Problem Solution #2

Calculation:

a. RG 58A/U: $\varepsilon_r = 2.3$, $a \approx 0.4mm$, $b \approx 1.4mm$.

$\mu_0 \approx 4\pi \times 10^{-7} H/m$,

$$l = (\mu_0 / 2\pi)\ln(b/a) \approx (4\pi \times 10^{-7} / 2\pi)\ln(1.4/0.4) \approx 2.51 \times 10^{-7} H/m$$

$$\varepsilon = \varepsilon_r \varepsilon_0, \quad \varepsilon_0 \approx 8.854 \times 10^{-12} F/m, \quad \varepsilon \approx 2.3 \times 8.854 \times 10^{-12} \approx 2.04 \times 10^{-11} F/m$$

$$c = 2\pi\varepsilon / \ln(b/a) = 2\pi \times 2.04 \times 10^{-11} / \ln(1.4/0.4) \approx 1.02 \times 10^{-10} F/m$$

They are almost the same as the lumped circuit transmission lines in the lab.
To model a 4 meter section of the cable,

$$L = 4 \times l = 4 \times 2.51 \times 10^{-7} = 1.004 \times 10^{-6} H$$

$$C = 4 \times c = 4 \times 1.02 \times 10^{-10} = 4.08 \times 10^{-10} F$$

so we should choose $\boxed{L = 1.004 \times 10^{-6} H}$ and $\boxed{C = 4.08 \times 10^{-10} F}$

20 sections:

$$L_{total} = 20 \times L = 20 \times 1.004 \times 10^{-6} \approx 2 \times 10^{-6} H$$
$$C_{total} = 20 \times C = 20 \times 4.08 \times 10^{-10} \approx 8.2 \times 10^{-9} F$$

80 meters of RG 58:

$$L_{total} = 80 \times l = 80 \times 2.5 \times 10^{-7} = 2 \times 10^{-6} H$$
$$C_{total} = 80 \times c = 80 \times 10^{-10} = 8 \times 10^{-9} F$$

It's almost equivalent.

b. $V = V_0 \cos(\omega s)$ where $s = t \pm (z/u)$

$$\frac{\partial s}{\partial z} = \pm\frac{1}{u}, \quad \frac{\partial s}{\partial t} = 1, \quad \frac{dV}{ds} = -V_0\omega\sin(\omega s)$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{dV}{ds}\frac{\partial s}{\partial z}\right) = \frac{\partial}{\partial z}\left[-V_0\omega\sin(\omega s)\frac{\partial s}{\partial z}\right] = -V_0\omega^2\cos(\omega s)\left(\pm\frac{1}{u}\right)\frac{\partial s}{\partial z} = -V_0\omega^2\cos(\omega s)\left(\pm\frac{1}{u}\right)^2$$
$$= -V_0\omega^2\cos(\omega s)\frac{1}{u^2}$$

$$\frac{\partial^2 V}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{dV}{ds}\frac{\partial s}{\partial t}\right) = \frac{\partial}{\partial t}\left[-V_0\omega\sin(\omega s)\frac{\partial s}{\partial t}\right] = -V_0\omega^2\cos(\omega s)\frac{\partial s}{\partial t} = -V_0\omega^2\cos(\omega s)$$

Since $$\frac{\partial^2 V(z,t)}{\partial z^2} - lc\frac{\partial^2 V(z,t)}{\partial t^2} = 0,$$

$$-V_0\omega^2\cos(\omega s)\frac{1}{u^2} - lc[-V_0\omega^2\cos(\omega s)] = 0,$$ This gives $\boxed{u = 1/\sqrt{lc}}$

For the RG 58 A/U cable,

$$u = 1/\sqrt{lc} = 1/\sqrt{2.51\times10^{-7}\times1.02\times10^{-10}} \approx \boxed{1.98\times10^{8}\, m/s}$$

The time delay for the 4 meter section is,

$$t = z/u = 4/1.98\times10^{8} \approx \boxed{2.02\times10^{-8}\, s}$$

Substitute $V = V_0\cos(\omega s)$ and $I = I_0\cos(\omega s)$, where $s = t \pm (z/u)$,

into transmission line equation $-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + c\frac{\partial V(z,t)}{\partial z}$, G=0.

$$I_0\omega\sin(\omega s)(\pm\frac{1}{u}) = -cV_0\omega\sin(\omega s)$$

$$Z_0 = \left|V_0/I_0\right| = 1/cu = \frac{1}{c(1/\sqrt{lc})} = \sqrt{l/c}$$

The characteristic impedance of the cable is,

$$Z_0 = \sqrt{2.51\times10^{-7}/1.02\times10^{-10}} \approx \boxed{49.6\Omega}$$