Fields and Waves I

Lecture 3

Input Impedance on Transmission Lines

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- Jeffrey Braunstein – Chung-Ang University, Seoul, Korea

Materials from other sources are referenced where they are used.
Those listed as Ulaby are figures from Ulaby's textbook.

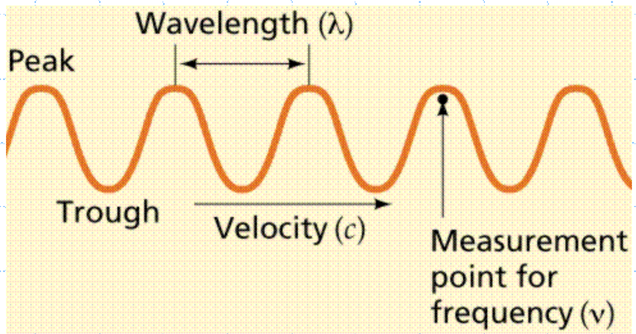
http://memory.loc.gov/ammem/index.html

Overview

Henry Farny *Song of the Talking Wire*

- Review
- Voltages and Currents on Transmission Lines
- Standing Waves
- Input Impedance
- Lossy Transmission Lines
- Low Loss Transmission Lines

What do we know so far?



- Solutions look like $A\cos(\omega t \mp \beta z)$

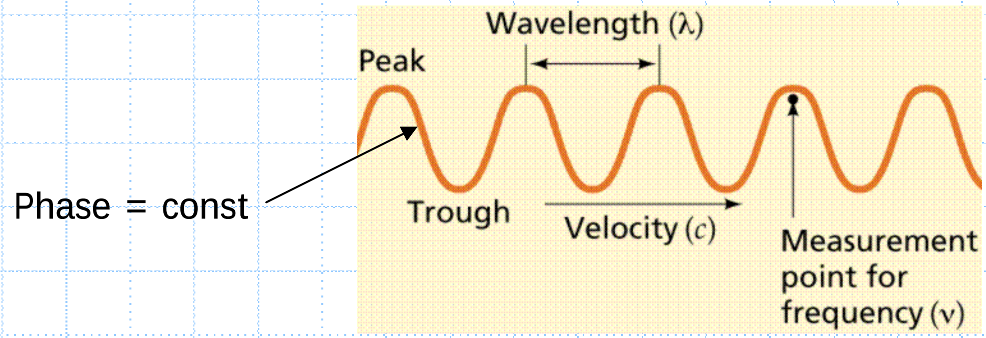
$$\beta = \frac{\omega}{u} = \omega\sqrt{lc} = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad u = \frac{1}{\sqrt{lc}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{u}{f} \qquad \varepsilon = \varepsilon_r\varepsilon_o \quad \mu = \mu_r\mu_o$$

Figure from http://www.emc.maricopa.edu/

Phase Velocity



- A simple way to find the phase velocity
  - Identify some feature of the sine wave. For this we choose constant phase

$$\omega t \mp \beta z = const$$

  - Determine it's velocity. Since the phase is a constant, we know that

$$\frac{\partial}{\partial t}(\omega t \mp \beta z) = 0 \qquad \omega \mp \beta \frac{\partial z}{\partial t} = 0$$

$$\frac{\partial z}{\partial t} = u = \frac{\omega}{\beta}$$

Phasor Notation

- For ease of analysis (changes second order partial differential equation into a second order ordinary differential equation), we use phasor notation.

$$f(z,t) = A\cos\left(\omega t \mp \beta z\right) = \mathrm{Re}\left(\left\{Ae^{\mp j\beta z}\right\}e^{j\omega t}\right)$$

- The term in the brackets is the phasor.

$$f(z) = Ae^{\mp j\beta z}$$

Phasor Notation

- To convert to space-time form from the phasor form, multiply by $e^{j\omega t}$ and take the real part.

$$f(z,t) = \mathrm{Re}(Ae^{\mp j\beta z}e^{j\omega t}) = A\cos(\omega t \mp \beta z)$$

- If A is complex

$$A = |A|e^{j\theta_A}$$

$$f(z,t) = \mathrm{Re}(|A|e^{j\theta_A}e^{\mp j\beta z}e^{j\omega t}) = |A|\cos(\omega t \mp \beta z + \theta_A)$$

Workspace

$V_o e^{j\theta_o} e^{\mp j\beta z}$

What is the phasor of the time derivative?

$$v(z,t) = \mathrm{Re}\left(\left[V_o e^{-j\beta z}\right] e^{j\omega t}\right)$$

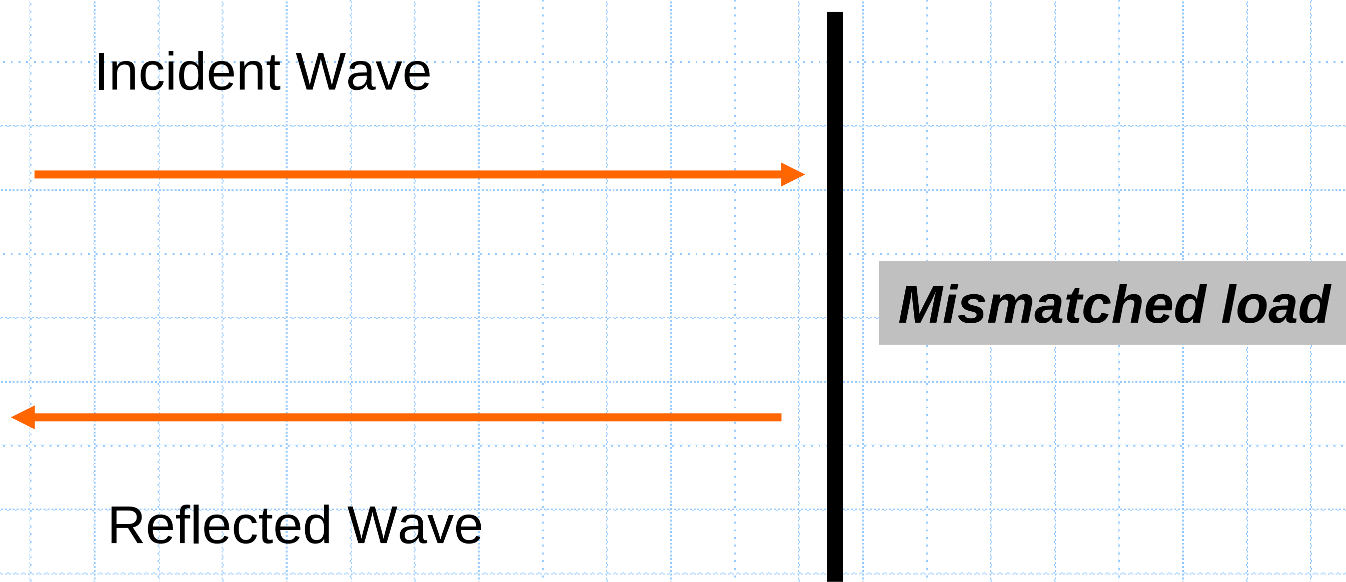
$$\frac{\partial}{\partial t} v(z,t) = \frac{\partial}{\partial t} \mathrm{Re}\left(\left[V_o e^{-j\beta z}\right] e^{j\omega t}\right) = ? \; \mathrm{Re}\left\{\left[V_o e^{-j\beta z}\right] \frac{\partial}{\partial t} e^{j\omega t}\right\}$$

const w.r.t time

$j\omega e^{j\omega t}$

$$= \mathrm{Re}\left[V_o e^{-j\beta z} (j\omega) e^{j\omega t}\right]$$

Phasors $\partial/\partial t \longrightarrow j\omega$



Standing wave due to interference

Transmission Lines – Phasor Voltage Solution

Phasor Form of the Wave Equation:

$$\frac{\partial^2 V}{\partial z^2} = l \cdot c \cdot \frac{\partial^2 V}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 V}{\partial z^2} = -\omega^2 \cdot l \cdot c \cdot V$$

where:

$$V = V^{\mp} \cdot e^{\pm j \cdot \beta \cdot z}$$

General Solution:

$$V = V^{+} e^{-j \cdot \beta \cdot z} + V^{-} e^{+j \cdot \beta \cdot z}$$

Transmission Lines - Standing Wave Derivation

$$V = V^{+} e^{-j\cdot\beta\cdot z} + V^{-} e^{+j\cdot\beta\cdot z}$$

Forward Wave

$$cos(\omega \cdot t - \beta \cdot z)$$

Backward Wave

$$cos(\omega \cdot t + \beta \cdot z)$$

TIME DOMAIN

V_{max} occurs when Forward and Backward Waves are in Phase

⇨ *CONSTRUCTIVE INTERFERENCE*

V_{min} occurs when Forward and Backward Waves are out of Phase

⇨ *DESTRUCTIVE INTERFERENCE*

Transmission Line Voltages and Currents

β, Z_o from line parameters.

