**Fields and Waves I**

**Name _______________ ECSE-2100 Spring 2001 Section __ __**

# FINAL EXAM

**1. Short Questions** (80 Points)
*Answer any* **8** *questions –* ***Circle the letters for the questions to be graded***

**a.** (10 Points) ____
**b.** (10 Points) ____
**c.** (10 Points) ____
**d.** (10 Points) ____
**e.** (10 Points) ____
**f.** (10 Points) ____
**g.** (10 Points) ____
**h.** (10 Points) ____
**i.** (10 Points) ____
**j.** (10 Points) ____
**k.** (10 Points) ____
**l.** (10 Points) ____
**m.** (10 Points) ____
**n.** (10 Points) ____

**Total** ________

**2. Electric Fields** (30 Points) ________

**3. Magnetic Fields** (30 Points) ________

**4. Transmission Lines** (30 Points) ________

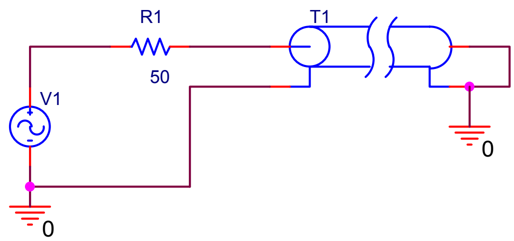
**5. Electromagnetic Waves** (30 Points) ________

**Total** (200 Points) ________

Henry Farny – *Song of the Talking Wire*
*Taft Museum of Art – Cincinnati*

## 1. Short Questions (50 Points)

a. (10 Points) Transmission Lines




A 500 meter long polyethylene ($\varepsilon_r = 2.25$) insulated transmission line has a short circuit at its load end. At low frequencies (near $\omega = 0$), does the line look like an inductor, a capacitor or a resistor? (Circle the correct answer.) Which of the following frequencies are low? (Again, circle the correct answers.)

f = 1 kHz. f = 10 kHz f = 1 MHz f = 10 MHz f = 1 GHz f = 10 GHz

b. (10 Points) Inductance

The center conductor of a coaxial cable has a radius equal to *a* while the outer conductor has an inner radius of *b* and an outer radius of *c*. The length of the cable is *l*. Which of the following is true? (Circle all that are correct)

i) Doubling *l* will double the inductance of the cable.

ii) Doubling *l* will halve the inductance of the cable.

iii) Doubling *l* will leave the inductance of the cable unchanged.

iv) Doubling *a* and *b* will double the inductance of the cable.

v) Doubling *a* and *b* will halve the inductance of the cable.

vi) Doubling *a* and *b* will leave the inductance of the cable unchanged.

vii) Increasing *a* (everything else the same) will increase the inductance.

viii) Increasing *a* (everything else the same) will decrease the inductance.

