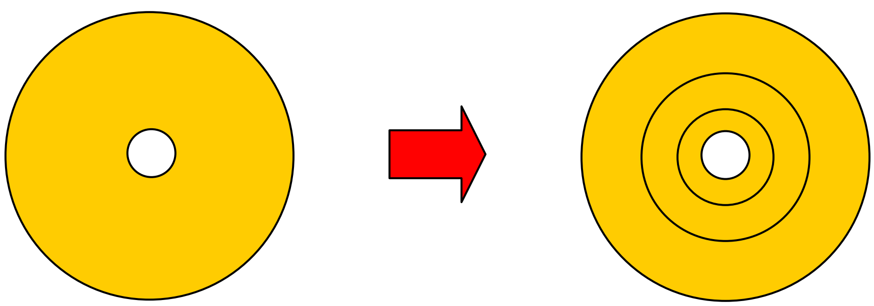
Homework 4

Due 8 March 2005

1. **Increasing the Breakdown Voltage:** This first question is a mini design project. Your first step is to find a commercial cable (coaxial or two wire line) for which you have the following information: the capacitance per meter, the type of material used for the insulator (to make it simpler to grade this problem, choose a cable that uses solid polyethylene (PE) for the insulator), the dimensions of the inner and outer conductors, and the maximum electric field or voltage that the cable can sustain. The latter parameter may not be given, but since you will know the insulating material, you can look up the value of its breakdown field.

   a. Using the parameters you have for your selected cable, calculate the capacitance from first principles. That is, find $\vec{D}$, $\vec{E}$, and V as functions of radius in the region between the two conductors and then use this information to determine the capacitance, either using the charge or the energy method. If you use the energy method it is not necessary to find the voltage.

We wish to improve on this cable in some way. Our specific goal will be to change the insulator so that the breakdown voltage increases while maintaining the same capacitance per unit length. We will assume that the inner and outer conductors remain the same. We will only change the insulator. The approach we will take will be to replace the PE with two or three other insulators as shown.



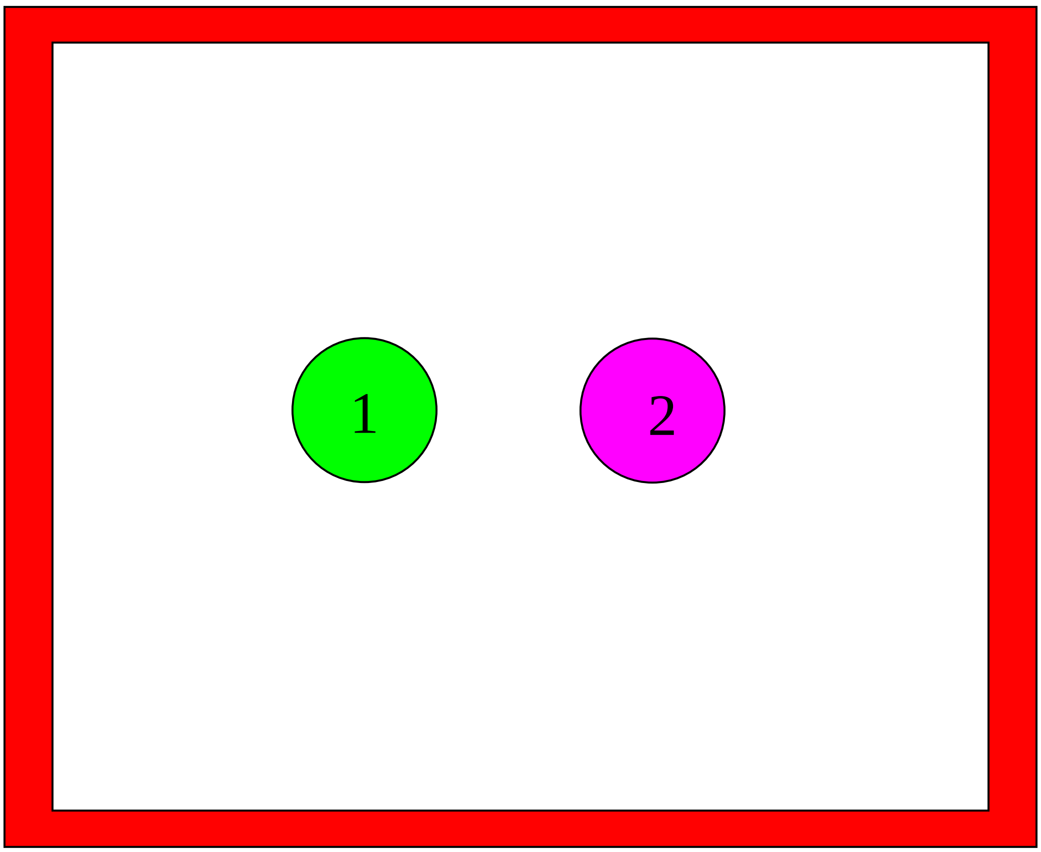
The center insulator will be chosen to have the largest breakdown voltage, the other one or two insulators will be chosen to obtain the same capacitance. Recall that each will have a different dielectric constant. To find material properties, you can go to MatWeb http://matweb.com/ which has almost any property one can think of for commercially available materials.

   b. Begin by solving for the capacitance of a coaxial cable with 2 insulating regions. Assume that the first insulator is in the region $a \le r \le b$ and the second insulator is in the region $b \le r \le c$ where a is the radius of the inner conductor and c is the radius of the outer conductor. The dielectric constant of region *1* is ε_1 and the dielectric constant of region *2* is ε_2. Again, do this from first principles and find $\vec{D}$, $\vec{E}$, and V as functions of radius in the region between the two conductors and then use this information to determine the capacitance, either using the charge or the energy method. If you use the energy method, it is not necessary to find the voltage.