- General Solution

$$v(z) = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$i(z) = \frac{V^+ e^{-j\beta z} - V^- e^{+j\beta z}}{Z_o} = \frac{V^+}{Z_o} e^{-j\beta z} - \frac{V^-}{Z_o} e^{+j\beta z}$$

- The latter expression is derived from

$$-j\omega l i(z) = \frac{\partial v(z)}{\partial z}$$

from

Can we easily explain the minus sign?

Can't ... analysis

Workspace

$$\frac{\omega}{\beta} = u = \frac{1}{\sqrt{\ell c}}$$

$$V(z) = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$-j\omega \ell\, i(z) = \frac{\partial V(z)}{\partial z} \quad (\text{Phasors})$$

$$= V^+(-j\beta)e^{-j\beta z} - V^-(+j\beta)e^{+j\beta z}$$

$$i(z) = V^+ \frac{\beta}{\omega \ell} e^{-j\beta z} - V^- \frac{\beta}{\omega \ell} e^{+j\beta z}$$

$$= \frac{V^+}{\omega \ell/\beta} e^{-j\beta z} - \frac{V^-}{\omega \ell/\beta} e^{+j\beta z}$$

$$Z_o = \frac{\omega \ell}{\beta} = \sqrt{\frac{\ell}{c}} = \sqrt{\frac{j\omega \ell}{j\omega c}}\ \Omega$$

Reflection Coefficient Derivation

Define the Reflection Coefficient:

$$\Gamma_L = \frac{V_m^-}{V_m^+}$$

$$\left|V_m^-\right| = \left|\Gamma_L\right| \cdot \left|V_m^+\right|$$

Maximum Amplitude when in Phase: $V_{max} = \left|V_m^+\right| + \left|V_m^-\right|$

$$\therefore V_{max} = \left|V_m^+\right| \cdot (1 + \left|\Gamma_L\right|)$$

Similarly: $V_{min} = \left|V_m^+\right| \cdot (1 - \left|\Gamma_L\right|)$

Standing Wave Ratio (SWR) = $\dfrac{V_{max}}{V_{min}} = \dfrac{1+\left|\Gamma_L\right|}{1-\left|\Gamma_L\right|}$

Another General Form for the Solution

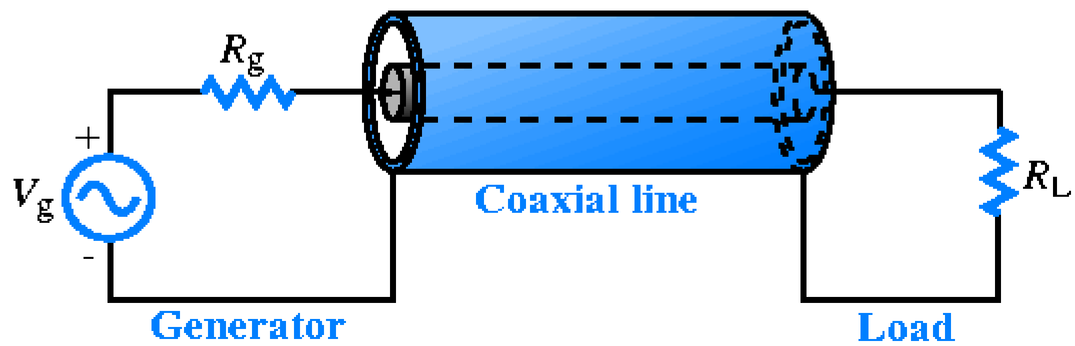
Note that we will be rewriting the solution in different forms

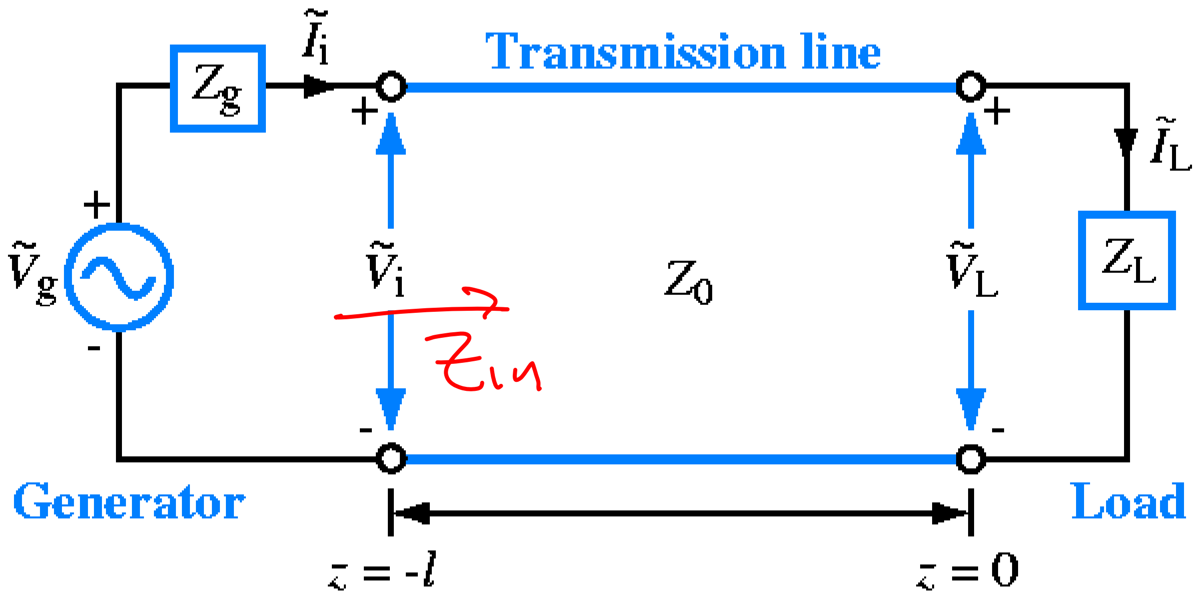
- Using the reflection coefficient

$$v(z) = V^{+}e^{-j\beta z} + \Gamma_L V^{+}e^{+j\beta z}$$

$$i(z) = \frac{V^{+}}{Z_o}e^{-j\beta z} - \frac{\Gamma_L V^{+}}{Z_o}e^{+j\beta z}$$

Transmission Lines





$Z_{in} = \frac{V_{in}}{I_{in}}$

$Z_{out} = Z_L$

Both from Ulaby

Reflection Coefficient

$e^{-j\beta z} \to 0 \qquad e^{+j\beta z} \to 0$

Let z=0 at the LOAD

$$\Rightarrow V_{load} = V^+ \cdot e^{-j\cdot\beta\cdot 0} + V^- \cdot e^{+j\cdot\beta\cdot 0}$$

$$= V^+ + V^-$$ ← Voltage at load

$$= V^+ \cdot (1 + \Gamma_L)$$

$$\frac{V_{load}}{I_{load}} = Z_L$$

$$I_{load} = \frac{V^+}{Z_o} - \frac{\Gamma_L V^+}{Z_o} = \frac{V^+}{Z_o}(1 - \Gamma_L)$$

Reflection Coefficient

This is a key relationship

- At the load

$$\frac{V_{load}}{I_{load}} = Z_L$$

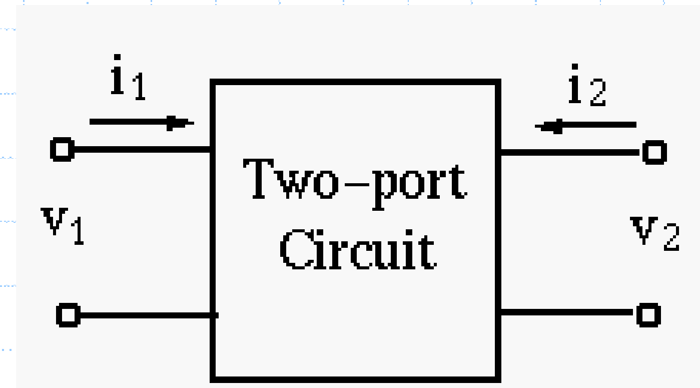
$$\frac{V^+(1+\Gamma_L)}{\frac{V^+}{Z_o}(1-\Gamma_L)} = Z_L$$

