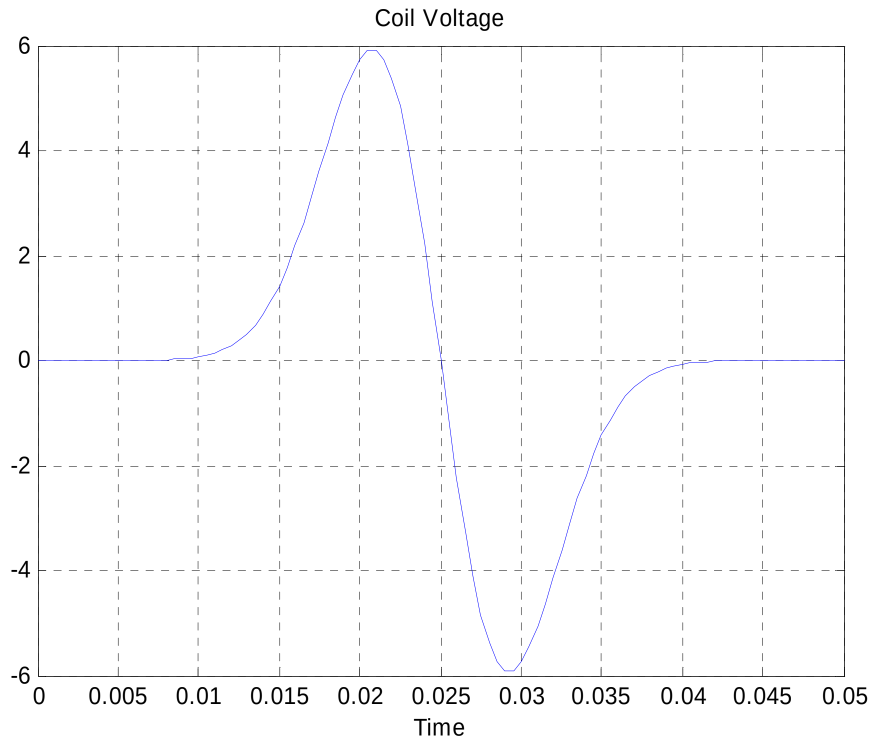
i. Using the simple relationship between the induced voltage and the flux linked by the coil, $V(t) = -\frac{d\Lambda}{dt}$, determine the magnetic flux linked by the coil $\Lambda(t)$ as a function of time during the entire period of the measurement. Circle the correct answer.

$\Lambda(t) = 4.14\exp\left(-\left(\frac{t}{\tau}\right)^2\right)$, $\Lambda(t) = 0.0414\exp\left(-\left(\frac{t}{\tau}\right)^2\right)$ or $\Lambda(t) = 6.9\exp\left(-\left(\frac{t}{\tau}\right)^2\right)$

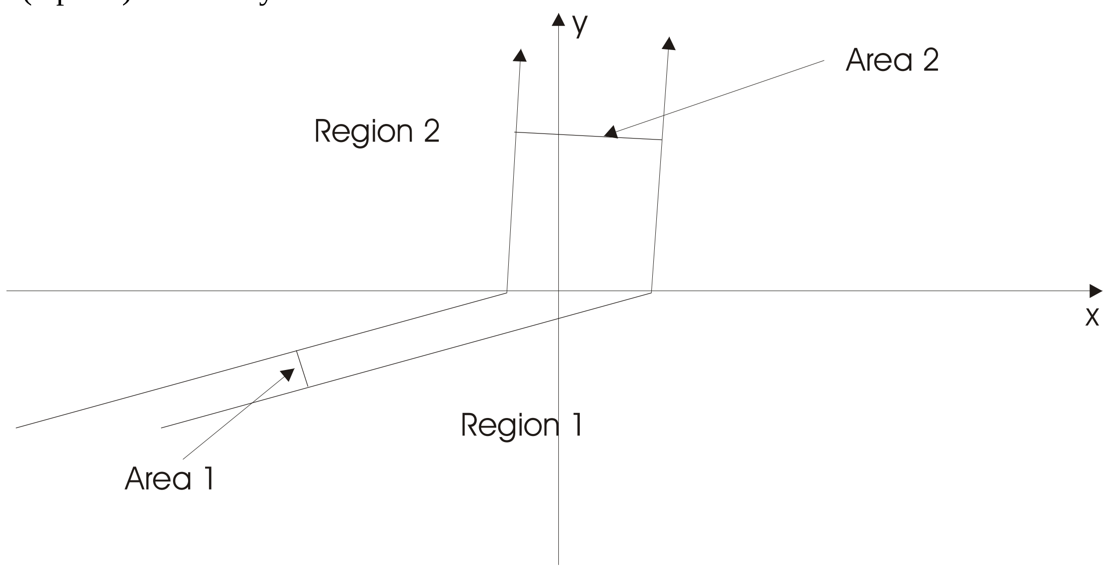
ii. Determine the average flux density $B_{axial}(t)$ of the magnet as a function of time. (The coil sensing the magnetic field is coaxial with the magnet axis, so you will be determining the average flux density in the axial direction.) Again, circle the correct answer.

