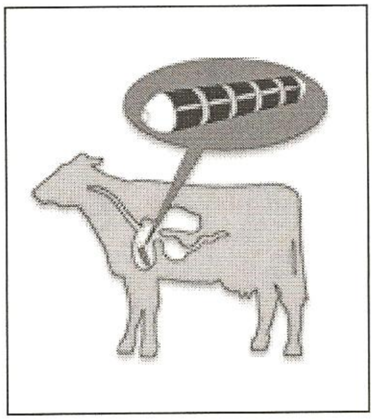
Rensselaer | EE EPE CSE

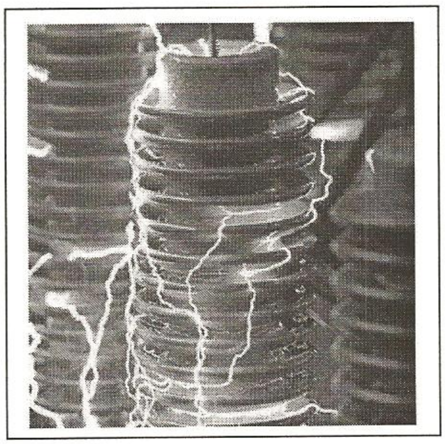
Department of Electrical, Computer, & Systems Engineering

Quiz 2
Fall 2009

Name Solution

Section ______





1. (12 Pts) ______

2. (12 Pts) ______

3. (5 Pts) ______

4. (26 Pts) ______

5. (20 Pts) ______

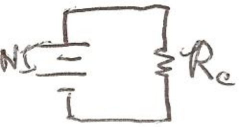
6. (25 Pts) ______

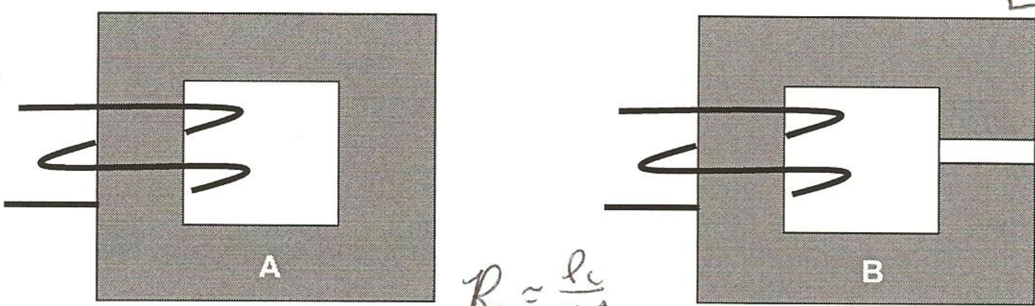
Total ______

Notes:
1. **Please read over all questions before you begin your work. There may be some information in a later question that helps you with an earlier question.**
2. For short answer questions, you may add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees & use 4 significant digits.

MULTIPLE CHOICE AND SHORT ANSWER QUESTIONS

1. Magnetic Fields (18 points)





$\Phi_m = \frac{NI}{R}$

$R_c = \frac{l_c}{\mu A}$ $\qquad R_c \simeq \frac{l_c}{\mu A}$ $\qquad R_g = \frac{l_g}{\mu_o A}$

$L = \frac{N\Phi_m}{I} = \frac{N^2 \cancel{I}}{\cancel{I} R} = \frac{N^2}{R}$

A square magnetic core is made with and without a gap, as shown. Both cores are made with a standard magnetic material such as soft iron and all dimensions and windings are identical, except for the gap. The saturation field for the core material is B_{max}. The effective length of the core in both cases is approximately l_c and the gap size is l_g. The permeability of the core $\mu >> \mu_o$ so that $\frac{l_c}{\mu} << \frac{l_g}{\mu_o}$. The cross-sectional area of the core materials is A.

a. (4 pts) Which of the two cases will have the larger inductance L?

$$L_c = \mu \frac{N^2 A}{l_c} > L_g \simeq \frac{\mu_o N^2 A}{l_g}$$

b. (4 pts) Which of the two cases can store the largest energy?

$$\frac{1}{2} L_c I_c^2 < \frac{1}{2} L_g I_g^2$$

c. (4 pts) Which of the following statements are true? Circle the letters of the true statements.

i. The force on the gap is in the direction to reduce the gap spacing l_g.
ii. The force on the gap is inversely proportional to the area of the gap.
iii. The force on the gap is inversely proportional to the gap spacing l_g.
Both OK iv. The force on the gap is proportional to B_{max}

This question is ambiguous since we may not be able to choose B_{max} ⇒ either is OK

Pressure is $\frac{B^2}{2\mu_o}$

For upper right $\quad B_g = \frac{\mu_o N I_g}{l_g} \qquad B_c = \frac{\mu N I_c}{l_c}$

$$I_g = \frac{l_g B_g}{\mu_o N} \qquad I_c = \frac{l_c B_c}{\mu N}$$

↳ Current for B_{max} goes as R.