= Z_o at match

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

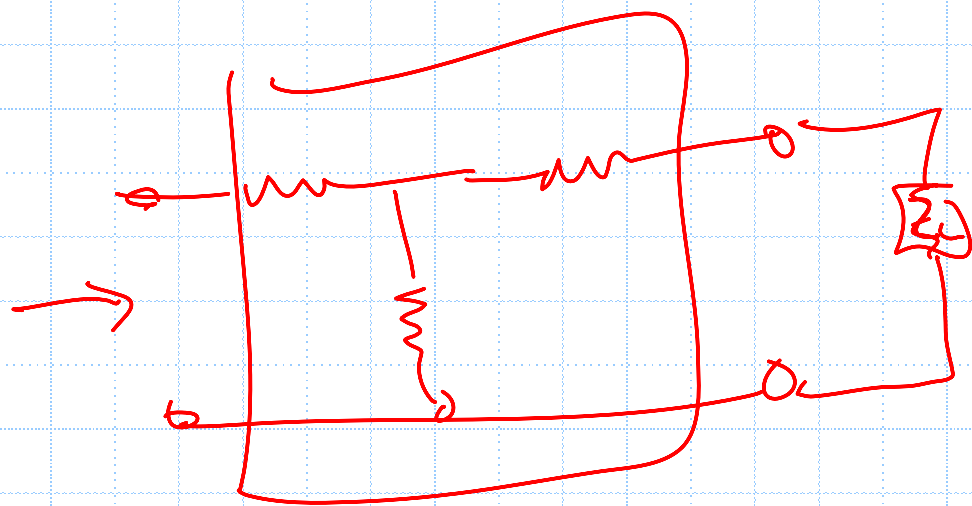
= 0 matched

Workspace

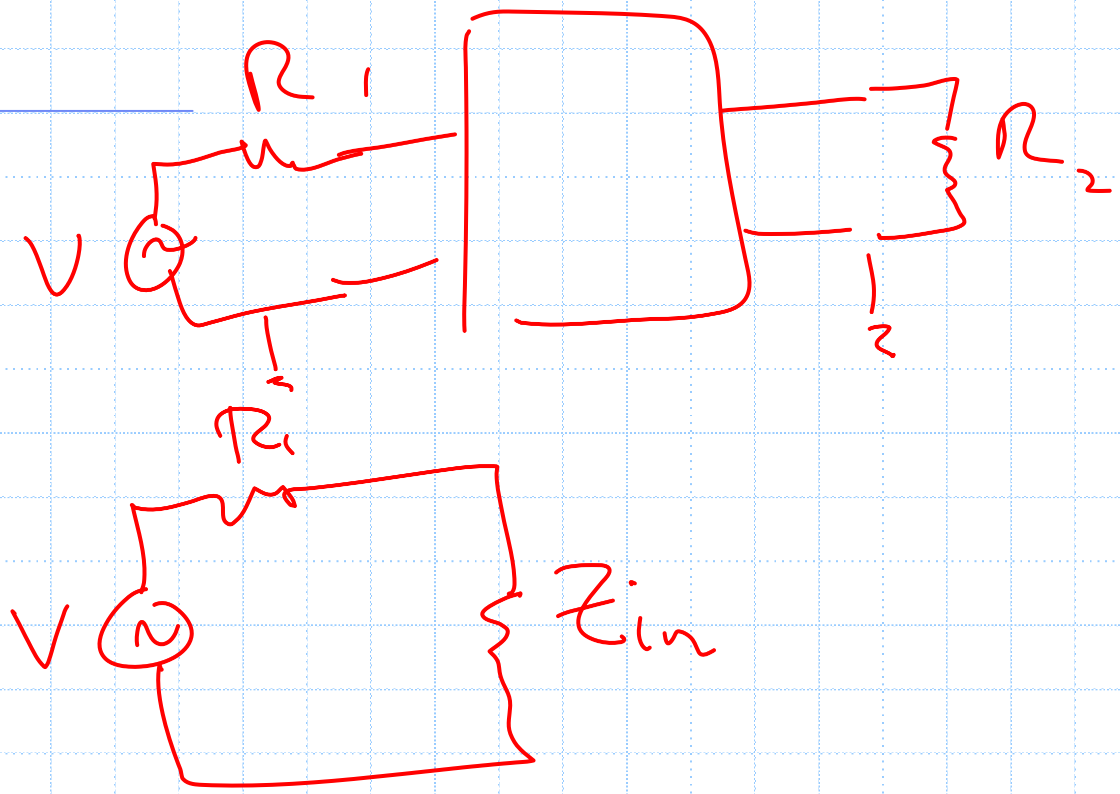


$Z_{in} = ?$ $\qquad$ $Z_{out} = ?$

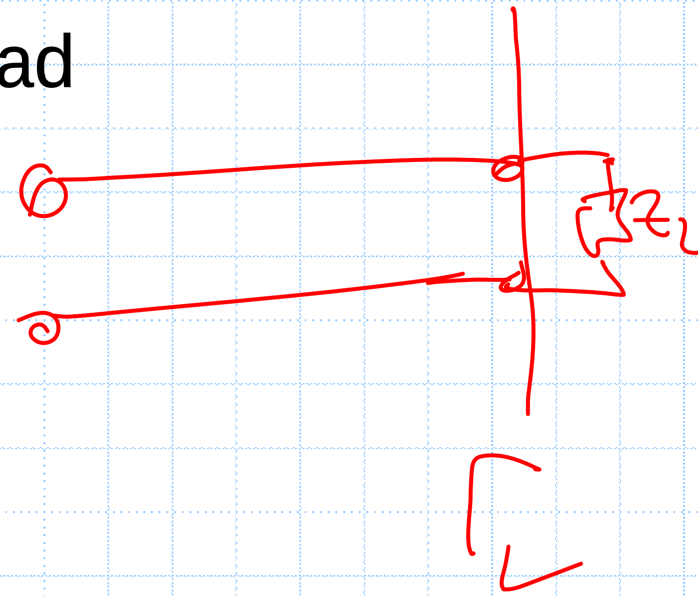
$Z_{in} = \frac{V_1}{i_1}$ $\qquad$ $Z_{out} = \frac{V_2}{i_2}$



Workspace

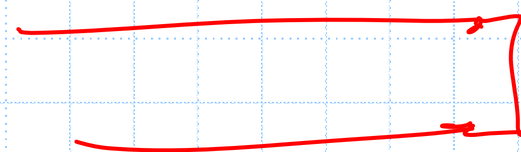


Workspace – Short Circuit Load



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_L = 0$$



$$= \frac{0 - Z_0}{0 + Z_0} = -1$$

Workspace – Open Circuit Load

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_L \to \infty$$

$$\to \frac{Z_L}{Z_L} = 1$$

Short Circuit Load

$V^- = -V^+$

- For $Z_L=0$, we have $\Gamma_L = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{0 - Z_o}{0 + Z_o} = -1$

$$v(z) = V^+ e^{-j\beta z} + \Gamma_L V^+ e^{+j\beta z} = V^+\left(e^{-j\beta z} - e^{+j\beta z}\right)$$

$$\begin{cases} e^{+j\beta z} = \cos\beta z + j\sin\beta z \\ e^{-j\beta z} = \cos\beta z - j\sin\beta z \end{cases}$$

$$\Rightarrow \quad v(z) = -V^+\left(j2\sin\beta z\right)$$

Short Circuit Load

- Convert to space-time form

$$v(z,t) = \mathrm{Re}\left(v(z)e^{j\omega t}\right) = \mathrm{Re}\left(V^{+}\left(-j2\sin\beta z\right)e^{j\omega t}\right)$$

$$\mathrm{Re}\left(\left(-j2\sin\beta z\right)e^{j\omega t}\right) = \mathrm{Re}\left(-2\sin\beta z\left(j\cos\beta z - \sin\beta z\right)\right)$$

$$v(z,t) = 2V^{+}\sin\beta z\sin\omega t$$

- This is a standing wave

Short Circuit Load

- What are the voltage maxima and minima?

