Homework 7

Due 19 April 2005

1. Plane Waves in Lossless Media

The magnetic field of a uniform plane in a lossless medium is given by $\vec{H}(z,t) = \hat{a}_x 100 \sin\left(6\pi 10^6 t + 0.04\pi z\right)$. Find:

a) The magnetic field phasor $\tilde{\vec{H}}(z)$
b) The direction of wave propagation
c) The frequency of the wave, *f*, and its period, *T*.
d) The phase velocity, u_p
e) The wavelength in the material, λ, and the propagation constant of the wave, β.
f) The relative permittivity of the material, ε_r, assuming the material is non-magnetic.
g) The electric field phasor, $\tilde{\vec{E}}(z)$
h) The electric field in time domain form, $\vec{E}(z,t)$

Finally, using Maple, Matlab or some similar tool, plot the electric and magnetic fields of the waves as a function of position at three times, $t = 0$, $t = T/3$, $t = 2T/3$.

2. Power Absorption in a Lossy Material

A uniform plane wave ($f = 3GHz$) is propagating in a huge block of ice. At $z = 0$, the average power density of the wave is $1kW/m^2$. We wish to investigate the heating of the ice by the wave. A reference containing information on the electrical properties of a wide variety of materials:
http://www.rfcafe.com/references/electrical/dielectric_constants_strengths.htm
Note that the data provided gives the loss tangent, not the imaginary part of the permittivity. While it has nothing to do with this problem, there is another good reference on the electrical properties of human tissue provided by a laboratory in Florence, Italy: http://niremf.ifac.cnr.it/tissprop/htmlclie/htmlclie.htm You can get a sense of the range of possible properties from these two sources.

Assume that the direction of wave propagation is +z. Determine the following:

a) The complex permittivity $\varepsilon_c = \varepsilon' - j\varepsilon''$.
b) The basic wave parameters ω, α, β, λ, and η_c.
c) The electric field phasor $\tilde{\vec{E}}(z)$ and the magnetic field phasor $\tilde{\vec{H}}(z)$.
d) The phase velocity, u_p.
e) The average power density (Poynting Vector) at $z = 1m$.
f) The average power deposited in a cubic meter of the material between $z = 0$ and $z = 1m$ using the integral of $\frac{1}{2}\text{Re}\,\vec{J}(z)\cdot\vec{E}^*(z)$ as in lecture 21.

Compare your answers to parts e) and f).