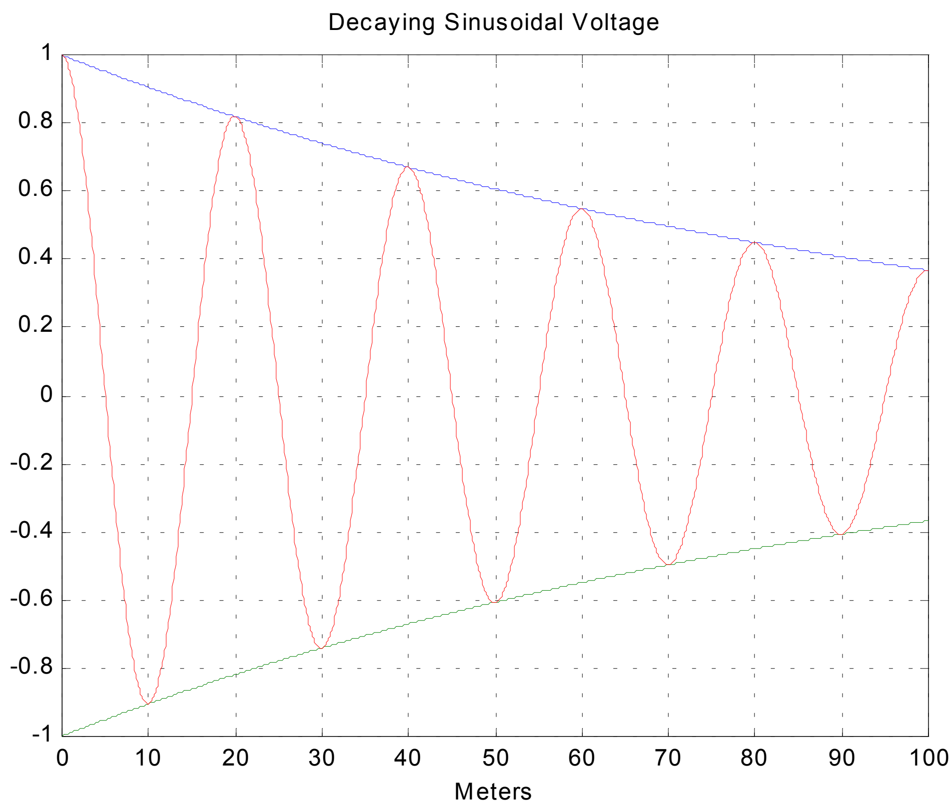
c. (10 Points) Good Conductor or Good Dielectric?

Typical conductivity for a combination of wet and dry soil (or earth) is about $\sigma = 10^{-3}$ S/m while the dielectric constant is about $\varepsilon_r = 9$. Circle the frequencies at which soil is a good conductor. Cross out the frequencies at which it is a good (low loss) dielectric.

1Hz 10Hz 100Hz 1kHz 10kHz 100kHz 1MHz 10MHz
100MHz 1GHz 10GHz 100GHz 1THz 10THz 100THz

d. (10 Points) Lossy Transmission Line

(Circle the correct answer.)
The following voltage signal is observed as a function of z on a lossy transmission line.




a. What are the values for $\alpha$ and $\beta$ for this voltage?

$\alpha = 1$ $\alpha = 0.1$ $\alpha = 0.01$ $\alpha = 0.001$

$\beta = \pi$ $\beta = \pi/2$ $\beta = \pi/5$ $\beta = \pi/10$

b. Write this voltage in phasor form.

e. (10 Points) Electric Fields

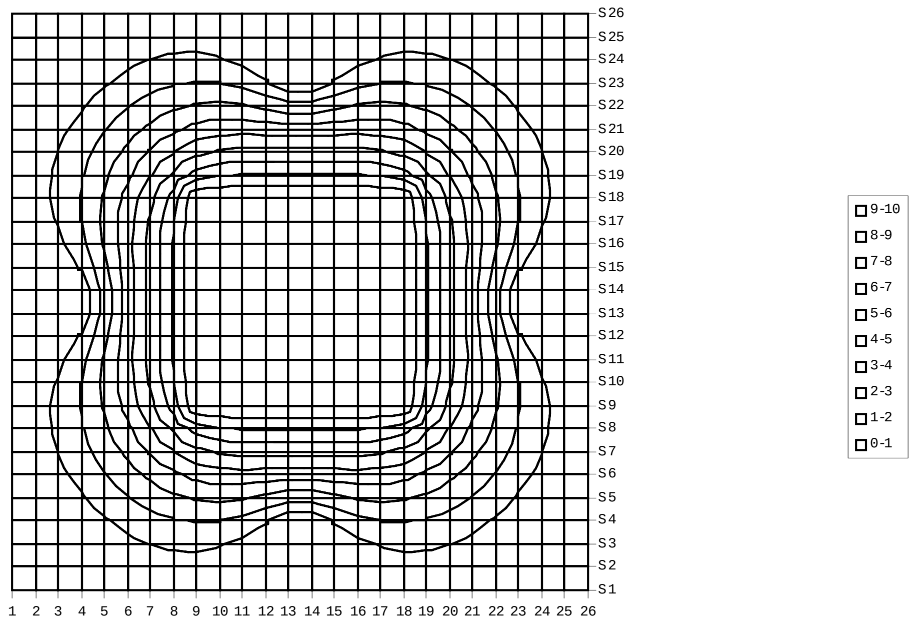
The potential structure of an oddly constructed cable is analyzed using the finite difference method we applied using Excel Spreadsheets. Use the information from this table to determine the capacitance per unit length for this cable. The dielectric used in this cable is glass ($\varepsilon_r = 4$) and the dimensions of the square inner conductor are 0.9 x 0.9 cm. The equipotentials for this cable are also shown in the next question.

