Name $\qquad$ Solution

Section $\qquad$ Both

## Short Answer Questions



1. (18 Pts) $\qquad$
2. (15 Pts) $\qquad$
3. (12 Pts) $\qquad$
4. (5 Pts) $\qquad$

## Regular Questions

5. (30 Pts) $\qquad$
6. (20 Pts) $\qquad$

Total

Notes:

1. Please read over all questions before you begin your work. There may be some information in a later question that helps you with an earlier question.
2. For short answer questions, you may add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians \& not degrees \& use 4 significant digits.


MULTIPLE CHOICE AND SHORT ANSWER QUESTIONS

1. Input Impedance of Lossless Transmission Lines (18 points)


Assume a sinusoidal source is connected to a lossless transmission line, as shown.
a. (6 pts) The transmission line load is an open circuit.
i. For what line lengths will the input impedance observed at the sending point end also be an open circuit? Circle all correct answers.

ii. For what line lengths will the input impedance observed at the sending point end be a short circuit? Circle all correct answers.
$0 \lambda / 8(\lambda / 43$
$3 \lambda / 8 \quad \lambda / 2$

$7 \lambda / 8$
$\lambda \quad 9 \lambda / 8$
(5 $1 / 411 \lambda / 8 \quad 3 \lambda / 2$
b. ( 6 pts ) A transmission line has length $\lambda / 4$, characteristic impedance $Z_{o}=100 \Omega$ and load $Z_{L}=j 100 \Omega$. What is the input impedance?
$Z_{I N}=\frac{Z_{o}{ }^{2}}{Z_{L}}=\frac{100^{2}}{j 100}=-j 100$
c. (6 pts) What is the input impedance of the network below? $Z_{\text {IN }}=$


At $B, Z_{\text {IN }}=Z_{o}=100 \Omega$ because the second line is matched. At A, $Z_{I N}=Z_{L}=100 \Omega$ because the first line is a multiple of a half wavelength.

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## 2. Lossy Transmission Line (15 points)

A sinusoidal voltage wave is propagating on a low loss transmission line. The voltage as a function of position appears as shown below. From this plot, determine the damping coefficient $\alpha=0.0026$ and the propagation constant $\beta=0.0236$


The red line in the figure shows the distance equal to $1.5 \lambda=400$ so that $\lambda=\frac{800}{3}=266.7$
$\beta=\frac{2 \pi}{\lambda}=\frac{3 \pi}{400}=0.0236$ and $100 e^{-\alpha 200}=0.6$ or $\alpha=0.0026$
If the measurements above were taken at in a transmission line with a propagation constant of $70 \%$ and capacitance per unit length of $31.75 \mathrm{pF} / \mathrm{m}$, what are the characteristic impedance, the inductance per unit length and the resistance per unit length of the line?
$u=0.7 \times 3 \times 10^{8}=2.1 \times 10^{8}=\frac{1}{\sqrt{l c}} Z_{o}=\sqrt{\frac{l}{c}}$ or $c=\frac{1}{u Z_{o}}$ or $Z_{o}=\frac{1}{u c}=150$
$l=c Z_{o}{ }^{2}=0.7 \mu H / m \quad r=2 Z_{o} \alpha=0.77 \Omega / m$


## 3. Standing Waves (12 points)

For all of the following questions, assume that a lossless transmission line has a characteristic impedance of $Z_{o}=300 \Omega$.


B

D


|  | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $Z_{L}$ | 300 | $\infty$ | 0 | 60 | 1500 |
| $\Gamma_{L}$ | 0 | 1 | -1 | $-2 / 3$ | $2 / 3$ |
| $S W R$ | 1 | $\infty$ | $\infty$ | 5 | 5 |

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a. (10 pts) For each of the five standing wave plots shown above, determine the load impedance $Z_{L}$, the reflection coefficient $\Gamma_{L}$ and the standing wave ratio $S W R$. Place the values you determine in the table.
b. (2 pts) Which of the standing wave plots looks like the current standing wave pattern for a short circuit load?

B (open circuit V)
Which of the standing wave plots looks like the current standing wave pattern for an open circuit load?

C (short circuit V)

## 4. Cultural Question (5 points)

What is the name of the organization that awarded the badge shown below for telegraphy in 1925? Hint: It is not the Boy Scouts. Ans: Girl Scouts


## REGULAR QUESTIONS

## 5. Sinusoidal Voltages on a Lossless Transmission Line (30 points)

Two identical, unidentified lossless transmission lines are connected in series, as shown below. A series of experiments is performed to determine their properties. For these experiments, both $R 1$ and $R 2$ are 50 Ohms.

a. The time delay observed for each of the lines is observed to be 200ns. The voltage at the input and output of each of the two lines is measured at frequencies from 1 MHz to 15 MHz . In the plot below, the top curve is measured at the input to T 2 , the middle curve at the input to T1 and the bottom curve at the load. Six peaks (maxima) are observed in the voltage at the input to T 2 at the following frequencies: $1.25 \mathrm{MHz}, 3.75 \mathrm{MHz}, 6.25 \mathrm{MHz}, 8.75 \mathrm{MHz}, 11.25 \mathrm{MHz}$, and 13.75 MHz . Six valleys (minima) are also observed at $2.5 \mathrm{MHz}, 5 \mathrm{MHz}, 7.5 \mathrm{MHz}, 10 \mathrm{MHz}$,
12.5 MHz , and 15 MHz . These frequencies and an enlarged voltage scale have been added on the right side of the figure for clarity.