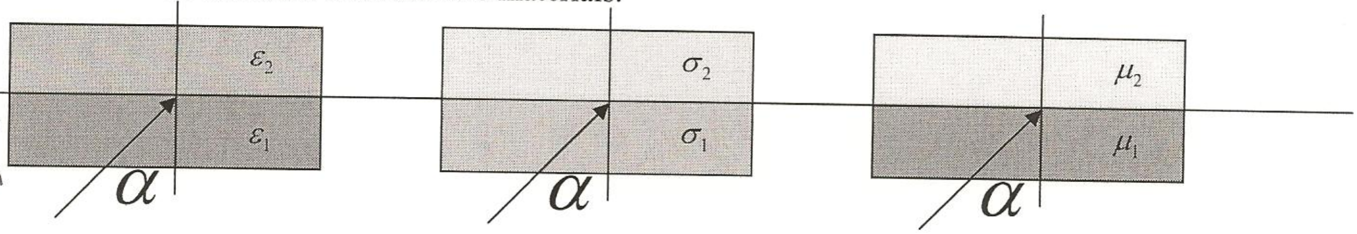
2. Boundary Conditions (12 points)

The boundary conditions for E fields, B fields and current density J are essentially identical at the interface between two materials.

$D_{1n} = D_{2n}$

$E_{1t} = E_{2t}$

$\tan\alpha = \dfrac{E_{1t}}{E_{1n}}$



Consider the three cases shown above with $\varepsilon_1 > \varepsilon_2$, $\sigma_1 >>> \sigma_2$ and $\mu_1 >> \mu_2$.

$\tan\alpha = \dfrac{E_{2t}}{D_{1n}/\varepsilon_1} = \dfrac{E_{2t}\,\varepsilon_1}{E_{2n}\,\varepsilon_2} = \tan\beta \dfrac{\varepsilon_1}{\varepsilon_2}$

a. (6 points) Given that the angle is α in the lower region in each case, determine the corresponding angle β (relative to the normal) in the upper region.

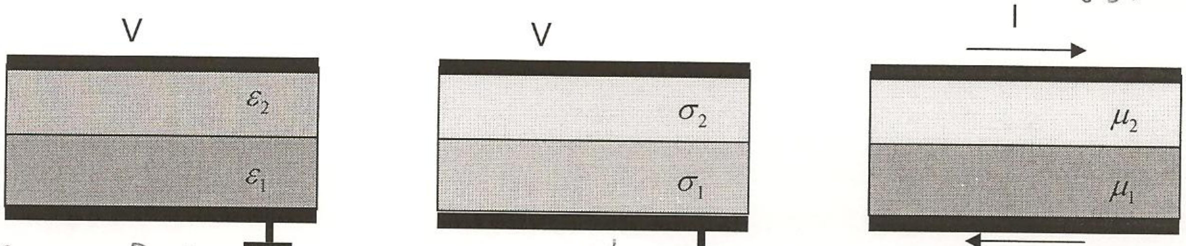
$\tan\beta = \tan\alpha \dfrac{\varepsilon_2}{\varepsilon_1}$

$\tan\beta = \tan\alpha \dfrac{\sigma_2}{\sigma_1}$

$\tan\beta = \tan\alpha \dfrac{\mu_2}{\mu_1}$

b. (6 points) Now assume the two materials, for each case, are found between conducting contacts at the top and the bottom as shown. Determine the capacitance, resistance and inductance. Note the voltages and currents for each case. The inductor case has currents directed right and left. The width and depth of the plates is *w*, and the separation between the plates is *d*.

Use W_m Method



Two Caps in Series

$\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{d/2}{\varepsilon W^2} + \dfrac{d/2}{\varepsilon_2 W^2}$

Two Res in series

$R = \dfrac{l}{\sigma A} = \dfrac{d/2}{\sigma_2 W^2} + \dfrac{d/2}{\sigma_1 W^2}$

$H = \dfrac{I}{W} = J_s$

$B_2 = \dfrac{\mu_2 I}{W} \quad B_1 = \dfrac{\mu_1 I}{W}$

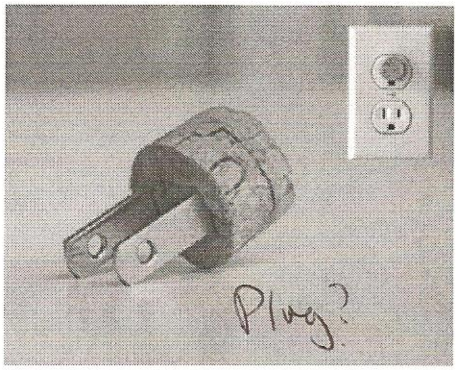
$\frac{1}{2} L I^2 = \left(\frac{1}{2}\dfrac{\mu_1 I^2}{W^2} + \frac{1}{2}\dfrac{\mu_2 I^2}{W^2}\right) W^2 \frac{d}{2}$