$$v(z,t) = 2V^{+} \sin \beta z \sin \omega t$$

$\beta z = 0, \pi, 2\pi$

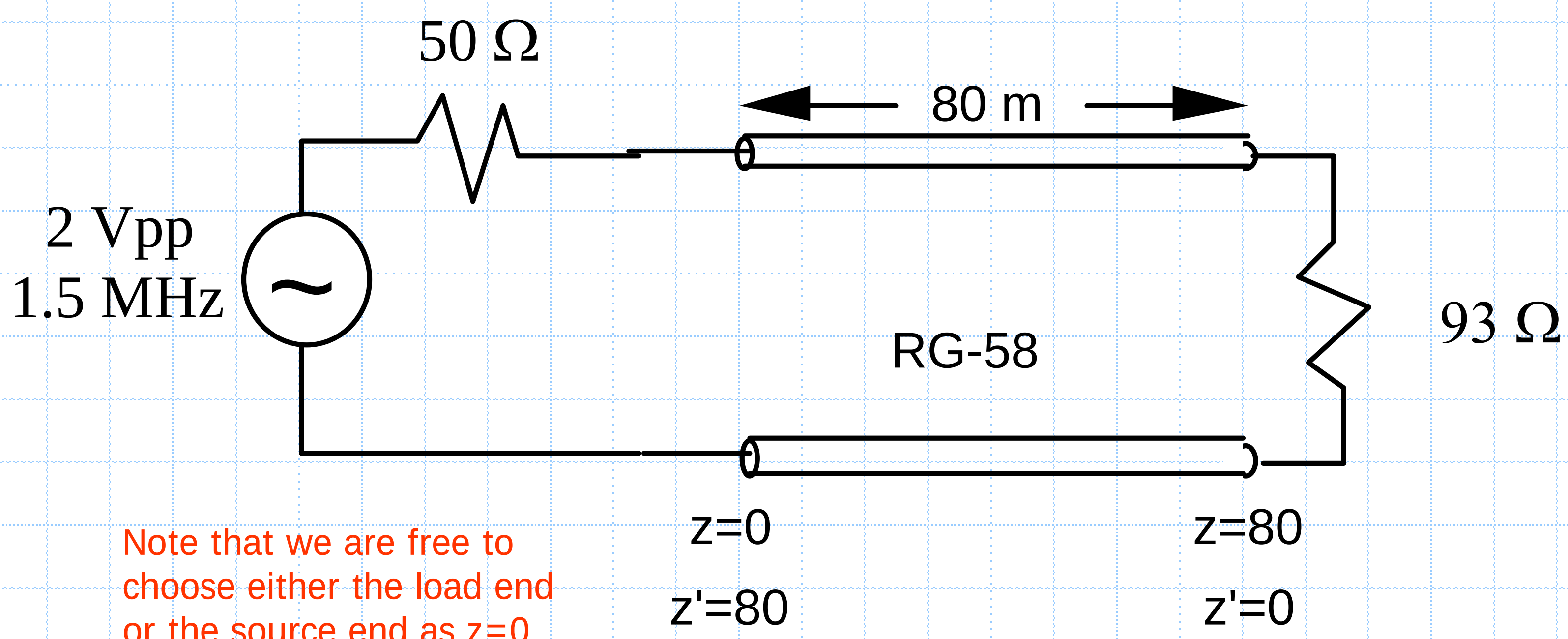
- Where are they?

$\beta = \frac{2\pi}{\lambda}$ $\quad \frac{2\pi}{\lambda} z = \pi$

$= \lambda/2$

- The standing wave pattern is the envelope of this function.

Reflection



Note that we are free to choose either the load end or the source end as z=0

Reflection Coefficient

- Determine the reflection coefficient at the load, Γ_L and the standing wave ratio, VSWR. Start with a short circuit load and then consider a 25 Ohm load. Then do an open circuit and 93 Ohm load.
- Assume that the forward traveling wave has an amplitude of 10 Volts. Sketch the standing wave pattern for voltage and current for the short circuit load. Include numbers for amplitudes and distances.
- Under what conditions do you get a voltage maximum at the load? a minimum? Can you answer this in general?
- If the load is a 3.3 nF capacitor, what is the reflection coefficient at the load? Where is the location of the first minimum? To answer this, we need a bit more development.

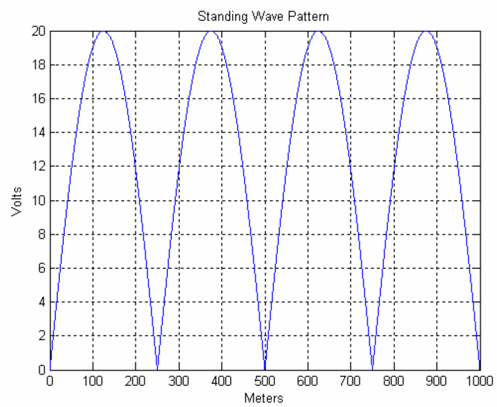
First Sketch the Standing Wave Patterns by Hand

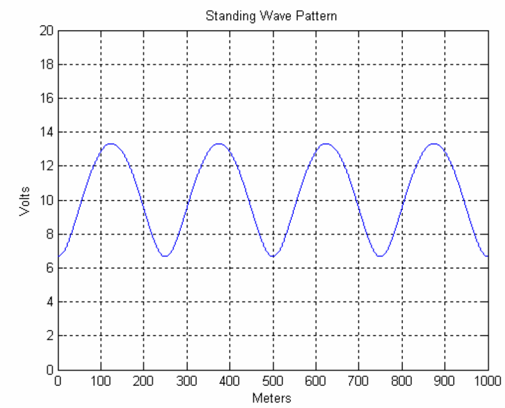
- The reflection coefficient

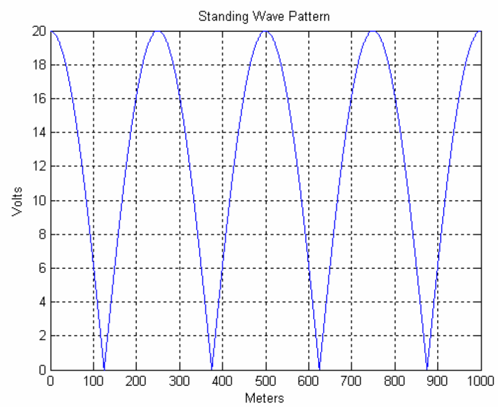
$$\Gamma_L = \frac{0-50}{0+50} = -1 \qquad \Gamma_L = \frac{25-50}{25+50} = -\frac{1}{3} = -0.333$$

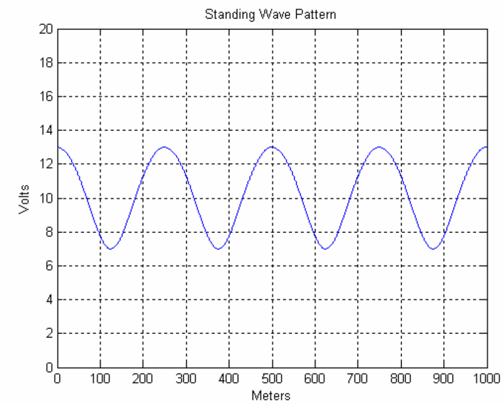
$$\Gamma_L = \frac{\infty-50}{\infty+50} = 1 \qquad \Gamma_L = \frac{93-50}{93+50} = \frac{43}{143} = +0.3$$

Using Matlab for the Voltage Standing Wave Patterns









Current Standing Wave Patterns

- Can we use what we just displayed to find the current standing wave patterns?

- Yes, because the reflection coefficient for current is always just the negative of the voltage reflection coefficient

Standing Wave Pattern

We have just seen that:

Minimum occurs at LOAD for $Z_L \rightarrow 0$

Is it also true that:

Maximum occurs at LOAD for $Z_L \rightarrow \infty$

Or, in general, that:

$\Gamma_L > 0 \quad \Rightarrow Z_L > Z_0$ *Max at LOAD*

$\Gamma_L < 0 \quad \Rightarrow Z_L < Z_0$ *Min at LOAD*

} *IF Z_L is REAL*

Standing Wave Pattern

If z=L at LOAD and z=0 at SOURCE,

$$\Gamma(z) = \Gamma_L \cdot e^{-j\cdot 2\cdot \beta\cdot (L-z)}$$

$$= \left|\Gamma_L\right| \cdot e^{j\cdot\theta_\Gamma} \cdot e^{-j\cdot 2\cdot\beta\cdot(L-z)}$$

Phase of the reflection coefficient

When Phase = π, the FIRST MINIMUM occurs

$$\theta_\Gamma - 2\cdot\beta\cdot(L-z) = \pi$$

$$\Rightarrow (L-z) = \frac{\lambda}{4} + \frac{\theta_\Gamma}{4\cdot\pi}\cdot\lambda$$

Other MINs are displaced by $\lambda/2$

A Repeat of HW2 Experiment (600kHz)

- What did you see at the 20 nodes?
  - Time Delay
  - Amplitude
- Did any of you try an open or short circuit?