$B_{axial} = 0.44 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$, $B_{axial} = 0.044 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$ or $B_{axial} = 73.2 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$

Answer: The derivative of the answer for the flux linked must be minus the voltage. The flux linked is the flux time N. Thus the flux is .0414/300. The radius of each loop is 0.01m so that the first answer is correct.



h. (1 point) Boundary Conditions



Shown above are two parallel magnetic field ($\vec{B}$) lines changing direction at the boundary between a magnetic material (with $\mu = \mu_r \mu_o$) and air ($\mu = \mu_o$).

i. Which of the following is correct? Circle the correct answers. (Region 1 is the magnetic material, Region 1 is air, Region 2 is the magnetic material, Region 2 is air)

ii. The angle between the y-axis and the field lines in Region 1 is 70°, while the angle between the field lines and the y-axis in Region 2 is 10°. What is the relative permeability of the magnetic material? Circle the correct answer.

$\mu_r = 156$ $\mu_r = 15.6$ $\mu_r = 312$ $\mu_r = 31.2$

iii. Which of the following is correct? Circle the correct answer. (The flux through area 1 is less than the flux through area 2, the flux through area 1 is equal to the flux through area 2, the flux through area 1 is greater than the flux through area 2)

Answer: $\mu_1 = \mu_2 \dfrac{\tan\theta_1}{\tan\theta_2} = 15.6$

j. (1 Point) Electromagnetic Waves

A uniform plane wave is incident normally on a dielectric interface between two media. Region 1 is air and region 2 is a dielectric region with $\varepsilon = 4\varepsilon_o$. The incident electric field is polarized in the x-direction and the magnetic field is polarized in the y-direction. The magnitude of the incident electric field is 100V/m and the frequency of the wave is 10MHz.

i. Write the incident magnetic field vector in phasor form. Be sure that your answer is in the form of a vector.

$$Answer:\ \vec{H} = \hat{y}\frac{100}{\eta_o}e^{-j\beta z}\ where\ \beta = \frac{\omega}{c} = .21\ and\ \eta_o = 120\pi$$

ii. What is the reflection coefficient Γ?

$$Answer:\ \Gamma = \frac{1-\sqrt{4}}{1+\sqrt{4}} = -0.333$$

k. (1 Point) Lossy Media

In a paper on the electrical loss properties of meat and other foodstuffs, the following table was published.

Dielectric properties (2450 MHz) of 5% aqueous solutions/suspensions of selected ingredients

Ingredient	ε''	ε'	d_p (cm)	$\tan\delta$	P_r
Deionised water	6.9	76.8	5.0	0.090	0.63
Antioxidant BHA	7.1	68.4	4.5	0.104	0.62
Soya concentrate	7.3	43.3	3.5	0.169	0.54
Natex rusk	7.5	47.9	3.6	0.157	0.56
Potato starch	8.3	77.8	4.2	0.106	0.63
Natex grits	9.0	57.7	3.3	0.156	0.59
Wheat flour	9.1	74.4	3.7	0.122	0.63
Boiled potato starch	9.2	76.1	3.7	0.121	0.63
Sucrose	9.4	76.1	3.6	0.124	0.63
Caseinate	9.9	74.3	3.4	0.133	0.63
Whey protein	10.2	74.2	3.3	0.137	0.63
Soya protein isolate	10.2	75.9	3.3	0.134	0.63
Rusk	10.3	77.2	3.3	0.133	0.63
Wheat gluten	10.3	74.5	3.3	0.139	0.63
Soya protein	10.9	76.3	3.1	0.143	0.63
Gluconolactone	11.5	76.1	3.0	0.151	0.63
Carrageenan	11.6	76.8	2.9	0.152	0.63
Sodium alginate	13.6	76.1	2.5	0.178	0.63
Sodium ascorbate	18.7	74.1	1.8	0.253	0.63
Red 2G	21.2	72.7	1.6	0.291	0.62
MSG	23.2	74.5	1.5	0.311	0.63
Sodium benzoate	24.9	73.1	1.4	0.340	0.63
Potassium sorbate	30.1	74.1	1.1	0.407	0.63
Phosphate (P22)	34.4	73.8	1.0	0.467	0.63
White pudding spice	42.8	71.2	0.80	0.602	0.62
Sodium sulphite anhydrous	44.7	70.7	0.77	0.632	0.62
Nitrate	44.9	70.9	0.76	0.633	0.62
Nitrite	53.1	69.6	0.65	0.763	0.62
Salt	65.5	67.3	0.53	0.973	0.61

ε'': dielectric loss factor; ε': dielectric constant; d_p: penetration depth; P_r: Power reflected.

Reference: J.G. Lyng et al. / Meat Science 69 (2005) 589–602

The dielectric constant and the loss factor are the usual material properties we have seen in this course. The penetration depth is equivalent to the skin depth and the reflected power indicates the fraction of the power reflected for a uniform plane wave incident normally on the surface of the liquid solution. The formulas used by the authors are

Reflected Power: $P_r = \left(\frac{\sqrt{\varepsilon'} - 1}{\sqrt{\varepsilon'} + 1}\right)^2$ Transmitted Power: $P_t = 1 - P_r$

Loss Tangent: $\tan\delta = \frac{\varepsilon''}{\varepsilon'}$ Penetration Depth: $d_p = \frac{\lambda\sqrt{2}}{2\pi}\left\{\varepsilon'\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{-\frac{1}{2}}$

Note that the properties of two of the foodstuffs are underlined: Wheat Gluten and Nitrite. For one of these, the formula used by the authors for reflected power is in error. Which one is incorrect and what is the correct answer? Their other formulas are correct.

