Problem Solution #4

1.

$$Q=\int \rho_v dv=\int_0^{2\pi} d\phi\int_0^{\pi}\sin\theta d\theta\int_0^{0.03}3r^2r^2dr=2\pi\times 2\times 3\times\frac{r^5}{5}\Big|_0^{0.03}=1.832\times10^{-7}C=183.2nC$$

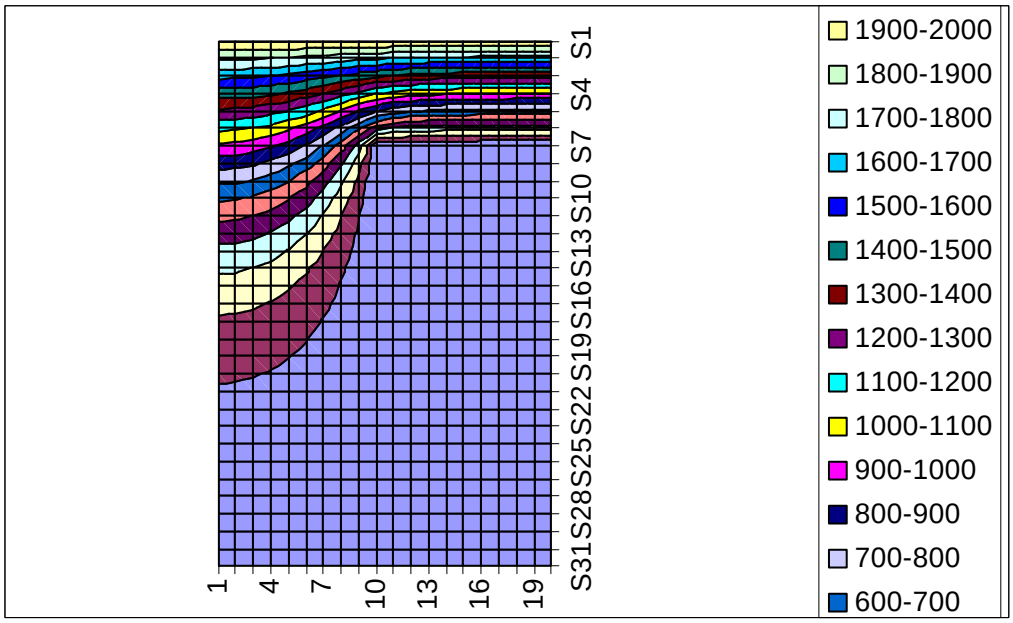
2.

$$\int \vec{E}\cdot d\vec{a}=\frac{1}{\varepsilon_0}\int \rho d^3r$$

$$2\pi rlE=\frac{1}{\varepsilon_0}\times 2\pi\times 0.05l\int_0^r r'\times r'dr'$$

$$\vec{E}=\frac{1.67\times10^{-2}r^2}{\varepsilon_0}\hat{r}$$

3.



For every cell on the top,

$$\vec{E}=-\nabla V \quad\rightarrow\quad V=-\int \vec{E}\cdot d\vec{l}=Ed \quad\rightarrow\quad \vec{D}=\varepsilon\vec{E} \quad\rightarrow\quad D=\rho_s \quad\rightarrow$$

$$Q=\rho_s A \quad\rightarrow\quad Q_T=\sum Q$$

$$C=\frac{Q_T}{V}$$

One side: $C\approx 130\,pF$

Two side: $C\approx 260\,pF$