Change the Frequency to 1.5MHz

a. $\beta = \frac{\omega}{v} = \frac{2\pi \times 1.5\times10^6 \, s^{-1}}{\frac{1}{\sqrt{\mu_0 2.3 \varepsilon_0}}} = \boxed{0.0477 \, m^{-1}}$ or $\beta = \omega\sqrt{\mu\varepsilon}$

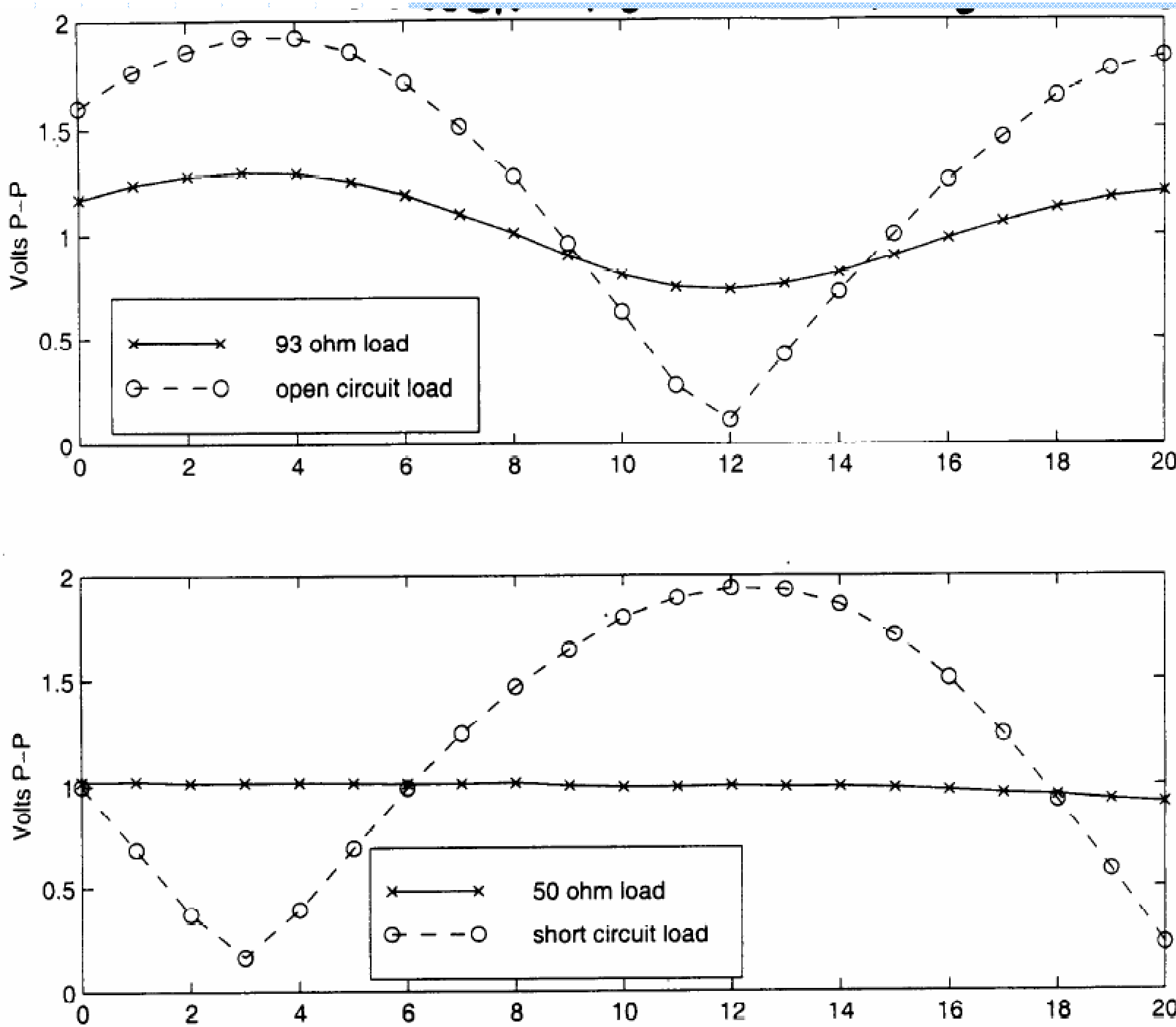
$\lambda = \frac{2\pi}{\beta} = \boxed{132 \, m}$

Distance between max and min $\hat{=}$ 32m $\Big| = \frac{32}{132}\lambda = 0.24\lambda \sim \boxed{\lambda/4}$

Distance between 2 maxima $= \boxed{\frac{\lambda}{2}}$

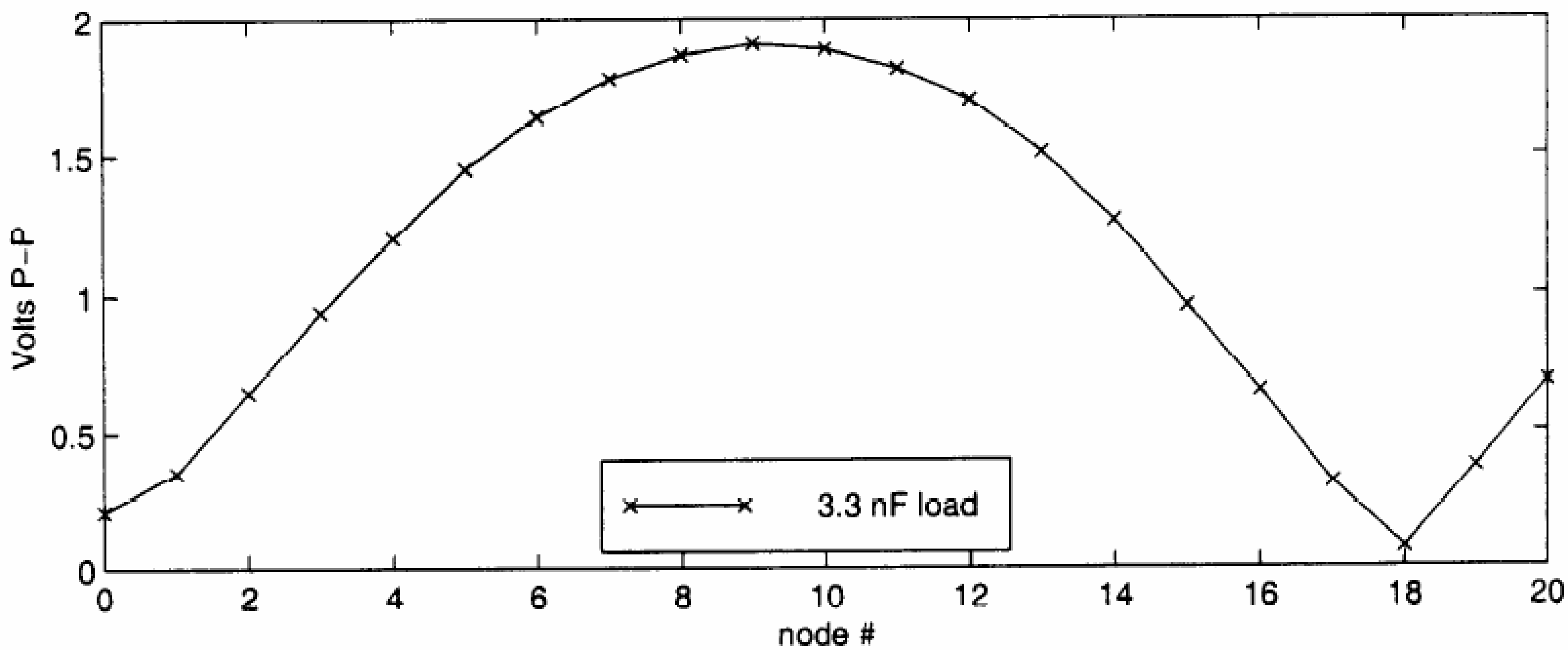
b. $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{93-50}{93+50} = \boxed{0.301}$

$VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \boxed{1.86}$ ← 5% higher than measured value



Volts P–P
93 ohm load
open circuit load
0
0.5
1
1.5
2
0
2
4
6
8
10
12
14
16
18
20
Volts P–P
50 ohm load
short circuit load
0
0.5
1
1.5
2
0
2
4
6
8
10
12
14
16
18
20

Capacitive Load?