Answer: The attenuation depth is the inverse of the attenuation constant: $\delta = \frac{2}{\omega\varepsilon''}\sqrt{\frac{\varepsilon'}{\mu}}$

The expression above can be re-written for a low-loss dielectric for comparison purposes as:

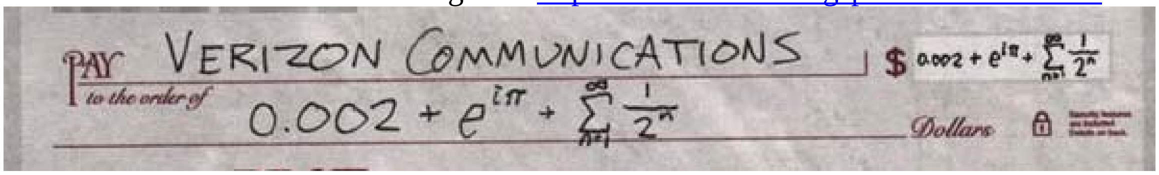
$$d_p = \frac{\lambda\sqrt{2}}{2\pi}\left\{\varepsilon'\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{-\frac{1}{2}} = \frac{\sqrt{2}}{2\pi f\sqrt{\mu\varepsilon'}}\frac{1}{\sqrt{\varepsilon'\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}} = \frac{\sqrt{2}}{2\pi f\sqrt{\mu\varepsilon''}}$$

$$\alpha = 2\pi f\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{1/2} = 2\pi f\sqrt{\frac{\mu}{2}}\left\{\varepsilon'\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{1/2} \textit{ so that}$$

$$\delta = \frac{1}{2\pi f}\sqrt{\frac{2}{\mu}}\left\{\varepsilon'\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{-1/2} = \frac{\lambda\sqrt{\mu\varepsilon'}}{2\pi}\sqrt{\frac{2}{\mu}}\left\{\varepsilon'\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{-1/2} = \frac{\lambda\sqrt{2\varepsilon'}}{2\pi}\left\{\varepsilon'\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{-1/2}$$

and we see there is an ε' *missing in the given expression. It was relatively easy to spot this error since the units are wrong for depth.*

l. (1 Point) In what is now getting to be a somewhat well known story on the internet, an engineer (or physicist, depending on the story) sent the following check to Verizon when they found that no one they talked to at Verizon could do the math to explain the roaming rate in Canada. There is even a blog on it http://verizonmath.blogspot.com/index.html



The question here is what is the value of this check?

Answer: This one was a gift, but the answer is pretty simple. The second term is -1 and the third term is +1 so that the total is 0.002.

m. Vocabulary (1 Point) Draw lines connecting the word or phrase on the left with its definition on the right. The definitions are essentially word for word from Wikipedia.

- Dissipation factor
- Faraday's Law
- Continuity equation
- Magnetic flux
- Reluctance
- Charge density
- Inductance
- Divergence
- Propagation constant
- Phase velocity

- The power loss rate for a wave propagating in some material.
- The magnetic analog of electrical resistance.
- The logarithmic rate of change, with respect to distance in a given direction, of the complex amplitude of any field component.
- The speed at which the phase of any one frequency component of the wave travels.
- A measure of quantity of magnetism, taking into account the strength and the extent of a magnetic field.
- The amount of electric charge in a line, surface, or volume.
- An empirical law expressing (local) charge conservation
- An operator that measures the magnitude of a vector field's source or sink at a given point
- The ratio of the magnetic flux to the current.
- The induced electromotive force or EMF in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit.

Note: this question was added to the final to remind you that you should be able to access a lot of good information through online sources like Wikipedia now that you have some basic understanding of electromagnetic theory. This is an exceptionally good way to expand one's knowledge base, as long as one remains reasonably skeptical about the accuracy of the information. As a general rule, most information found online is wrong, but the truth is relatively easy to identify if a few sources are considered.

2. Electric Fields (3 Points)

A uniform volume charge distribution with $\rho_v = \rho_o$ exists in the spherical region $r < R$. There are no conductors or other materials involved in this problem.

a) Using the integral form of Gauss' Law, determine the electric field of this charge distribution in the region $r > R$ and in the region $r < R$.

Answer: (Remember this is spherical, not cylindrical … a common mistake.) The Gaussian surface is a spherical surface of radius r. For r > R all of the charge is enclosed by this surface or $Q_{encl} = \rho_o \frac{4\pi R^3}{3}$ *while for r < R* $Q_{encl} = \rho_o \frac{4\pi r^3}{3}$. *The left hand side of Gauss' Law is the flux passing through the surface or* $\Phi_E = D_r 4\pi r^2$. *Setting the two sides equal gives us*

For r>R: $D_r = \frac{4\pi R^3 \rho_o}{3(4\pi r^2)} = \frac{R^3\rho_o}{3(r^2)} = \varepsilon_o E_r$

and for r<R: $D_r = \frac{4\pi r^3 \rho_o}{3(4\pi r^2)} = \frac{\rho_o r}{3} = \varepsilon_o E_r$

b) Determine the electric scalar potential at $r = a$, $V(a)$. Assume that the voltage goes to zero as $r \to \infty$.