Vol

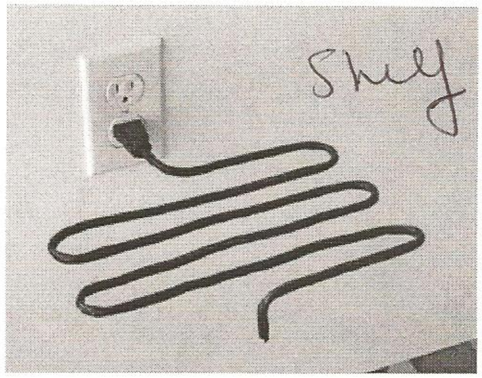
$L = (\mu_1 + \mu_2)\dfrac{d}{2}$

Note: Check Table 2-1 in Ulaby

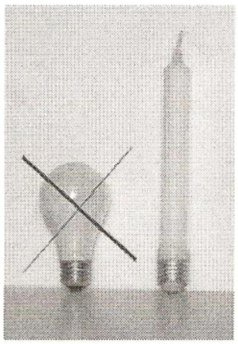
3. Cultural Question (5 points)



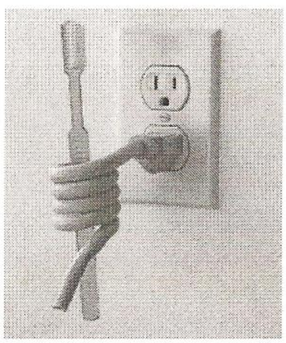
Plug?



Shelf



Candle lamp



These objects were created by Scott Amron a couple of years ago. Explain what he had in mind in each case. (source: http://www.dielectric.org)

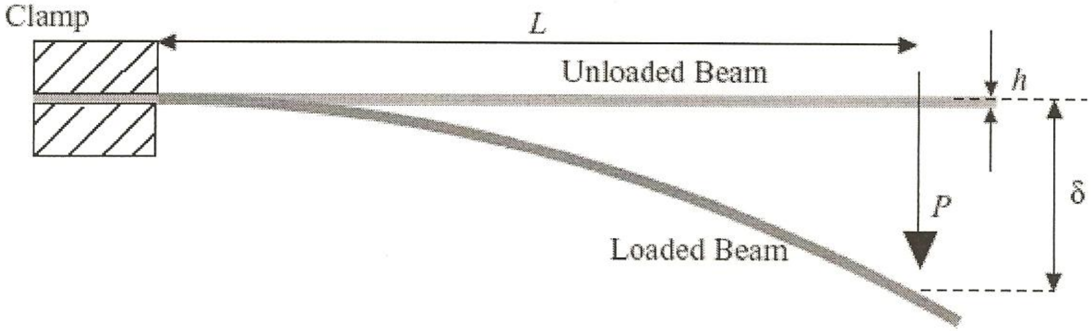
Toothbrush Holder

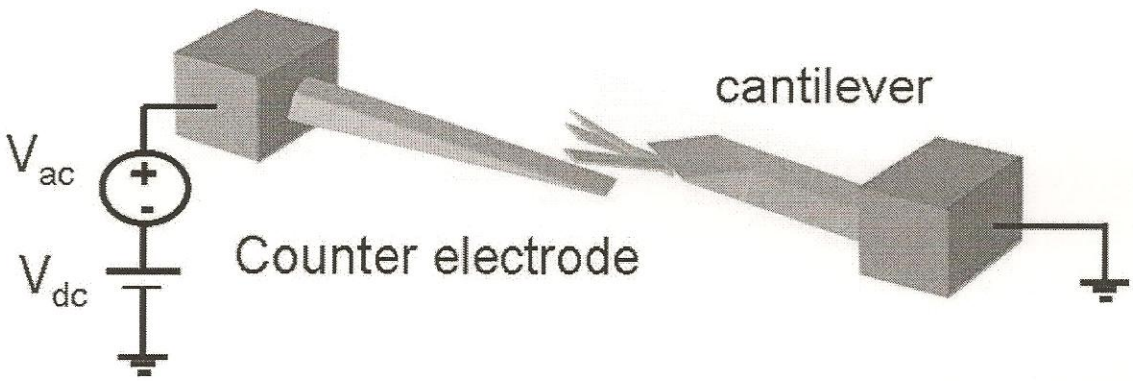
Uses for things with no power.