Capacitive Load

- Now we can answer the question about where the first minimum is located for a capacitive load.

$$\Gamma_L = \frac{\frac{1}{j\omega C} - Z_o}{\frac{1}{j\omega C} + Z_o} = \frac{1 - j\omega C Z_o}{1 + j\omega C Z_o} = 1\exp\left(\tan^{-1}\left(\frac{-\omega C Z_o}{1 - (\omega C Z_o)^2}\right)\right)$$

$$\frac{a - jb}{a + jb} = \frac{a - jb}{a + jb}\frac{a - jb}{a - jb} = \frac{a^2 - j2ab - b^2}{a^2 + b^2} = \frac{(a^2 + b^2)\exp\left(\tan^{-1}\left(\frac{-2ab}{a^2 - b^2}\right)\right)}{a^2 + b^2} = \exp\left(\tan^{-1}\left(\frac{-2ab}{a^2 - b^2}\right)\right)$$

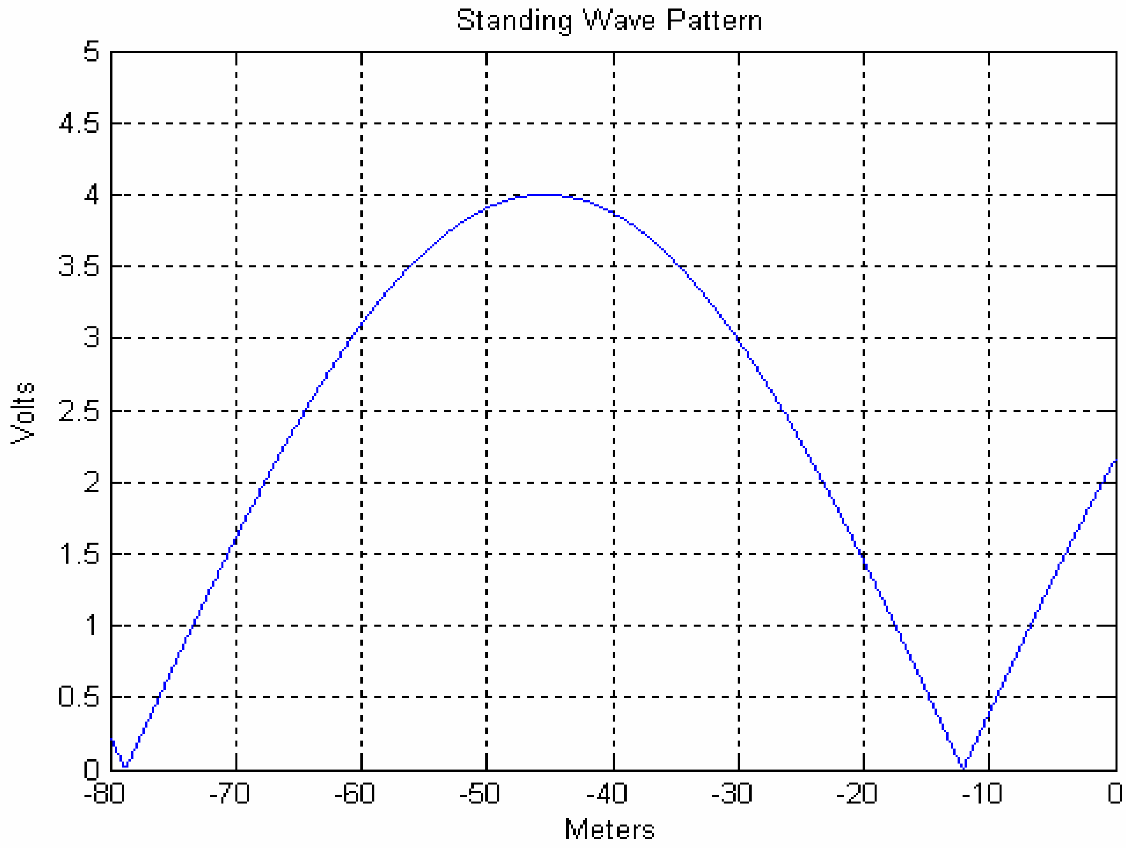
Capacitive Load

- For the specific case here

$$\Gamma_L = \frac{\dfrac{1}{j\omega C} - Z_o}{\dfrac{1}{j\omega C} + Z_o} = \frac{-j32.2 - 50}{-j32.2 + 50} = -0.4 - j0.91 = 1e^{-j0.6362\pi}$$

$$L - z = \frac{\lambda}{4} + \frac{\theta_\Gamma}{2\pi}\lambda = \frac{\lambda}{4} + \frac{-.6362\pi}{4\pi}\lambda = \frac{\lambda}{4} - .1591\lambda = 0.09\lambda$$