To complete your design, you need to identify two new dielectric materials, one of which has a higher breakdown voltage than PE and the other has a dielectric constant that compensates for the first dielectric in the capacitance. You can choose the distance *b* to get the capacitance you need. If you cannot make two insulators work, you will have to try three.

c. What are the final parameters of your design? Demonstrate that the capacitance is the same and the breakdown voltage is larger.

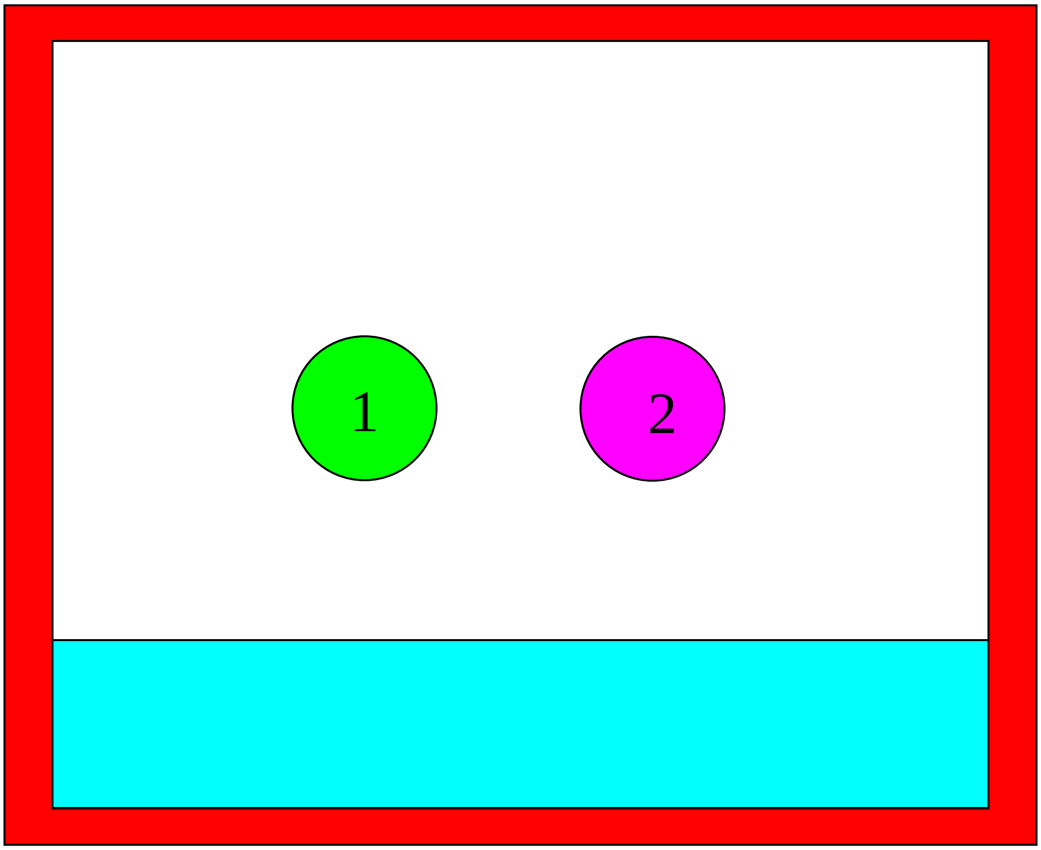
2. Using a Spreadsheet to Find Capacitance: Assume that you have the following two dimensional configuration of conductors (all dielectrics are air).



Assume that the voltage on conductor 1 is *-100V* and on conductor 2 is *+100V*. The diameter of each is *10mm*, the outer box is *50mm* by *50mm*, and the distance between the conductors is also *10mm*.

a. Use the spreadsheet method to find the capacitance per unit length of this shielded, two wire transmission line.
b. Produce a plot showing at least 8 equipotentials. Sketch a representative set of electric field lines on this plot.
c. Find the charge per unit length.
d. Using a standard analytic formula, find the capacitance per unit length of the two wire line without the shield.

e. Assume that about half of the lower half of the insulating region is filled with a dielectric with $\varepsilon = 81\varepsilon_o$. Repeat all steps and find the capacitance per unit length.



3. Poisson's Equation: The electric scalar potential in a spherical region $0 \le r \le a$ is given by $V(r) = V_o$. This region is known to be filled with charge with an unknown density distribution $\rho = \rho(r)$. Determine the charge distribution responsible for this potential.