Anything OK

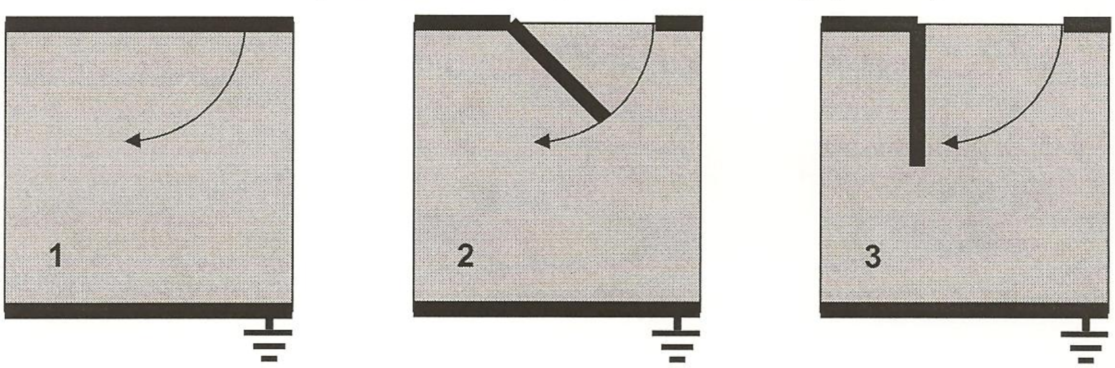
4. **Electric Field and Capacitance** (24 points)

This problem addresses several aspects of electric fields in the context of the small cantilever beams used in accelerometers. A generic cantilever beam is shown below along with a diagram showing how the capacitance of the beam can be used to measure its motion.





The problem considered here is inspired by these figures, but the configuration is rather different. Assume that there is an approximately parallel plate capacitor structure that has a movable element in it (like the cantilever), with the following geometry.



Note that a section of the top electrode opens up, which mimics the changes that occur when the top electrode is a cantilever beam. A DC voltage is applied to the top electrode. The insulator is air (or the beam would not be able to move) and the total dimensions of the structure are *0.3mm* by *0.3mm*.

$.3mm = .0003 m$

divide by 30 cells

$\Rightarrow \frac{.0003}{30} = .00001 m$

A spreadsheet is used to determine the potentials for these three configurations. The arrays for each case are shown on the following pages and the voltages for the cells just inside the left, right, and bottom boundaries are listed, along with their averages. (Labeled A, B and C)

a. (5 points) Before using the spreadsheet data to find the capacitance, first assume the capacitor is an ideal parallel plate configuration and find its capacitance per unit length from its dimensions and insulator properties.

$$C = \frac{\varepsilon_o A}{d} = \frac{\varepsilon_o (.3\,mm)}{.3\,mm} = \varepsilon_o$$

b. (12 points) Now, use the information from the spreadsheet solution to determine the capacitance for each of the three cases, C_1, C_2 & C_3.

C_1 V_{ave} on bottom $= .333$

$$E_{ave} = \frac{.333}{10^{-5}} = 3.3 \times 10^{4} \frac{V}{m}$$

$$D = \varepsilon_o E = 3.3 \times 10^{4}\, \varepsilon_o$$

$$\rho_s = D$$

$$Q = (.0003\,m)(3.3 \times 10^{4})\,\varepsilon_o$$

$$= 10\,\varepsilon_o$$

$$C = \frac{10\,\varepsilon_o}{10_{\text{voltage}}} = \varepsilon_o$$

(See also attached spreadsheet)

C_2 $V_{ave} = .405$

$$\Rightarrow C_2 = \frac{4.05}{3.33} C_1$$

$$= 1.21\,\varepsilon_o$$

C_3 $V_{ave} = .424$

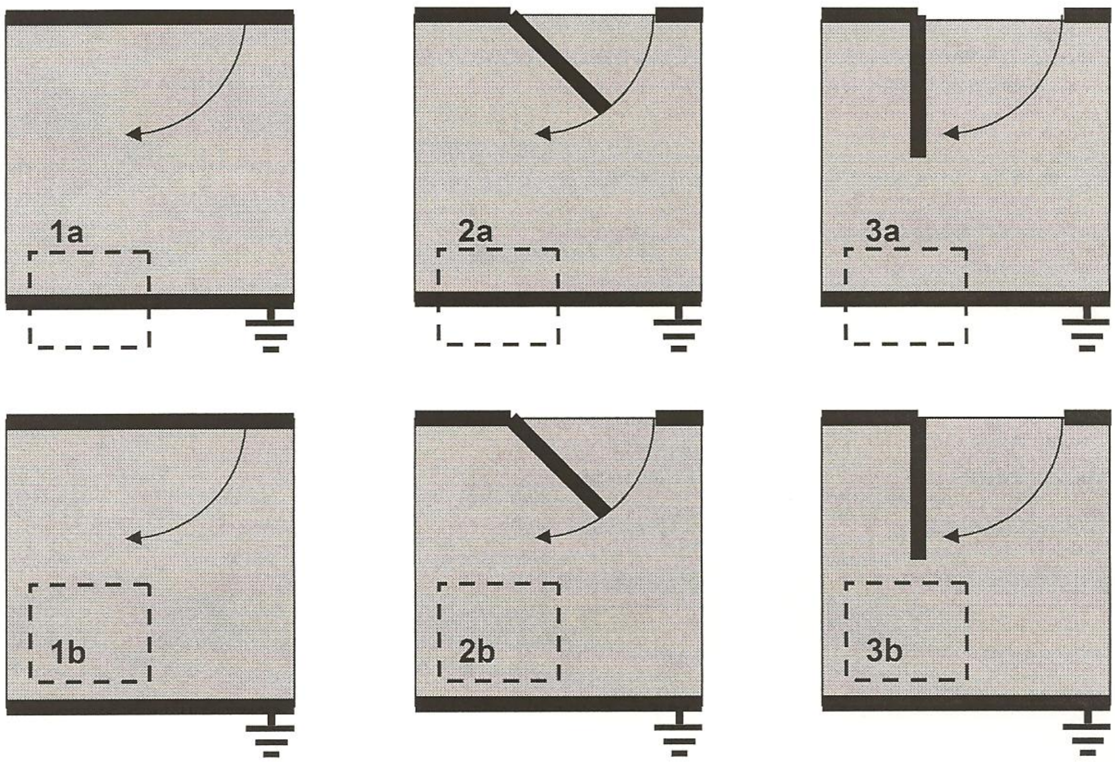
$$\Rightarrow C_3 = \frac{4.24}{3.33} C_1$$