Answer: $V(a) = -\int_\infty^a \frac{R^3\rho_o}{3\varepsilon_o (r^2)} dr = \frac{R^3\rho_o}{3\varepsilon_o a}$ *for a > R. for a < R, we need another term.*

$$V(a) = \frac{R^3\rho_o}{3\varepsilon_o R} - \int_R^a \frac{r\rho_o}{3\varepsilon_o} dr = \frac{R^2\rho_o}{3\varepsilon_o} - \frac{\rho_o}{3\varepsilon_o}\left(\frac{a^2}{2} - \frac{R^2}{2}\right)$$

c) Determine the electric field energy stored in the region $r > R$ <u>or</u> in the region $r < R$ (your choice).

Answer: For r>R

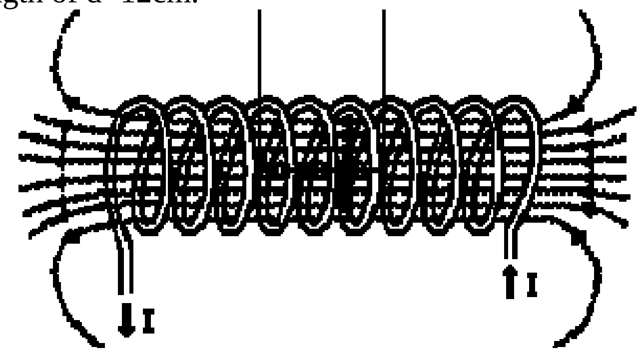
$$4\pi \int_R^\infty \left(\frac{1}{2\varepsilon_o}\left(\frac{R^3\rho_o}{3(r^2)}\right)^2\right) r^2 dr = 4\pi \frac{1}{2\varepsilon_o}\left(\frac{R^3\rho_o}{3}\right)^2 \int_R^\infty \frac{1}{r^2} dr = \frac{4\pi R^6 \rho_o^2}{18\varepsilon_o}\left(\frac{1}{R}\right) = \frac{2\pi R^5 \rho_o^2}{9\varepsilon_o}$$

For r < R $4\pi \int_0^R \left(\frac{1}{2\varepsilon_o}\left(\frac{\rho_o r}{3}\right)^2\right) r^2 dr = 4\pi \frac{1}{2\varepsilon_o}\left(\frac{\rho_o}{3}\right)^2 \int_0^R r^4 dr = \frac{4\pi\rho_o^2}{18\varepsilon_o}\frac{R^5}{5} = \frac{2\pi\rho_o^2 R^5}{45\varepsilon_o}$

Note that the first answer should give us the capacitance of a sphere, which it does if we note that $= \frac{2\pi R^5 \rho_o^2}{9\varepsilon_o} = \frac{1}{2}\frac{(4\pi)^2 R^6 \rho_o^2}{9\varepsilon_o 4\pi R} = \frac{1}{2}\frac{Q^2}{\varepsilon_o 4\pi R}$ *where Q is the total charge.*

3. Magnetic Fields (3 points)

A simple solenoid is constructed by winding N=100 turns of wire around a hollow cylindrical tube. For simplicity, assume that the tube is a toilet paper tube with a radius of a = 2cm and a length of d=12cm.



a. Making the assumption of no fringing effects (that is, treating the solenoid as a piece of an infinitely long cylindrical solenoid), determine the magnetic flux density **B** everywhere in space. For now, leave your answer in terms of a, d, and N. Do not plug in the numbers.

Answer: $B = \dfrac{\mu_o NI}{d}$

b. What is the magnetic energy stored in the field of this inductor? Again, leave your answer in terms of the parameters a, d, and N.

Answer: $W_m = (d\pi a^2)\left(\dfrac{B^2}{2\mu_o}\right)$

c. Determine the inductance of this solenoid. First, express your answer in terms of the parameters and then plug in the given numbers.

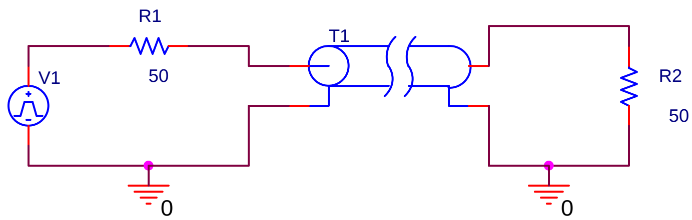
Answer:

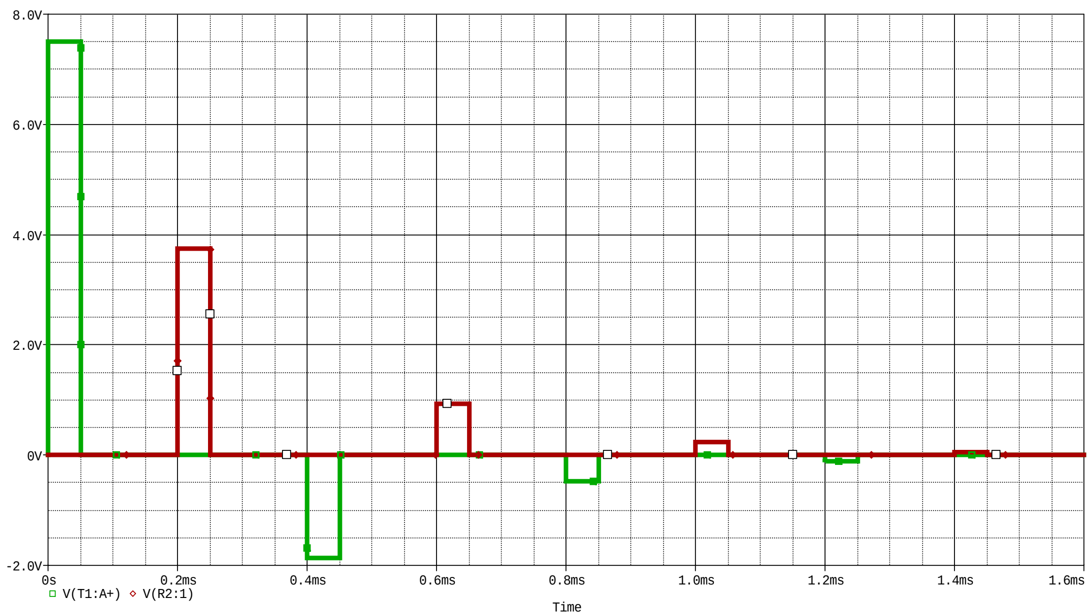
$$\frac{1}{2}LI^2 = W_m = (d\pi a^2)\left(\frac{B^2}{2\mu_o}\right)$$