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 1 | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 | 4 | 4 | 5 | 5 | 5 | 4 | 4 | 3 | 3 | 2 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 5 | 4 | 3 | 2 | 2 | 1 | 0 | 0 |
| 0 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 1 | 0 |
| 0 | 1 | 1 | 2 | 3 | 4 | 6 | 7 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 7 | 6 | 4 | 3 | 2 | 1 | 1 | 0 |
| 0 | 0 | 1 | 2 | 3 | 5 | 6 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 8 | 6 | 5 | 3 | 2 | 1 | 0 | 0 |
| 0 | 0 | 1 | 2 | 3 | 5 | 6 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 8 | 6 | 5 | 3 | 2 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 3 | 5 | 6 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 8 | 6 | 5 | 3 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 2 | 4 | 6 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 8 | 6 | 4 | 2 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 2 | 4 | 6 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 8 | 6 | 4 | 2 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 3 | 5 | 6 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 8 | 6 | 5 | 3 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 2 | 3 | 5 | 6 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 8 | 6 | 5 | 3 | 2 | 1 | 0 | 0 |
| 0 | 0 | 1 | 2 | 3 | 5 | 6 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 8 | 6 | 5 | 3 | 2 | 1 | 0 | 0 |
| 0 | 1 | 1 | 2 | 3 | 4 | 6 | 7 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 7 | 6 | 4 | 3 | 2 | 1 | 1 | 0 |
| 0 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 1 | 0 |
| 0 | 0 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 5 | 4 | 3 | 2 | 2 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 | 4 | 4 | 5 | 5 | 5 | 4 | 4 | 3 | 3 | 2 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 1 | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Since there are no decimals shown for the numbers in the table, many of the cells do not appear to satisfy Laplace's equation. Find a cell and its neighbors that show the correct relationship and one that does not.

f. (10 Points) Electric Fields

A set of equipotentials for the cable of the previous problem are shown below.




You should be able to tell the shapes of the inner and outer conductors. Sketch the conductors on the figure (note that part of the outer conductor has been drawn in to help you get started). *Carefully* draw at least 10 electric field lines with arrows to show their direction. Indicate where you think the electric field is the largest in the figure.

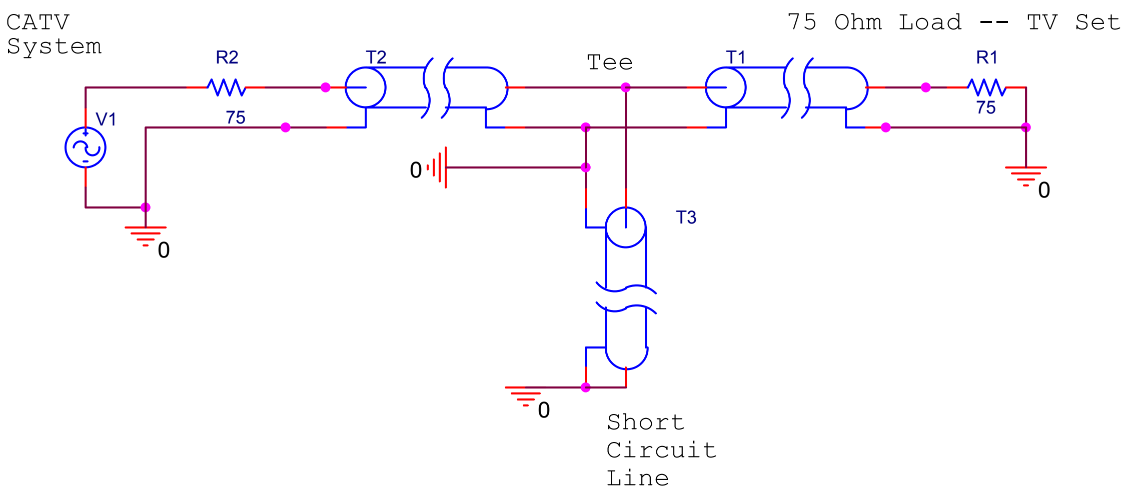
(*Requires the solution from the previous problem*.) Determine the electric flux per unit length for this cable.