$$= 1.27\,\varepsilon_o$$

c. (4 points) Discuss the differences between the three capacitances. That is, which one is largest and why, etc.

The fully open case is the largest capacitance because it puts the average position of the top electrode closest to the bottom.

d. Gauss' Law (3 points) For the three configurations, two Gaussian surfaces (shown dashed) are used to evaluate the flux integral from Gauss' Law $\oint \vec{D} \cdot d\vec{S}$.

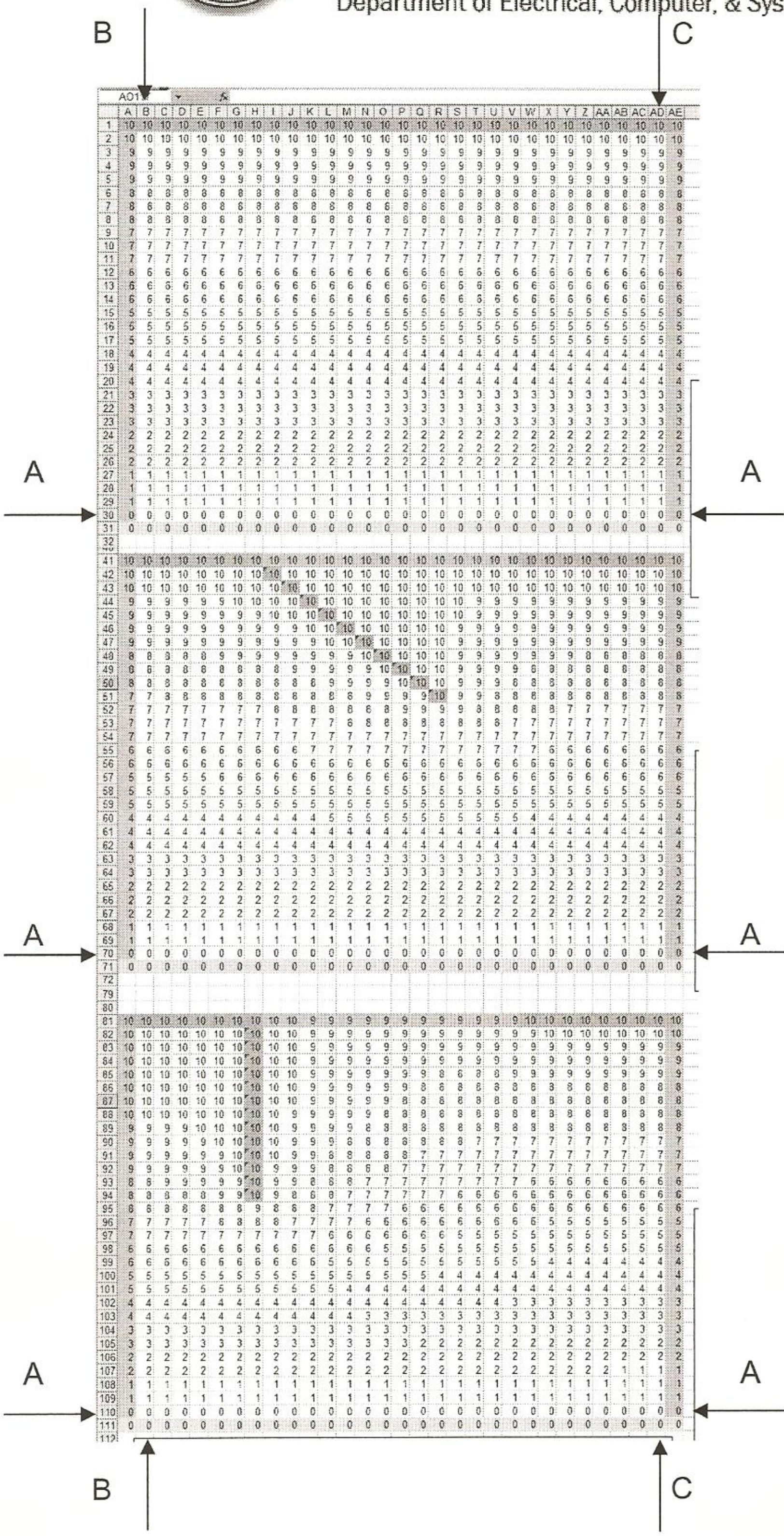


For the six distinct cases, put the flux integrals in order from largest to smallest. If some are equal, indicate that too.