$$L = (d\pi a^2)\left(\frac{\left(\frac{\mu_o N}{d}\right)^2}{\mu_o}\right) = \frac{\mu_o N^2 \pi a^2}{d} = \frac{(4\pi 10^{-7})(100)^2 \pi(.02)^2}{.12} = 0.13mH$$

4. Transmission Lines (3 Points)

A lossless transmission line is driven by a pulsed voltage source and connected to a resistive load, as shown. The source produces a 50 microsecond pulse. The voltages at the input and output end of the transmission line are shown on the same plot. Neither the characteristic impedance Z_o nor the insulating material used in the cable are known. However, we do know that the length of the line is 50 kilometers. (Ideal conditions are assumed.)





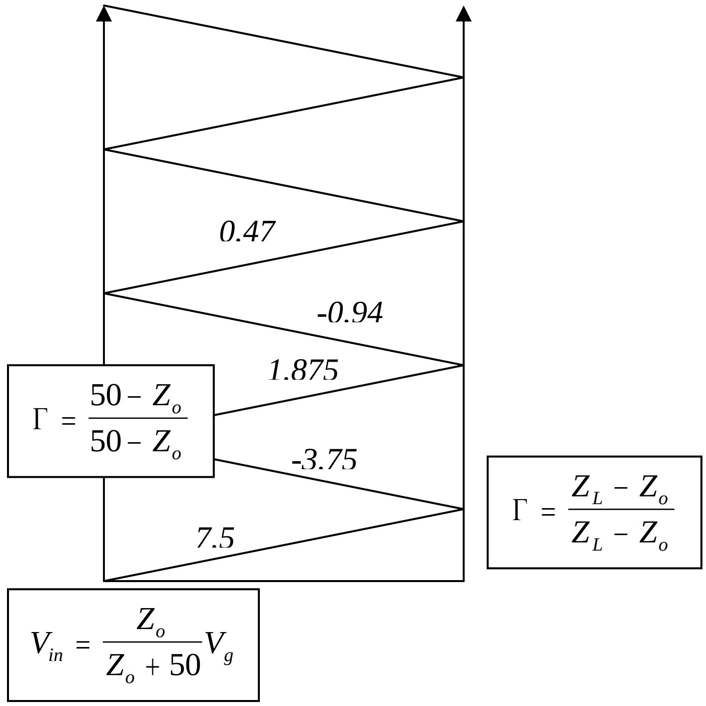
a. First, identify the input and output voltages in the diagram above. Then, determine the time it takes for the pulse to propagate from the input to the output of the transmission line. From this information, determine the relative permittivity ε_r of the insulating material used in the cable.

Answer: The green pulses are the input since they occur immediately. The time to go from one end to the other is 0.2ms. The phase velocity on the line must be 50km/0.2ms=2.5x10^8 *m/s which is the speed of light divided by 1.2. Thus the permittivity is* $\varepsilon_r = 1.2^2 = 1.44$

The internal impedance of the source is 50 Ohms. The voltage divider action at the input end, results in a first pulse of 7.5V. We do not know what the source voltage is.

b. Determine the characteristic impedance Z_o of the transmission line. To determine this parameter, you must first generate the bounce or reflection diagram for this transmission line and then use the voltage data given above. Draw the bounce diagram on the back of the preceding page or on the next page.

Ans: $V_{Load} = 7.5(1+\Gamma) = 3.75$ *so that* $\Gamma = -0.5$. *This also means that the pulse propagating backwards has a magnitude of -3.75. At the input end, we have* $-3.75(1+\Gamma) = -1.875$ *so that again* $\Gamma = -0.5$ *which now allows us to find* Z_o*=150 Ohms. We could also have found this from the reflection coefficient at the load. Either one will work. To check to see if we have the correct numbers, we will regenerate the bounce diagram. Adding the pulse ampl. at each end gives the same numbers as we see in the PSpice plot. QED.*



c. Now that you have determined the characteristic impedance of this line, determine its input impedance if it was driven by a sinusoidal source with a frequency of 25kHz. Begin by first determining the propagation constant β. Then find Z_{in}.

Answer:

$\beta = \frac{\omega}{u} = \frac{2\pi 25x10^3}{2.5x10^8} = 2\pi 10^{-4} = 6.28x10^{-4}$ $\beta d = 6.28x10^{-4}(50x10^3) = 31.4 = 10\pi$ *which is a multiple of half wavelengths so that* Z_{in}*=50 Ohms*

4. Capacitance and Laplace's Equation (3 Points)

a. The two-dimensional parallel plate structure shown below consists of two electrodes with conducting fins aligned with the z-direction (into the paper). The region in the lower half of the diagram is filled with an insulator. Two cases are shown, one in which $\varepsilon_r = 20$ and one in which $\varepsilon_r = 1$. First, identify, which case is which.