g. (10 Points) Electromagnetic Waves (Circle the correct answers.)

1. The closest distance between a maximum and a minimum in an electric field standing wave pattern is ($\lambda/4$, $\lambda/2$, $\lambda$, $2\lambda$).

2. A randomly polarized (equal amounts of perpendicular and parallel polarization) uniform plane wave is propagating in air and is incident obliquely at 75° on the water/air surface at z=0 (for water $\varepsilon_r = 1.5$ at optical frequencies). Which of the following is correct?
i) There is no transmitted wave, the transmitted wave propagates with an angle greater than 75°, the transmitted wave propagates at an angle less than 75°.
ii) The reflected wave will be mostly parallel polarized, the reflected wave will be mostly perpendicular polarized, the reflected wave will be equally perpendicular and parallel polarized.

h. (10 Points) Transmission Lines

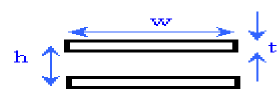


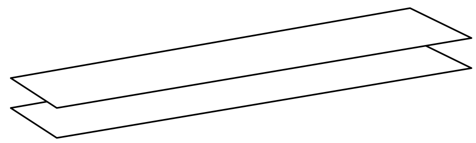

A short-circuited, 0.5 meter long transmission line with $Z_o = 75\Omega$ and $v_{ph} = 2 \times 10^8$ m/s connected to a CATV line with a Tee, as shown. This is like the blocker configuration we used in Project 2. Which of the following channels will be blocked by this line? (The frequencies are given in MHz.)

| Channel | 2 | 11 | 19 | 37 | 53 | 62 |
|---|---|---|---|---|---|---|
| Frequency | 55.25 | 199.25 | 151.25 | 301.25 | 397.25 | 451.25 |

i. (10 points) Resistance, Inductance and Capacitance

An air insulated parallel plate transmission line is constructed as shown in the figure. Assume that the length of the line is given by *l*.





i) Determine the resistance of this line. Circle the correct answer.

$$R = \frac{2l}{\sigma hw} \qquad R = \frac{l}{\sigma wt} \qquad R = \frac{l}{\sigma hw} \qquad R = \frac{2l}{\sigma wt}$$

ii) Determine the inductance of this line. Circle the correct answer.

$$L = \frac{\mu_o t2l}{w} \qquad L = \frac{\mu_o hl}{w} \qquad L = \frac{\mu_o h2l}{w} \qquad L = \frac{\mu_o tl}{w}$$

iii) Determine the capacitance of the line. Circle the correct answer.