3a	2a	1a	1b	2b	3b

1b, 2b, 3b: all = 0

Charge density is higher in the larger Cap case.



A	B	C
0.333333	10	10
0.333333	9.666667	9.666667
0.333333	9.333333	9.333333
0.333333	9	9
0.333333	8.666667	8.666667
0.333333	8.333333	8.333333
0.333333	8	8
0.333333	7.666667	7.666667
0.333333	7.333333	7.333333
0.333333	7	7
0.333333	6.666667	6.666667
0.333333	6.333333	6.333333
0.333333	6	6
0.333333	5.666667	5.666667
0.333333	5.333333	5.333333
0.333333	5	5
0.333333	4.666667	4.666667
0.333333	4.333333	4.333333
0.333333	4	4
0.333333	3.666667	3.666667
0.333333	3.333333	3.333333
0.333333	3	3
0.333333	2.666667	2.666667
0.333333	2.333333	2.333333
0.333333	2	2
0.333333	1.666667	1.666667
0.333333	1.333333	1.333333
0.333333	1	1
0.333333	0.666667	0.666667
0.333333	0.333333	0.333333
0.333333	0	0
0.333333	5	5 Ave

The data along the lower boundary is shown in column A; the left boundary in column B; the right boundary in column C. This permits the full set of significant digits to be seen.

4.

A	B	C
0.398949	10	10
0.398949	9.789456	9.776112
0.399166	9.575505	9.551006
0.399593	9.355093	9.323297
0.400216	9.125737	9.091371
0.401011	8.885584	8.853436
0.401952	8.633384	8.607649
0.403004	8.368413	8.352253
0.404128	8.090392	8.085724
0.405279	7.799408	7.806892
0.406413	7.495836	7.515022
0.407482	7.180285	7.20986
0.408442	6.853529	6.891617
0.409252	6.516457	6.560915
0.409877	6.170027	6.218688
0.410291	5.815217	5.86607
0.410479	5.452996	5.504297
0.410437	5.084294	5.134608
0.410173	4.709981	4.758191
0.409709	4.330857	4.376131
0.409075	3.947644	3.989399
0.408311	3.560985	3.598836
0.407463	3.17145	3.205165
0.406581	2.779534	2.808994
0.405714	2.385672	2.410836
0.404909	1.990243	2.011118
0.404209	1.593579	1.610199
0.403651	1.19597	1.208382
0.403262	0.79768	0.805927
0.403063	0.398949	0.403063
0.403063	0	0
0.405294	5.517876	5.533389

Ave

Half open case

A	B	C
0.478175	10	10
0.478175	9.9557	9.683055
0.477591	9.908847	9.367896
0.476415	9.85675	9.055805
0.474636	9.796441	8.747306
0.472241	9.72453	8.442199
0.469227	9.637062	8.139735
0.465602	9.529393	7.838843
0.461394	9.396116	7.538318
0.456654	9.231114	7.236961
0.45145	9.027844	6.933669
0.445872	8.780022	6.627485
0.440021	8.482868	6.317623
0.434004	8.13484	6.003468
0.42793	7.739262	5.684576
0.421902	7.304356	5.360657
0.416014	6.840567	5.031565
0.410348	6.357865	4.697276
0.404974	5.864307	4.357874
0.399948	5.365724	4.013532
0.395312	4.865991	3.664499
0.3911	4.367477	3.311082
0.387334	3.871488	2.953636
0.384028	3.37861	2.592553
0.381191	2.888964	2.22825
0.378828	2.402377	1.861164
0.376939	1.918494	1.491745
0.375524	1.436855	1.120453
0.374582	0.956935	0.747753
0.374112	0.478175	0.374112
0.374112	0	0
0.424375	6.370935	5.207196

Ave

Fully open case

5. Gauss' Law and E Fields (20 points)

A cylindrical charge distribution is given by $\rho_v(r) = \rho_{vo}$ for $0 \le r \le a$; $\rho_v(r) = -\rho_{vo}$ for $a \le r \le 2a$; the regions $2a \le r < 3a$ and $3a < r < \infty$ are charge free; and there is a surface charge density at $r = 3a$ so that the total charge per unit length is zero.

a. (8 points) Determine the total charge per unit length Q_1 in the region $0 \le r \le a$; the total charge per unit length Q_2 in the region $a \le r \le 2a$; and the surface charge density ρ_S at $r = 3a$.