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From the measurements above, determine the characteristic impedance of the two identical lines and the reflection coefficient at the load $\Gamma_{L}$. (5 pts)

When the length of the combination of both lines is equal to an odd multiple of quarter wavelengths, we have $Z_{I N}=\frac{Z_{o}{ }^{2}}{Z_{L}}=\frac{Z_{o}{ }^{2}}{50}$

From the voltage divider action at the input, $10 \frac{Z_{I N}}{Z_{I N}+50} 8$ where the latter voltage is read from the plot at 3.125 MHz . Combining these expressions gives us $Z_{o}=100 \Omega$ The frequencies where this happens are between those listed.

The reflection coefficient is then $\Gamma_{L}=\frac{50-100}{50+100}=-\frac{1}{3}$

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b. Now determine the total capacitance and total inductance for each of the two identical lines. (5 pts)

The time delay is $t_{d}=\frac{d}{u}$ and the total capacitance divided by the time delay is $\frac{C}{t_{d}}=\frac{t_{d} d}{d} u=c u=\frac{1}{Z_{o}}$ The total capacitance is then $C=\frac{t_{d}}{Z_{o}}=2 n F$ The total inductance is $L=Z_{o}{ }^{2} C=20 \mu H$
c. Assume that the one fact that is known about the lines is their insulating material - polyethylene. ( See Ref below from Picwire) Using the velocity factor for polyethylene, determine the velocity of propagation and the length of each identical cable. (5 pts)

The velocity of propagation is
$u=\frac{2}{3} 3 \times 10^{8}=2 \times 10^{8} \mathrm{~m} /$ s and the
length of each cable is
$d=u t_{d}=40 \mathrm{~m}$

| MATERIAL | @ 1.0 GHz | VOP | Delay |
| :---: | :---: | :---: | :---: |
| Typical Insulation Materials |  |  |  |
| Cellular TFE | 1.38 | 85\% | 1.2 |
| FEP | 2.1 | 69\% | 1.47 |
| Silicone Rubber | 3.6-2.1 | 53-69\% | 1.92-1.47 |
| TFE | 2.1 | 69\% | 1.47 |
| Polyethylene | 2.3 | 66\% | 1.55 |
| PVC | 8.2-3.0 | 5-58\% | 2.9-1.75 |
| Nyton | 4.5-3.6 | 47-53\% | 2.16-1.92 |
| Non-Typical Insulation Materiais* |  |  |  |
| Snow (Fresh) | 1.2 | 91\% | 1.1 |
| Vaseline | 2.2 | 68\% | 1.49 |
| Beeswax | 2.8 | 60\% | 1.69 |
| ICe | 3.2 | 56\% | 1.8 |
| Glass | 8.2-3.8 | 35-51\% | 2.9-2.0 |
| Water (Distilled) | 82 | 11\% | 9.2 |
| *Theoretical values, if cables were constructed of these materiats |  |  |  |

Table 1. Electrical parameters of various materials.
d. Write the general expression for the input impedance seen by the source at the input to T1, simplifying it as much as possible. Then evaluate it at both $f=3.75 \mathrm{MHz}$ and $f=5 \mathrm{MHz}$. ( 5 pts )
$Z_{\text {IN }}=\frac{50+j 100 \tan \omega t_{d}}{100+j 50 \tan \omega t_{d}}=50 \Omega$ for both frequencies, because the lines are either $\frac{\lambda}{2}$ or $\frac{\lambda}{4}$ long and thus their combination will always be a multiple of $\frac{\lambda}{2}$.
e. What is the voltage at the input to T 1 and the output to T 2 at these two frequencies? Using this information, determine the average power delivered to the input of T1 and to the load. (5 pts)

At both frequencies, the lines are matched, so the input voltage is 5 V . The average power is therefore $P=\frac{5^{2}}{2(50)}=250 \mathrm{~mW}$
f. Find the propagation constant $\beta$ and wavelength $\lambda$ for the two frequencies. (5 pts)
$\beta=\frac{2 \pi f}{u}$ or $\beta=\frac{2 \pi 3.75 \times 10^{6}}{2 \times 10^{8}}=.0375 \pi$ and $\beta=\frac{2 \pi 5 \times 10^{6}}{2 \times 10^{8}}=.05 \pi$
$\lambda=\frac{2 \pi}{\beta}$ or $\lambda=\frac{2 \pi}{.0375 \pi}=53.33$ and $\lambda=\frac{2 \pi}{.05 \pi}=40$

Checking for each line, does $\beta d=\omega t_{d}$ ? For the latter case $\beta d=.05 \pi 40=2 \pi\left(5 \times 10^{6}\right)(200 n s)=\omega t_{d}$ which checks.

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6. Pulses on Transmission Lines ( 20 pts )


Using the same configuration as in the previous problem, a 10 V , 50 ns pulse was input to line $T 3$ and measured at the input to $T 4$, with the result shown below. (These are the same lines as in the previous problem, but have different names to keep PSpice happy.) Also, $R 3$ and $R 4$ are the same.


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a. Using the information in this plot, determine the characteristic impedance of the two identical lines and then fill in the bounce diagram for the complete configuration. Use the information on the velocity of propagation from the previous problem. (10 pts)

a. Determine and plot the voltages at the input to $T 3$ and at the load, as functions of time. (10 pts)


The green pulses are at the input and the red at the output. The first green pulse is from the voltage divider and thus is 6.67 V . The second green pulse is equal to the incident pulse (the first one) times $\left(\Gamma_{L}\left(1+\Gamma_{g}\right)\right)$ and thus is equal to -1.5 V . The first red pulse (at the load) is equal to the incident pulse times $\left(1+\Gamma_{L}\right)$ and thus is equal to 4.44 V . The second red pulse is equal to the incident pulse times $\Gamma_{L} \Gamma_{g}\left(1+\Gamma_{L}\right)$ and so on $\ldots$.