AI7 fx 1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
2	9.9	9.9	9.9	9.9	10	10	9.9	9.8	9.7	9.6	9.5	9.5	9.4	9.4	9.4	9.4	9.4	9.5	9.5	9.6	9.7	9.8	9.9	10	9.9	9.8	9.8	9.7	9.7	9.7	9.7
3	9.7	9.7	9.8	9.8	9.9	10	9.8	9.6	9.4	9.2	9.1	8.9	8.9	8.8	8.8	8.8	8.9	8.9	9.1	9.2	9.4	9.6	9.8	10	9.8	9.7	9.5	9.4	9.4	9.3	9.3
4	9.6	9.6	9.6	9.7	9.8	10	9.7	9.3	9	8.8	8.5	8.4	8.3	8.2	8.2	8.2	8.2	8.4	8.5	8.8	9	9.3	9.7	10	9.7	9.5	9.3	9.1	9	8.9	8.9
5	9.3	9.3	9.4	9.6	9.8	10	9.5	9.1	8.6	8.3	8	7.8	7.6	7.5	7.5	7.5	7.6	7.8	8	8.3	8.6	9.1	9.5	10	9.6	9.3	9	8.7	8.6	8.5	8.5
6	9	9	9.1	9.4	9.7	10	9.4	8.7	8.2	7.7	7.4	7.1	6.9	6.7	6.7	6.7	6.9	7.1	7.4	7.7	8.2	8.7	9.3	10	9.5	9	8.6	8.3	8	7.9	7.9
7	8.6	8.6	8.8	9.1	9.5	10	9.1	8.4	7.7	7.1	6.6	6.3	6.1	5.9	5.9	5.9	6.1	6.3	6.6	7.1	7.7	8.4	9.1	10	9.3	8.6	8.1	7.7	7.4	7.3	7.3
8	8	8	8.3	8.7	9.3	10	8.9	7.9	7	6.4	5.8	5.4	5.2	5	4.9	5	5.2	5.4	5.8	6.3	7	7.9	8.9	10	9	8.2	7.5	7	6.7	6.5	6.5
9	7.3	7.3	7.5	8.1	8.9	10	8.5	7.2	6.2	5.5	4.9	4.4	4.1	4	3.9	4	4.1	4.4	4.9	5.4	6.2	7.2	8.5	10	8.7	7.5	6.7	6.1	5.8	5.6	5.6
10	6.2	6.2	6.5	7.2	8.3	10	7.9	6.3	5.2	4.4	3.7	3.3	3	2.8	2.8	2.8	3	3.3	3.7	4.3	5.2	6.3	7.9	10	8	6.6	5.7	5	4.6	4.5	4.5
11	4.8	4.8	5.1	5.8	7.2	10	6.8	5	3.9	3.1	2.5	2	1.7	1.6	1.5	1.6	1.7	2	2.4	3	3.8	5	6.7	10	6.9	5.3	4.3	3.7	3.3	3.2	3.2
12	[illegible]	[illegible]	3.4	3.9	4.5	5.2	4.2	3.1	2.2	1.5	1	0.6	0.3	0.2	0.1	0.2	0.3	0.6	1	1.5	2.2	3	4.1	5	4.2	3.3	2.6	2.1	1.8	1.7	1.7
13	[illegible]	[illegible]	1.4	1.7	1.9	2.1	1.6	1	0.4	-0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-0	0.3	0.9	1.5	1.9	1.6	1.2	0.7	0.4	0.1	0	0
14	[illegible]	[illegible]	-1	-1	-0	-1	-1	-1	-2	-2	-2	-3	-3	-3	-3	-3	-3	-3	-2	-2	-2	-1	-1	-1	-1	-1	-1	-1	-2	-2	-2
15	[illegible]	[illegible]	-3	-3	-3	-3	-3	-3	-4	-4	-4	-4	-4	-5	-5	-5	-4	-4	-4	-4	-4	-3	-3	-3	-3	-3	-3	-3	-4	-4	-4
16	[illegible]	[illegible]	-5	-5	-5	-5	-5	-5	-5	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
17	[illegible]	[illegible]	-7	-7	-7	-7	-7	-7	-7	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-7	-7	-7	-7	-7	-7	-7	-7	-7	-7	-7
18	[illegible]	[illegible]	-9	-9	-9	-9	-9	-9	-9	-9	-10	-10	-10	-10	-10	-10	-10	-10	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9
19	[illegible]	[illegible]	-9	-9	-9	-9	-9	-9	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9
20	[illegible]	[illegible]	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
21	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
22	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
23	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
24	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
25	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
26	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
27	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
28	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
29	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
30	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
31	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
32																															

Insulator case, since the E field is clearly very small in the green area.