$$Q_1 = \rho_{vo} \pi a^2 \qquad Q_2 = -\rho_{vo}\left(\pi (2a)^2 - \pi a^2\right) = -\rho_{vo}(3\pi a^2)$$

$$\rho_s = \frac{\rho_{vo}\pi a^2 - 3\pi a^2 \rho_{vo}}{2\pi(3a)} = \frac{-2\pi a^2 \rho_{vo}}{2\pi 3a} = -\frac{a}{3}\rho_{vo}$$

b. (8 points) Determine the electric flux density $\vec{D}(r)$ for all values of r.

$$D_r 2\pi r = Q_{encl}$$

for $0 \le r \le a$ $\quad D_r = \dfrac{\rho_{vo}\pi r^2}{2\pi r} = \rho_{vo}\dfrac{r}{2}$

$a \le r \le 2a$ $\quad D_r = 2\dfrac{\rho_{vo}\pi a^2}{2\pi r} - \dfrac{\rho_{vo}\pi r^2}{2\pi r} = \dfrac{\rho_{vo}a^2}{r} - \dfrac{\rho_{vo} r}{2}$

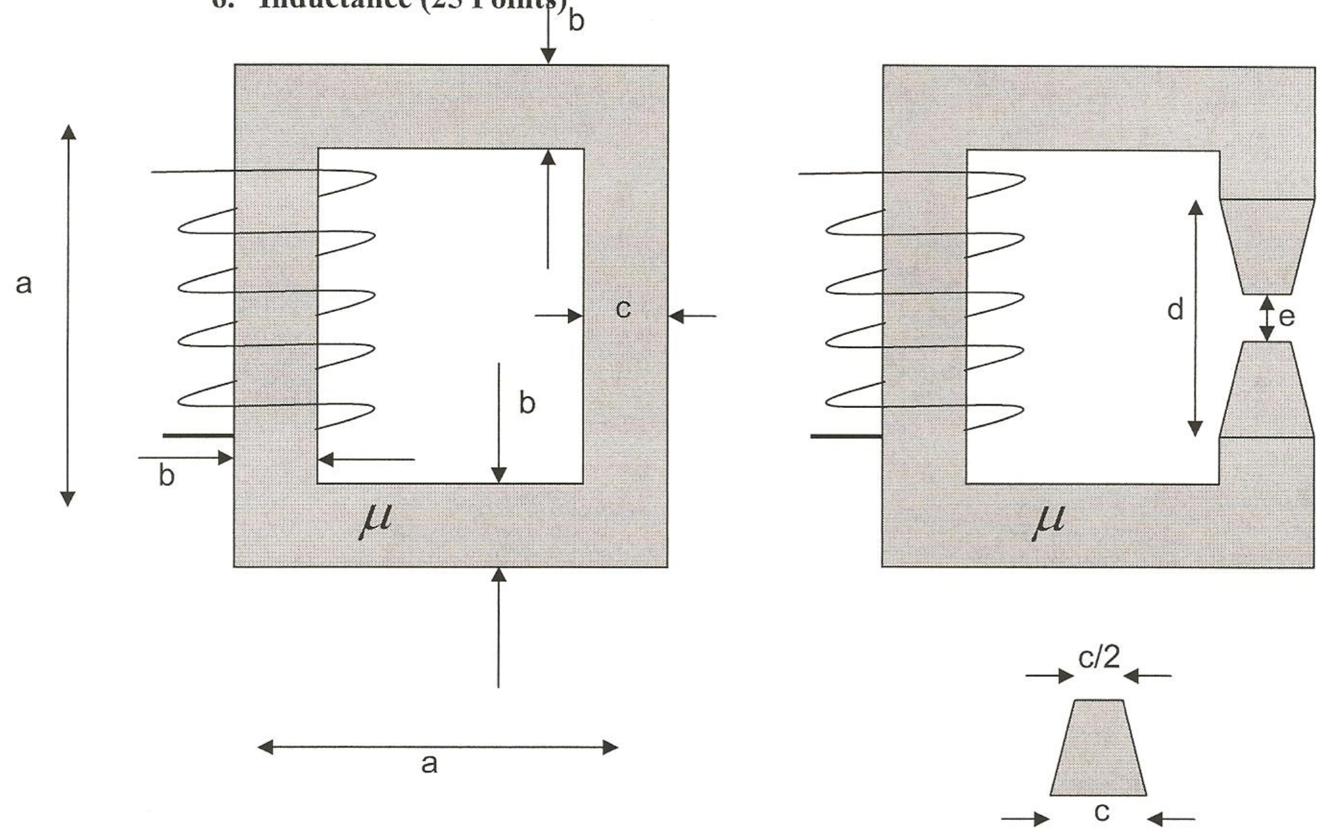
$2a \le r \le 3a$ $\quad D_r = -\dfrac{2\pi a^2 \rho_{vo}}{2\pi r} = -\dfrac{a^2\rho_{vo}}{r}$

$3a \le r$ $\quad D_r = 0$

c. (4 points) Evaluate the electric field $\vec{E}(r) = \vec{E}(3a)$ at $r = 3a$.