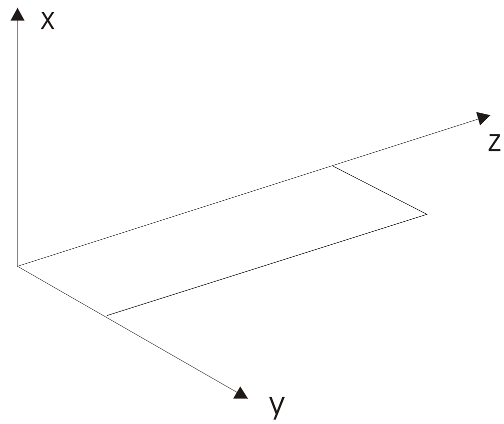
$$C = \frac{\varepsilon_o wl}{h} \qquad C = \frac{\varepsilon_o t2l}{h} \qquad C = \frac{\varepsilon_o w2l}{h} \qquad C = \frac{\varepsilon_o tl}{h}$$

iv) Determine the characteristic impedance $Z_o$ of the line, assuming that the resistance is small enough to neglect (lossless line approximation):

$$Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}\frac{w}{h}} \qquad Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o 2}\frac{h}{w}} \qquad Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}\frac{h}{w}} \qquad Z_o = \sqrt{\frac{\mu_o 2}{\varepsilon_o}\frac{w}{h}}$$

j. (10 points) Transmission Lines (Circle the correct answers)

The voltage on a parallel plate transmission line like the one in question i is given in phasor form as $\hat{V}(z) = V_m^+ \exp(-j\beta z) + V_m^- \exp(+j\beta z)$. Which of the following is the magnetic field in the region between the plates? Assume that the coordinate directions are as shown below and that the plate dimensions are the same as in question g. Circle the correct answer.




$$\hat{H}_y = \frac{V_m^+}{Z_o w}\exp(-j\beta z) + \frac{V_m^-}{Z_o w}\exp(+j\beta z)$$

$$\hat{H}_y = \frac{V_m^+}{Z_o h}\exp(-j\beta z) + \frac{V_m^-}{Z_o h}\exp(+j\beta z)$$

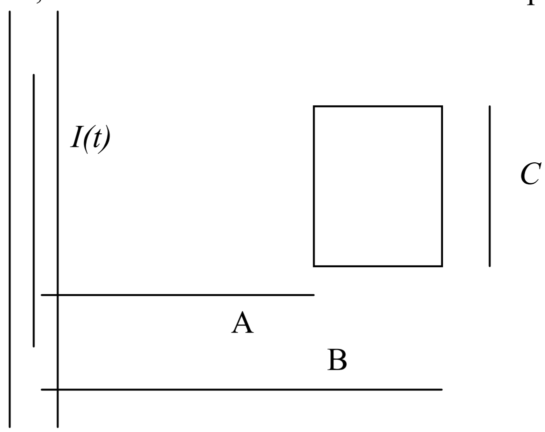
$$\hat{H}_y = \frac{V_m^+}{Z_o w}\exp(-j\beta z) - \frac{V_m^-}{Z_o w}\exp(+j\beta z)$$

$$\hat{H}_y = \frac{V_m^+}{Z_o h}\exp(-j\beta z) - \frac{V_m^-}{Z_o h}\exp(+j\beta z)$$

where *h* is the separation between the plates, *w* is the width of the plates and $Z_o$ is the characteristic impedance of the transmission line.

k. (5 Points) Faraday's Law

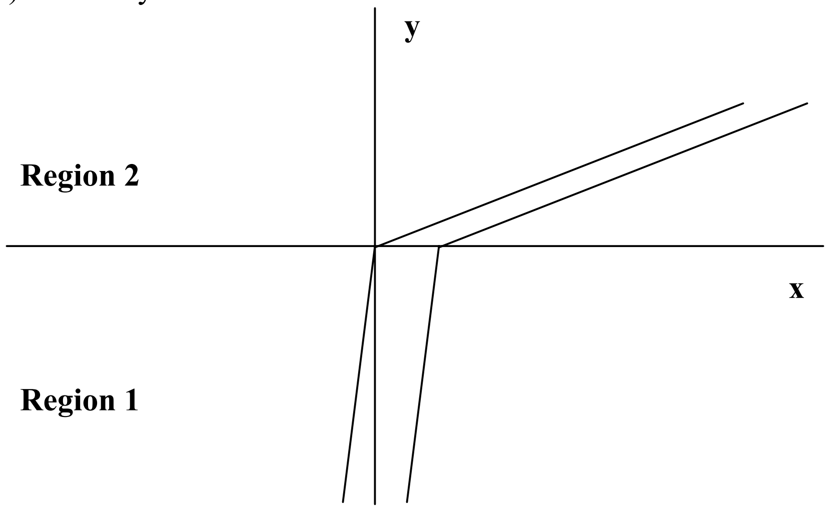
A long straight wire carries a current $I(t) = I_o \cos\omega t$. A rectangular N-turn loop is located as shown below with the wire in the plane of the loop. If the resistance of the N-turn loop is equal to $R$, what is the current induced in the loop?