Note that the two electrodes are at voltages of +10V and -10V.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
35																															
36	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
37	9.9	9.9	9.9	9.9	10	10	9.9	9.8	9.8	9.7	9.6	9.6	9.6	9.6	9.5	9.6	9.6	9.6	9.6	9.7	9.8	9.8	9.9	10	9.9	9.9	9.9	9.8	9.8	9.8	9.8
38	9.8	9.8	9.8	9.9	9.9	10	9.8	9.7	9.5	9.4	9.3	9.2	9.1	9.1	9.1	9.1	9.1	9.2	9.3	9.4	9.5	9.7	9.8	10	9.9	9.8	9.7	9.6	9.6	9.6	9.6
39	9.7	9.7	9.7	9.8	9.9	10	9.7	9.5	9.3	9.1	8.9	8.8	8.7	8.6	8.6	8.6	8.7	8.8	8.9	9.1	9.3	9.5	9.7	10	9.8	9.7	9.5	9.4	9.3	9.3	9.3
40	9.6	9.6	9.6	9.7	9.8	10	9.6	9.3	9	8.7	8.5	8.3	8.2	8.1	8.1	8.1	8.2	8.3	8.5	8.7	9	9.3	9.6	10	9.7	9.5	9.3	9.2	9.1	9	9
41	9.4	9.4	9.4	9.6	9.8	10	9.5	9.1	8.6	8.3	8	7.8	7.6	7.5	7.5	7.5	7.6	7.8	8	8.3	8.6	9.1	9.5	10	9.7	9.3	9.1	8.9	8.7	8.7	8.7
42	9.1	9.1	9.2	9.4	9.7	10	9.4	8.8	8.2	7.8	7.5	7.2	7	6.9	6.8	6.9	7	7.2	7.5	7.8	8.2	8.8	9.4	10	9.5	9.1	8.8	8.5	8.3	8.2	8.2
43	8.7	8.7	8.8	9.1	9.5	10	9.2	8.4	7.8	7.2	6.8	6.5	6.3	6.2	6.1	6.2	6.3	6.5	6.8	7.2	7.8	8.4	9.2	10	9.4	8.8	8.4	8	7.8	7.7	7.7
44	8.2	8.2	8.4	8.7	9.3	10	8.9	8	7.2	6.6	6.1	5.8	5.5	5.4	5.3	5.4	5.5	5.8	6.1	6.6	7.2	8	8.9	10	9.1	8.4	7.9	7.5	7.2	7.1	7.1
45	7.5	7.5	7.7	8.1	8.9	10	8.5	7.3	6.4	5.8	5.3	4.9	4.6	4.5	4.4	4.5	4.6	4.9	5.3	5.8	6.4	7.3	8.5	10	8.7	7.8	7.2	6.8	6.5	6.4	6.4
46	6.6	6.6	6.8	7.2	8.1	10	7.7	6.4	5.5	4.8	4.3	3.9	3.6	3.5	3.4	3.5	3.6	3.9	4.3	4.8	5.5	6.4	7.7	10	7.9	6.9	6.3	5.9	5.7	5.5	5.5
47	5.5	5.5	5.6	5.9	6.3	6.7	5.9	5	4.3	3.7	3.2	2.8	2.5	2.3	2.3	2.3	2.5	2.8	3.2	3.7	4.3	5	5.9	6.6	6.2	5.6	5.2	4.9	4.7	4.6	4.6
48	4.2	4.2	4.3	4.4	4.6	4.6	4.1	3.6	3	2.5	2	1.6	1.3	1.1	1	1.1	1.3	1.6	2	2.5	3	3.6	4.1	4.5	4.4	4.2	3.9	3.7	3.6	3.5	3.5
49	2.9	2.9	2.9	3	3	2.9	2.6	2.1	1.7	1.2	0.7	0.3	-0	-0	-0	-0	-0	0.3	0.8	1.2	1.7	2.1	2.5	2.8	2.9	2.8	2.7	2.5	2.5	2.4	2.4
50	1.5	1.5	1.5	1.5	1.5	1.3	1.1	0.7	0.3	-0	-1	-1	-1	-2	-2	-2	-1	-1	-1	-0	0.4	0.8	1.1	1.3	1.4	1.4	1.4	1.3	1.3	1.2	1.2
51	0.2	0.2	0.2	0.1	0.1	-0	-0	-1	-1	-1	-2	-2	-3	-3	-4	-3	-3	-2	-2	-1	-1	-1	-0	-0	0.1	0.1	0.1	0.1	0	0	0
52	-1	-1	-1	-1	-1	-1	-2	-2	-2	-3	-3	-4	-5	-5	-6	-5	-5	-4	-3	-3	-2	-2	-1	-1	-1	-1	-1	-1	-1	-1	-1
53	-2	-2	-2	-2	-3	-3	-3	-3	-3	-4	-4	-5	-6	-7	-10	-7	-6	-5	-4	-4	-3	-3	-3	-2	-2	-2	-2	-2	-2	-2	-2
54	-4	-4	-4	-4	-4	-4	-4	-4	-4	-5	-5	-6	-7	-8	-10	-8	-7	-6	-5	-5	-4	-4	-4	-4	-3	-3	-4	-4	-4	-4	-4
55	-5	-5	-5	-5	-5	-5	-5	-5	-5	-6	-6	-7	-8	-9	-10	-9	-8	-7	-6	-6	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
56	-6	-6	-6	-6	-6	-6	-6	-6	-6	-7	-7	-8	-8	-9	-10	-9	-8	-7	-7	-6	-6	-6	-6	-5	-6	-6	-6	-6	-6	-6	-6
57	-8	-8	-8	-8	-7	-7	-7	-7	-7	-7	-8	-8	-9	-9	-10	-9	-9	-8	-7	-7	-7	-6	-6	-6	-6	-7	-7	-7	-8	-8	-8
58	-9	-9	-10	-9	-8	-8	-8	-8	-8	-8	-8	-8	-9	-9	-10	-9	-9	-8	-8	-8	-7	-7	-7	-7	-7	-7	-8	-9	-10	-9	-9
59	-10	-10	-10	-9	-9	-8	-8	-8	-8	-8	-9	-9	-9	-10	-10	-10	-9	-9	-8	-8	-8	-8	-8	-8	-8	-8	-8	-9	-10	-10	-10
60	-10	-10	-10	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-10	-10	-10	-9	-9	-9	-8	-8	-8	-8	-8	-8	-9	-9	-9	-10	-10	-10
61	-10	-10	-10	-10	-9	-9	-9	-9	-9	-9	-9	-9	-10	-10	-10	-10	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-10	-10	-10	-10
62	-10	-10	-10	-10	-10	-9	-9	-9	-9	-9	-9	-9	-10	-10	-10	-10	-10	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-10	-10	-10	-10
63	-10	-10	-10	-10	-10	-10	-9	-9	-9	-9	-10	-10	-10	-10	-10	-10	-10	-10	-9	-9	-9	-9	-9	-9	-9	-9	-10	-10	-10	-10	-10
64	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
65	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
66	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10