Standing Wave Pattern for Capacitive Load



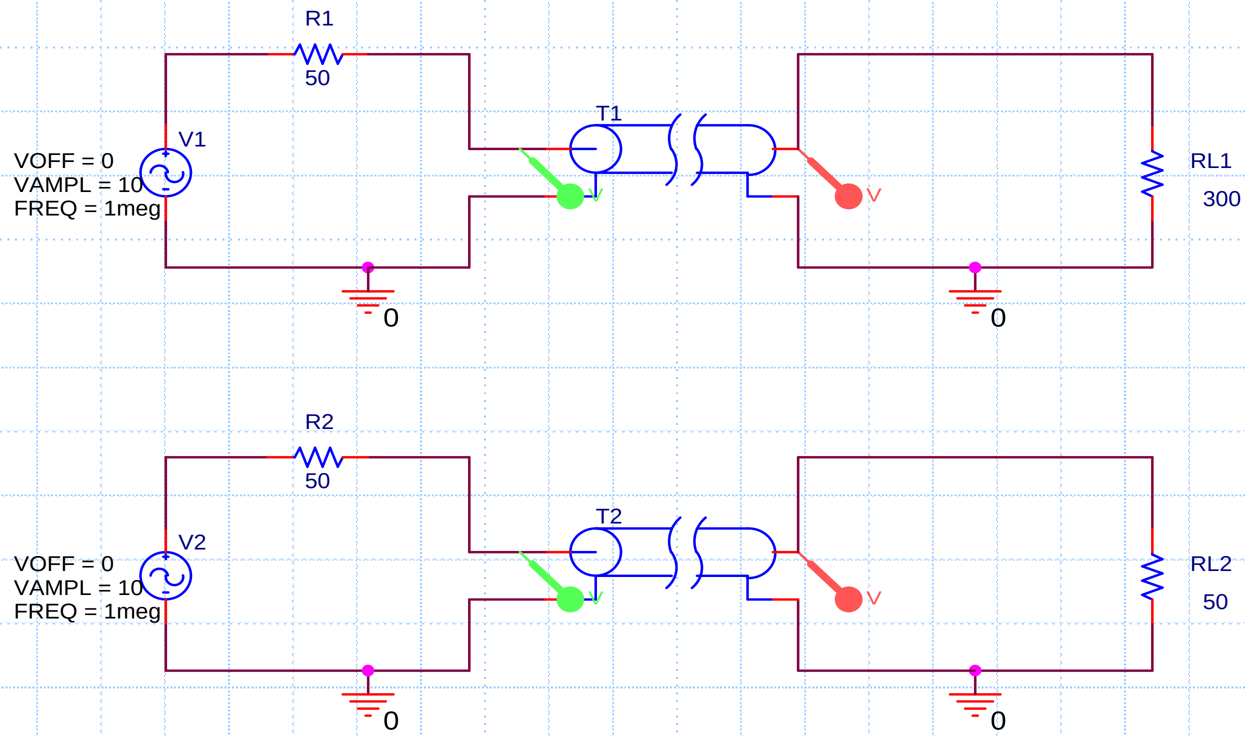
Reflection Coefficient

$$\Gamma_L = 1e^{-j0.6362\pi}$$

Load End

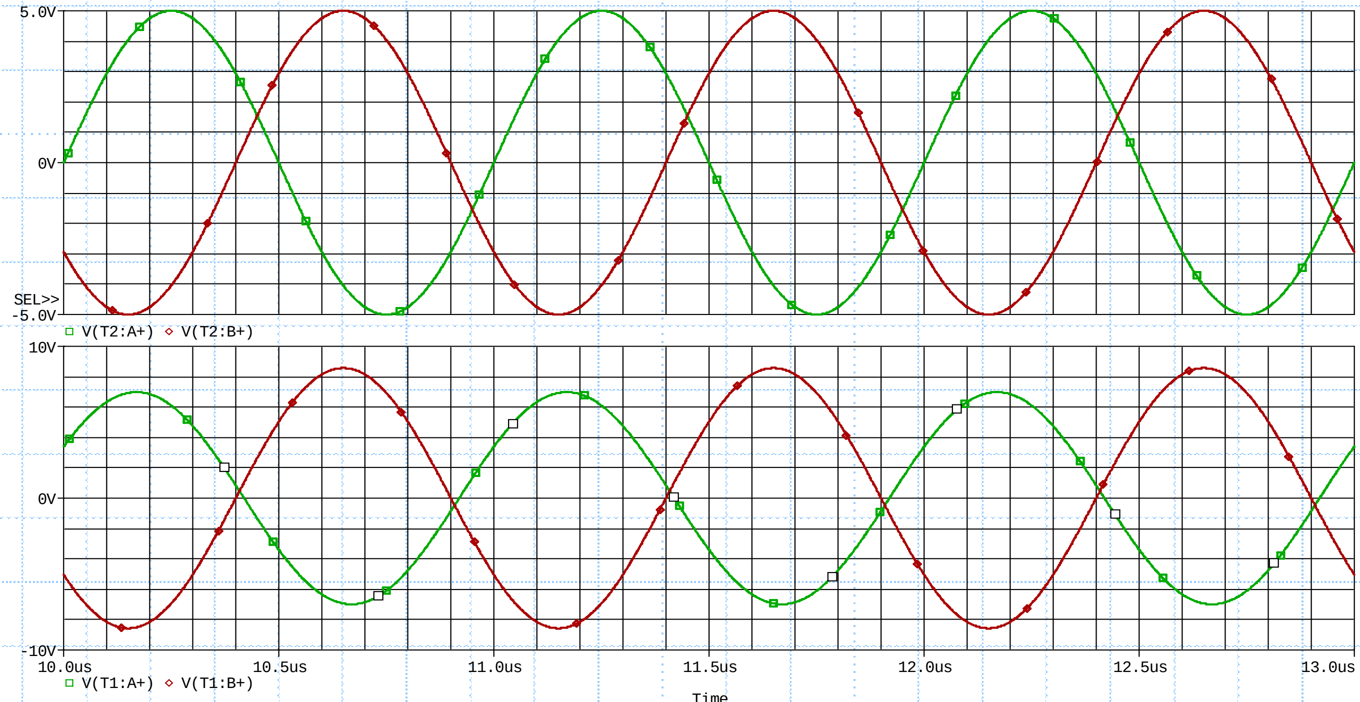
PSpice – Input Impedance

- For the same source and line, but different load:



Changing the Load

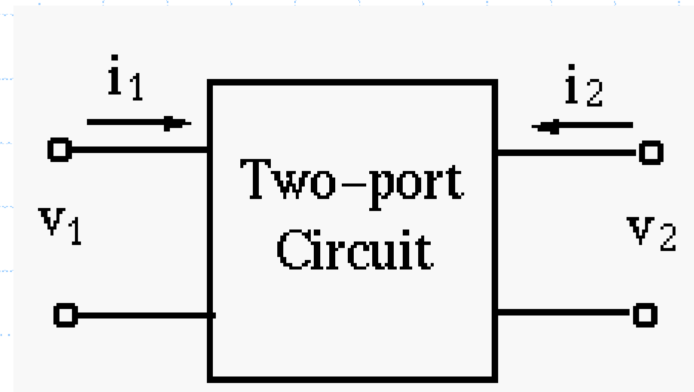
- The voltages and currents at the input change → the input impedance changes.



Changing the Length and Line Properties

- From the standing wave patterns or the expressions for the voltages and the currents on the line, we can see that the ratio of the voltage to the current will depend on the length of the line and the line properties.

Workspace



$$V(z) = V^+\left(e^{-j\beta z} + \Gamma_L e^{+j\beta z}\right)$$

$$= V^+ e^{-j\beta z}\left(1 + \underbrace{\Gamma_L e^{j2\beta z}}_{\Gamma(z)}\right)$$

Input Impedance

What does $Z_{in} = \frac{V_{in}}{I_{in}}$, look like?

When Z_L is complex, so is Γ_L. To address the input impedance, we need to generalize the reflection coefficient.

Define:

$$\Gamma(z) = \frac{V^- \cdot e^{+j \cdot \beta \cdot z}}{V^+ \cdot e^{-j \cdot \beta \cdot z}} = \frac{V^-}{V^+} \cdot e^{j \cdot 2 \cdot \beta \cdot z}$$

$$= \Gamma_L \cdot e^{j \cdot 2 \cdot \beta \cdot z}$$ if z =0 at LOAD

Another Form for the General Solution

- Using the Generalized Reflection Coefficient

$$v(z) = V^{+} e^{-j\beta z}\left(1 + \Gamma(z)\right)$$

$$i(z) = \frac{V^{+}}{Z_o} e^{-j\beta z}\left(1 - \Gamma(z)\right)$$

$$\Gamma(z) = \Gamma_L e^{j2\beta z}$$

Input Impedance

Previously, we have seen:

$$\hat{V}(z) = V^+(z) + V^-(z) = V^+ \cdot e^{-j \cdot \beta \cdot z} \cdot (1 + \Gamma(z))$$

What about I?

$$\hat{I}(z) = \frac{V^+(z)}{Z_o} - \frac{V^-(z)}{Z_o} = \frac{V^+}{Z_o} \cdot e^{-j \cdot \beta \cdot z} \cdot (1 - \Gamma(z))$$

Also,

$$\Gamma(z) = \Gamma_L \cdot e^{-2 \cdot j \cdot \beta \cdot (L - z)}$$

Input Impedance

Form the Ratio (the generalized impedance):

$$\frac{\hat{V}(z)}{\hat{I}(z)} = Z_o \cdot \frac{1+\Gamma(z)}{1-\Gamma(z)} = Z(z)$$

input

We are primarily interested in z=0 value

- treat connection to rest of circuit as 2 port with,

$$Z_{in}(z=0) = Z_o \cdot \frac{1+\Gamma(z=0)}{1-\Gamma(z=0)}$$

Input Impedance

After lots of algebra, one can show:

$$Z_{in}(z=0) = Z_o \cdot \frac{Z_L + j \cdot Z_o \cdot \tan(\beta \cdot L)}{Z_o + j \cdot Z_L \cdot \tan(\beta \cdot L)}$$

Special Case example: $Z_L=0$ (short circuit)

$$Z_{in}(z=0) = Z_o \cdot \frac{0 + j \cdot Z_o \cdot \tan(\beta \cdot L)}{Z_o + j \cdot 0 \cdot \tan(\beta \cdot L)} = j \cdot Z_o \cdot \tan(\beta \cdot L)$$

Input Impedance - SHORT CIRCUIT

$$Z_{in}(z=0) = j \cdot Z_o \cdot \tan(\beta \cdot L)$$

$\beta = \omega\sqrt{\ell c}$

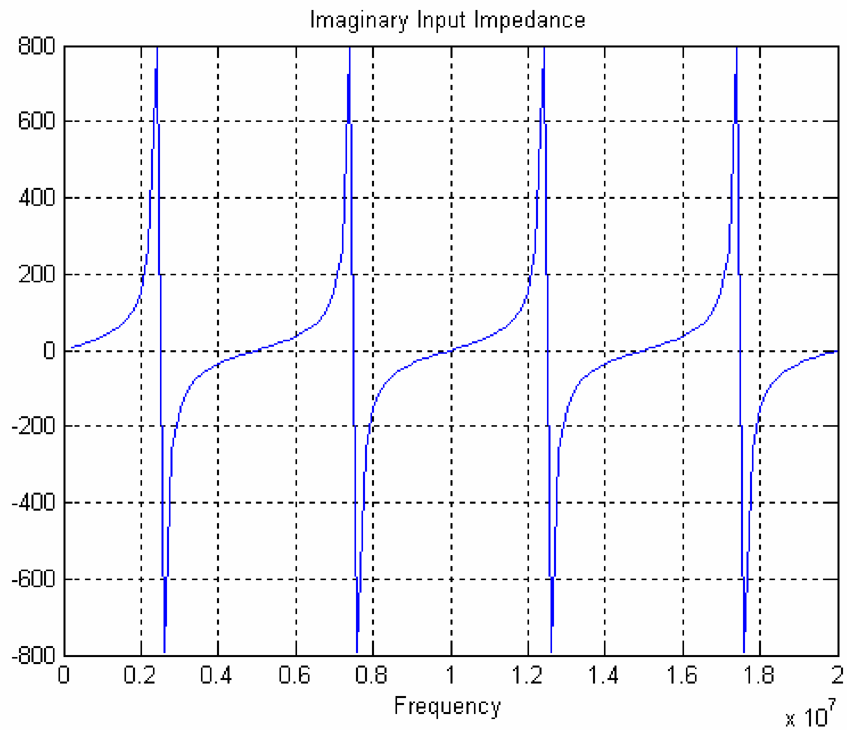
Can change Z_{in} by changing these two parameters