$$I(t) = -\frac{N\mu_o C}{2\pi R}\ln\left(\frac{B}{A}\right)\omega I_o \cos\omega t \qquad I(t) = \frac{N\mu_o R}{2\pi C}\ln\left(\frac{B}{A}\right)\omega I_o \sin\omega t$$

$$I(t) = \frac{N\mu_o C}{2\pi R}\ln\left(\frac{B}{A}\right)\omega I_o \sin\omega t \qquad I(t) = \frac{N\mu_o C}{2\pi R}\ln\left(\frac{B}{A}\right) I_o \cos\omega t$$

l. (10 points) ) Boundary Conditions and Flux




Shown above are two parallel magnetic field ( $\vec{B}$ ) lines changing direction at the boundary between a magnetic material (with $\mu = \mu_r\mu_o$ ) and air ( $\mu = \mu_o$ ).

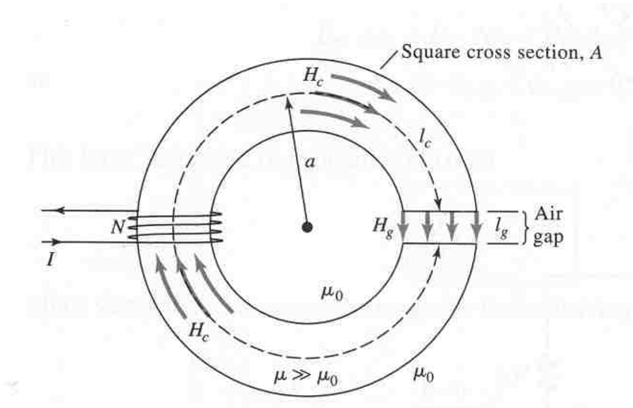
i) Which of the following is correct? Circle the correct answers. (Region 1 is the magnetic material, Region 1 is air, Region 2 is the magnetic material, Region 2 is air)

ii) The angle between the y-axis and the field lines in Region 1 is 20$^o$, while the angle between the field lines and the y-axis in Region 2 is 76$^o$. What is the relative permeability of the magnetic material? Circle the correct answer.

$\mu_r = 110$ $\mu_r = 11$ $\mu_r = 220$ $\mu_r = 22$

iii) The magnetic flux between the two field lines in region 1 is (greater than, less than or equal to) the flux between the field lines in region 2. The magnetic flux density between the two field lines in region 1 is (greater than, less than or equal to) the magnetic flux density between the two field lines in region 2.

m. (10 Points) Magnetic Circuits



(a)

A square cross-section toroidal core (area = $A$) with a small air gap is wound with $N$ turns carrying a current $I$. The permeability of the core material is large $\mu >> \mu_o$. The mean radius of the core is equal to $a$.

i) Draw the magnetic circuit for this case and determine the values of all of the sources and reluctances.

ii) Determine the inductance of this core with a small gap.

n. (10 Points) Miscellaneous Basic Information

Indicate which of the following statements are true or false.

i) There is no electric field inside a perfect conductor.

ii) A voltage is constant on a perfect conductor.

iii) In a conductor with finite conductivity $\sigma$ , the current flows in the same direction as the electric field.

iv) The electric field vector is directed towards positive charge and away from negative charge.

v) The electric and magnetic field vectors of a uniform plane wave are perpendicular to one another and to the direction of propagation of the wave.