Sheet1 Sheet2 Sheet3

Ready Calculate

b. Next, use the information given in the plots to estimate the charge per unit length for this structure (consider both cases). Remember that this is a two-dimensional configuration, so that the dimension of the surfaces into the page should be considered to be 1meter. You should also assume a cell spacing of 1mm.

Answer: Average charge per unit length can be determined by evaluating either the normal component of D at the upper or lower conductor or, somewhat easier, evaluate the electric flux passing through a surface in between the conductors. We will take the latter approach.

No insulator: V_{ave} in row 49 = 1.77, V_{ave} in row 50 = 0.4, voltage difference = 1.37 , electric field = 1370, from which $D = 1370\varepsilon_o$ *and this is also the average charge density, so that the charge per unit length is* $Q = 41.2\varepsilon_o$

Insulator case: V_{ave} in row 14 = -1.59, V_{ave} in row 15 = -3.55, voltage difference = 1.96 , electric field = 1960, from which $D = 1960\varepsilon_o$ *and this is also the average charge density, so that the charge per unit length is* $Q = 59\varepsilon_o$

c. Now, determine the capacitance per unit length for this structure (both cases).

Answer: For the insulator: $C = 2.95\varepsilon_o$ *and for no insulator:* $C = 2.06\varepsilon_o$

Material Properties (from Ulaby Appendices)

Dielectric Materials

Material	ε_r	Material	ε_r
Vacuum	1	Dry Soil	2.5-3.5
Air (sea level)	1.006	Plexiglas	3.4
Styrofoam	1.03	Glass	4.5-10
Teflon	2.1	Fused Quartz	3.8-5
Petroleum Oil	2.1	Bakelite	5
Wood (Dry)	1.5-4	Porcelain	5.7
Paraffin	2.2	Formica	6
Polyethylene	2.25	Mica	5.4-6
Polystyrene	2.6	Ammonia	22
Paper	2-4	Sea Water	72-80
Rubber	2.2=4.1	Distilled Water	81
Most Metals	1	All are low frequency values at room temperature (20°C)	

Conductors

Material	$\sigma\, S/m$	Material	$\sigma\, S/m$
Conductors		*Semiconductors*	
Silver	$6.2\text{x}10^{7}$	Pure Germanium	2.2
Copper	$5.8\text{x}10^{7}$	Pure Silicon	$4.4\text{x}10^{-4}$
Gold	$4.2\text{x}10^{7}$	*Insulators*	
Aluminum	$3.5\text{x}10^{7}$	Wet Soil	10^{-2}
Tungsten	$1.8\text{x}10^{7}$	Fresh Water	10^{-3}
Zinc	$1.7\text{x}10^{7}$	Distilled Water	10^{-4}
Brass	$1.5\text{x}10^{7}$	Dry Soil	10^{-4}
Iron	$1\text{x}10^{7}$	Glass	10^{-12}
Bronze	$1\text{x}10^{7}$	Hard Rubber	10^{-15}
Tin	$0.9\text{x}10^{7}$	Paraffin	10^{-15}
Lead	$0.5\text{x}10^{7}$	Mica	10^{-15}
Mercury	$0.1\text{x}10^{7}$	Fused Quartz	10^{-17}
Carbon	$3\text{x}10^{4}$	Glass	10^{-17}
Sea Water	4		
Animal Body	0.3	All are low frequency values at room temperature (20°C)	

Magnetic Materials

Material	μ_r	Material	μ_r
Diamagnetic		Titanium	$1.0002 \approx 1$
Bismuth	$0.99983 \approx 1$	Platinum	$1.0003 \approx 1$
Gold	$0.99996 \approx 1$	*Ferromagnetic*	*Nonlinear*
Mercury	$0.99997 \approx 1$	Cobalt	250
Silver	$0.99998 \approx 1$	Nickel	600
Copper	$0.99999 \approx 1$	Mild Steel	2000
Water	$0.99999 \approx 1$	Iron (Pure)	4000-5000
Paramagnetic		Silicon Iron	7000
Air	$1.000004 \approx 1$	Mu Metal	~100,000
Aluminum	$1.00002 \approx 1$	Purified Iron	~200,000
Tungsten	$1.00008 \approx 1$	Only typical values, actual values depend on material variety	