- Fix β, vary L - different effects
- Vary β, fix L - get same effects

Note that L is the length of the Transmission Line

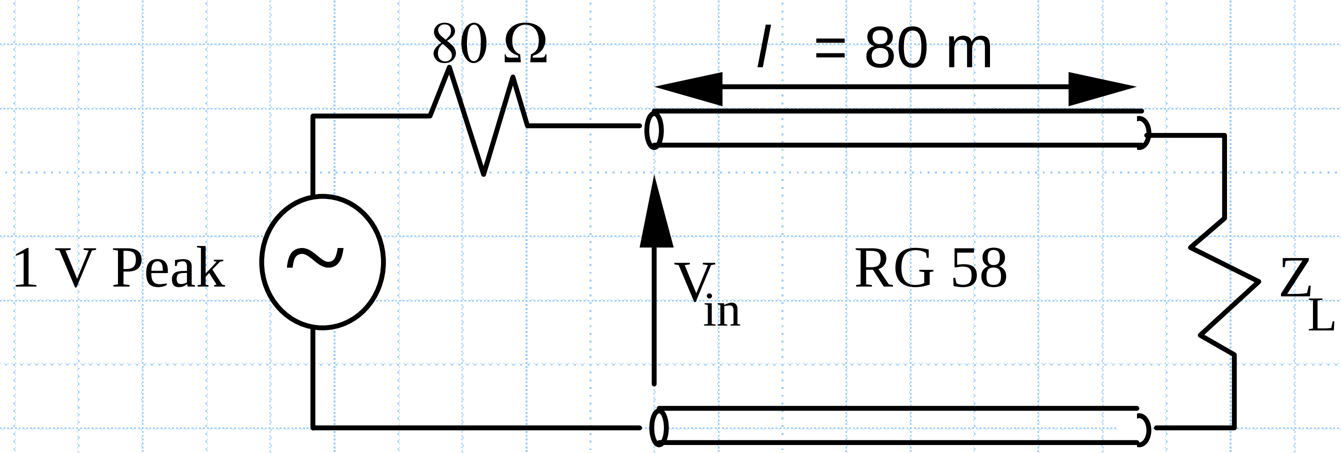
Input Impedance – Short Circuit

- For varying frequency, the input impedance is imaginary and can achieve any value.



Input Impedance - TL

- Consider some other cases



Input Impedance - TL

Open Circuit Case

$$Z_L = \infty$$

$$Z_L = Z_o \frac{Z_L + j \cdot Z_o \cdot \tan(\beta \cdot L)}{Z_o + j \cdot Z_L \cdot \tan(\beta \cdot L)}$$

small

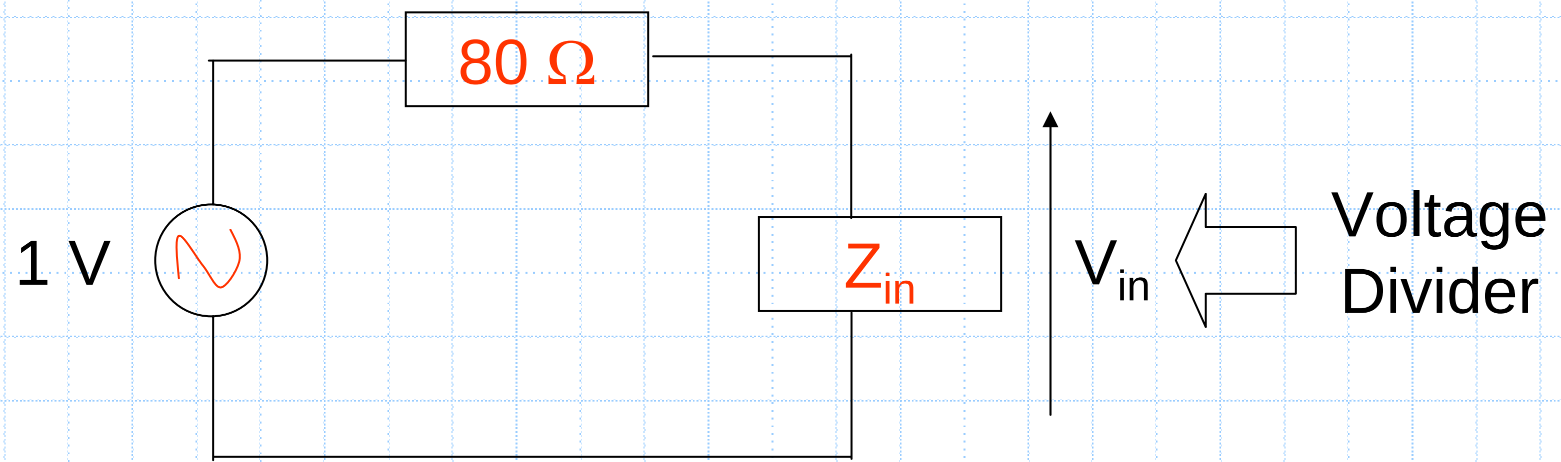
small

$$= Z_o \frac{Z_L}{j Z_L \tan \beta L} = -j Z_o \cot \beta L$$

Z_L = 93Ω - lots of complex algebra, but straight forward

Using the Input Impedance

We know Z_{in} (z=0) - treat as 2-PORT



$$Power = \frac{1}{2} Re\{V_{in} \times I_{in}^*\} = \frac{1}{2} Re\left\{\frac{V_{in} \times V_{in}^*}{Z_{in}^*}\right\} = \frac{1}{2} Re\left\{\frac{|V_{in}|^2}{Z_{in}^*}\right\}$$

Using the Input Impedance

- In a Lossless Transmission Line, P_{in} flows into the Transmission Line and it is dissipated at the LOAD

$$P_{in} = \frac{1}{2} \cdot \frac{|V_L|^2}{Z_L}$$

What is the voltage at the load?

$$\hat{V}(z) = V^+ \cdot e^{-j \cdot \beta \cdot z} \cdot (1 + \Gamma(z))$$

$$\hat{V}(z=0) = V_{in} = V^+ \cdot e^{-j \cdot \beta \cdot z} \cdot (1 + \Gamma(z=0))$$

$$\Rightarrow V^+ = \frac{V_{in}}{1 + \Gamma(0)}$$

Can then plug back and get the full phasor expression

Using the Input Impedance

- The full form of the voltage

$$V(z) = V^{+}e^{-j\beta z} + V^{-}e^{+j\beta z}$$

- All information is now available to determine the voltage and current everywhere on the line. You will be doing this on the project.

Special Cases

- Recall that the standing wave pattern repeated every half wavelength. Thus, we expect that this will also happen for Z_{in}. First, consider the trivial case of $L=0$.

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta L}{Z_o + jZ_L \tan \beta L} = Z_L$$

- Now let the line be a half wavelength long

$$\tan \beta L = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right) = \tan(\pi) = 0 \qquad Z_{in} = Z_o \frac{Z_L + 0}{Z_o + 0} = Z_L$$

Special Cases

- Thus, for a line that is exactly an integer number of half wavelengths long

$$Z_{in} = Z_L$$

- Thus, if you have a transmission line with the wrong characteristic impedance, you can match the load to the source by selecting a length equal to a half wavelength.

Special Cases

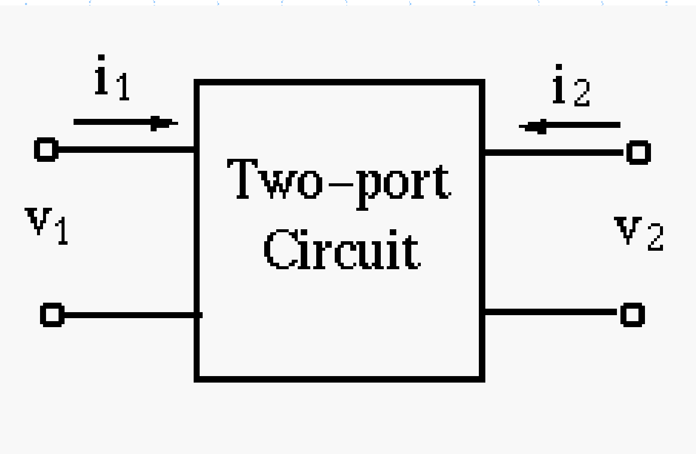
- If the line is an odd multiple of a quarter wavelength, we also get an interesting result.

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta L}{Z_o + jZ_L \tan \beta L} = Z_o \frac{jZ_o \tan \beta L}{jZ_L \tan \beta L} = \frac{Z_o^{\ 2}}{Z_L}$$

$$\tan \beta L = \tan \frac{2\pi}{\lambda}\frac{\lambda}{4} = \tan \frac{\pi}{2} \to \infty$$

- Thus, such a transmission line works like an impedance transformer and has a real input impedance.

Today's Major Result



- Input Impedance

$$Z_{in}(z=0) = Z_o \cdot \frac{Z_L + j \cdot Z_o \cdot \tan(\beta \cdot L)}{Z_o + j \cdot Z_L \cdot \tan(\beta \cdot L)}$$