vi) The current induced in a conductor by a time-varying magnetic field is directed in such a way to reduce the total magnetic field.

vii) For a pulsed voltage source, the input impedance of transmission line is equal to the characteristic impedance of the line.

viii) For a sinusoidally varying voltage source, the input impedance of a long, lossy transmission line is approximately equal to the characteristic impedance of the line.

ix) For a uniform plane wave incident obliquely on a dielectric interface, it is possible for the average power carried by the transmitted wave to be larger than the average power carried by the incident wave.

x) The skin effect causes the centers of large diameter wires to carry no current at high frequencies.

**2. Electric Fields** (30 Points)

A uniform volume charge distribution with $\rho_v = \rho_o$ exists in the spherical region $r < a$. There is also a uniform surface charge density $\rho_s = -\rho_{so}$ located on the spherical surface at $r = b$. There are no conductors or other materials involved in this problem.

a) What relationship must exist between $\rho_o$ and $\rho_{so}$ if the total charge in this configuration is zero?

b) Using the integral form of Gauss' Law, determine the electric field of this combination of a positive and a negative charge, everywhere in space (for all radii).

c) Using the differential form of Gauss' Law, verify that your answer to part (b) is consistent with your source distribution in part (a).

d) Determine the electric scalar potential at $r = a$, $V(a)$. Assume that the voltage goes to zero as $r \to \infty$.

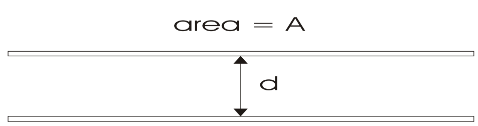
e) Determine the electric field energy stored in the region between $r = a$ and $r = b$.

**3. Inductance and Capacitance** (30 points)

Given below are several expressions for inductance or capacitance. Also given are the names associated with these expressions. You are to identify which expression goes with which name by filling in the blank next to the formula. Also, you must identify each of the terms in the expression. Identify the material constants and give typical values for the given type of device. For geometric parameters, you are to draw a simple picture of the geometry and label the parameters. An example is provided to show you how this is to be done.

A. Parallel Plate Capacitor
B. Coaxial Cable – Capacitance per unit length
C. Spherical Capacitor
D. Coaxial Cable – Inductance per unit length
E. Toroidal Inductor
F. Solenoidal Inductor

a. $\frac{\varepsilon A}{d}$ Farads Geometry ____A____




$\varepsilon = \varepsilon_r \varepsilon_o$ is the permittivity of the insulator. A typical value for the relative permittivity $\varepsilon_r$ is 2.1.

b. $\frac{\mu N^2 h}{2\pi} \ln\frac{b}{a}$ Henries Geometry ____________

c. $\dfrac{2\pi\varepsilon}{\ln\frac{b}{a}}$ Farads per meter Geometry ____________

d. $\dfrac{\mu}{2\pi}\ln\dfrac{b}{a}$ Henries per meter Geometry ____________

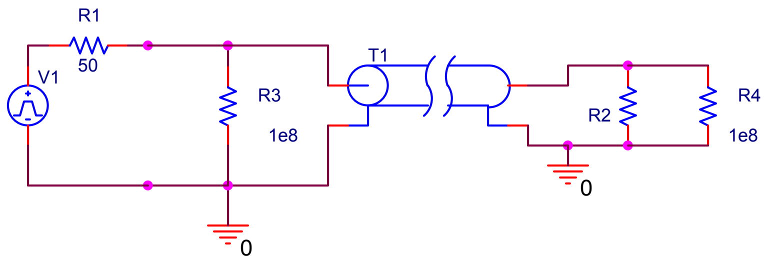
e. $\dfrac{\mu N^2 A}{d}$ Henries Geometry ____________