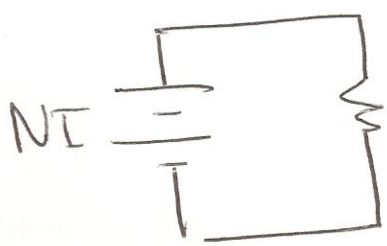
$$E(3a) = \frac{-a^2\rho_{vo}}{\varepsilon_0 3a} = \frac{-a\rho_{vo}}{3\varepsilon_0}$$

6. Inductance (25 Points)



Consider the two cores shown above. For both cases, the depth into the page of each part of the core is equal to *w*. For the one at the left, three legs have the same thickness *b* with the fourth having a thickness *c*. For the one at the right, again, three legs have the same thickness *b*. However, the 4th leg is more complex.

a. (5 points) Determine the reluctance, the flux and the inductance of the configuration at the left. Also, draw the magnetic circuit diagram.



$$R = \frac{3a}{\mu b w} + \frac{a}{\mu c w}$$

$$\phi_m = \frac{NI}{R} \qquad L = \frac{N\phi_m}{I} = \frac{N^2}{R}$$

b. (5 points) For the configuration at the right, let is do this in more of a step-by-step process. First determine the reluctance of the core with the section of length *d* removed.

$$\mathcal{R}_{core} = \frac{3a}{\mu b w} + \frac{a-d}{\mu c w}$$

c. (5 points) Next determine the reluctance of the air gap.

$$\mathcal{R}_{gap} = \frac{e}{\mu_0 \frac{c}{2} w}$$

d. (5 points) Now, the more difficult task, determine the reluctance of the tapered cap, which is shown again below the main figure to make the dimensions clearer. Double this value to get the total reluctance of the caps.

Two methods ① Assume average width is $3c/4$ & $x_0 = \frac{d-e}{2}$

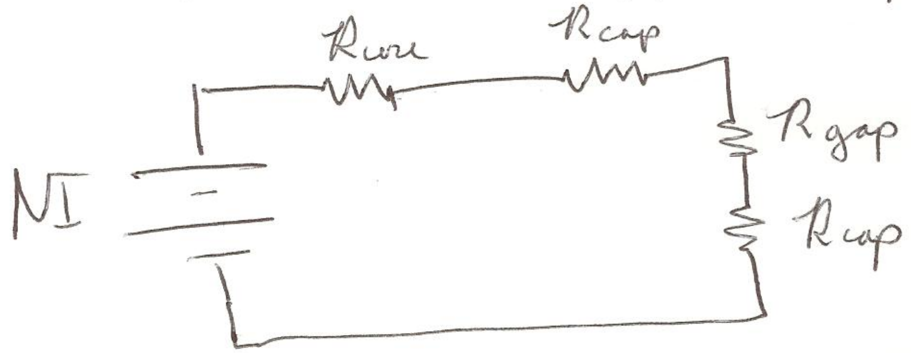
$$\Rightarrow \mathcal{R}_{cap} \approx \frac{x_0}{\mu \frac{3c}{4} w} = \frac{4}{3}\frac{x_0}{\mu c w}$$

This is not exact but OK since we are also neglecting fringing

Next Page for complete integral.

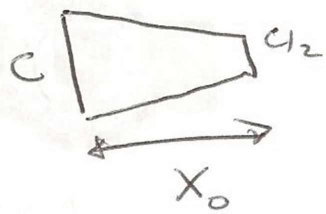
e. (5 points) Finally, determine the total reluctance, the flux, and the inductance of this configuration and draw the magnetic circuit diagram.

$$\mathcal{R} = \mathcal{R}_{core} + \mathcal{R}_{gap} + \mathcal{R}_{cap} + \mathcal{R}_{cap}$$



$$\Phi_m = \frac{NI}{\mathcal{R}}$$

$$L = \frac{N^2}{\mathcal{R}}$$



$$R_{cap} = \int \frac{dl}{\mu A(l)}$$

$$= \int_0^{x_0} \frac{dx}{\mu w \left(c - \frac{c}{2}\frac{x}{x_0}\right)}$$

$$= \frac{x_0}{\mu w c} \int_0^{x_0} \frac{dx}{x_0 - \frac{x}{2}}$$

let $y = x_0 - \frac{x}{2}$

$dy = -\frac{1}{2}dx$

$\Rightarrow dx = -2\,dy$

$$R_{cap} = \frac{-2x_0}{\mu w c} \int_{x_0}^{x_0/2} \frac{dy}{y}$$

$$= \frac{\cancel{x_0}}{\mu w c} 2 \ln 2 = \frac{x_0}{\mu w c} 1.38$$

Very similar to $1.33 \frac{x_0}{\mu w c}$

from approx result