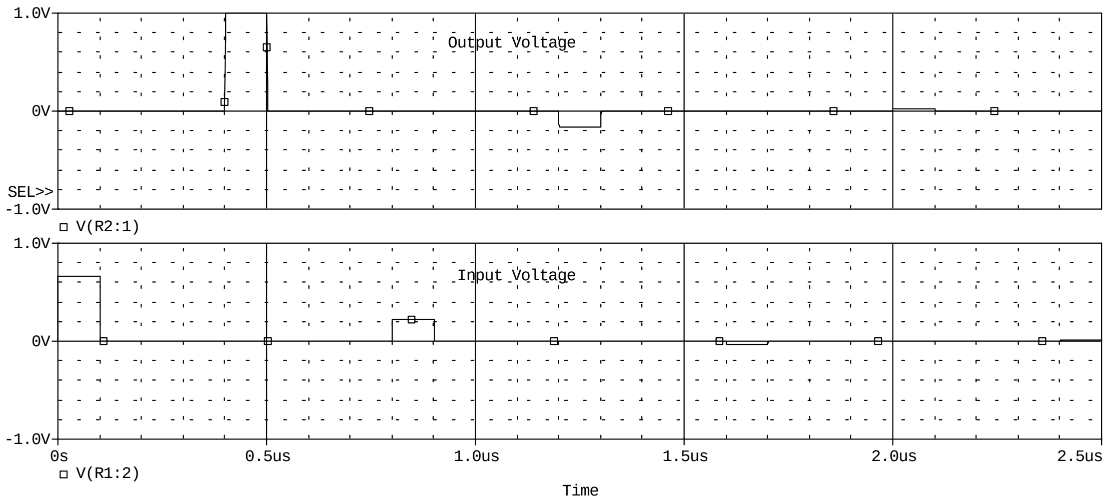
f. $\dfrac{4\pi\varepsilon ab}{b-a}$ Farads Geometry ____________

g. Derive any one of the expressions b – f.

**4. Transmission Lines** (30 Points)

A lossless transmission line is driven by a pulsed voltage source and connected to a resistive load, as shown. The source produces a 100 nanosecond, 1 volt pulse. The voltages at the input and output end of the transmission line are shown. Neither the characteristic impedance $Z_o$ , the load impedance $R2$, nor the insulating material used in the cable are known. However, we do know that the length of the line is 50 meters.




a. Determine the time it takes for the pulse to propagate from the input to the output of the transmission line. From this information, determine the relative permittivity $\varepsilon_r$ of the insulating material used in the cable.

b. Determine the characteristic impedance $Z_o$ of the transmission line and the load impedance $R2$. To determine these parameters, you must first generate the bounce or reflection diagram for this transmission line and then use the voltage data given above. Draw the bounce diagram on the back of the preceding page or on the next page.

c. Now that you have determined the characteristic impedance of this line, find the lowest non-zero frequency for which the input impedance is equal to the load impedance. Begin by first determining the propagation constant $\beta$. Then find the frequency and show that $Z_{in} = R2$.

**5. Electromagnetic Waves** (30 Points)


A uniform plane wave is incident normally on a dielectric interface between two media. Region 1 is air and region 2 is a dielectric region with an unknown permittivity $\varepsilon = ?$. The incident electric field is polarized in the x-direction and the magnetic field is polarized in the y-direction. The magnitude of the incident electric field is 1v/m and the frequency of the wave is 10MHz. It is observed that 25% of the average power is reflected.

a. Write the incident electric and magnetic field vectors in phasor form. Be sure that your answers are in the form of a vector.

b. Write the reflected electric and magnetic field vectors in phasor form in terms of the given information and an unknown reflection coefficient $\Gamma$. (You will figure out the value of $\Gamma$ in the next question.) Again be sure that your answers are in the form of a vector.

c. Determine the ratio of the reflected average power to the incident average power and then find the reflection coefficient $\Gamma$ .

d. From the reflection coefficient $\Gamma$ , determine the permittivity $\varepsilon = \varepsilon_r\varepsilon_o$ of region 2.

d. Write the transmitted electric and magnetic field vectors in phasor form.