Homework 7
Fields and Waves I
Fall 2007

1. For copper the conductivity is $\sigma_{cu} = 5.7 \times 10^7 \, S/m.$ and for Teflon $\sigma_{teflon} = 3.0 \times 10^{-9} \, S/m$. At 1 MHz show that one is a “good conductor” and one is a “good insulator”.

Solution :

Copper :

$$\frac{\sigma}{\omega\,\epsilon} = \frac{5.8 \times 10^7}{2\,\pi \times 10^6 \times \frac{1}{36\pi} \times 10^{-9}} = 1.04 \times 10^{12} \gg 1$$

So copper is a good conductor.

Teflon :

$$\frac{\sigma}{\omega\,\epsilon} = \frac{3 \times 10^{-9}}{2\,\pi \times 10^6 \times 2.1 \times \frac{1}{36\pi} \times 10^{-9}} = 2.57 \times 10^{-5} \ll 1$$

So Teflon is a good insulator.

2. Find the magnetic field in free space if the electric field is given by
$E = E_m \sin(\alpha x) \cos\left(\omega t - \beta z\right) \hat{a}_y$.
Show that in order for these fields to satisfy Maxwell’s equations we must have
$\alpha^2 + \beta^2 = \mu_0 \varepsilon_0 \omega^2$
In phasor notation,

$$E^{\sim} = E_m \sin(\alpha x)\, e^{-j\,\beta z}$$

$$\nabla \times E^{\sim} = -j\,\omega\mu H^{\sim}$$

$$\nabla \times E^{\sim} = \left(-\frac{\partial E_y}{\partial z}\right) x^{\rightarrow} + \left(\frac{\partial E_y}{\partial x}\right) z^{\rightarrow}$$

$$= \left(j\beta E_m \sin \alpha x \;\, e^{-j\,\beta z}\right) x^{\rightarrow} + \left(\alpha E_m \cos \alpha x \, e^{-j\,\beta z}\right) y^{\rightarrow}$$

$$H^{\sim} = -\frac{\beta}{\mu_0 \omega} \left(E_m \sin \alpha x \, e^{-j\,\beta z}\right) x^{\rightarrow} + \frac{j\alpha}{\mu_0 \omega} \left(E_m cos \alpha x \, e^{-j\,\beta z}\right) z^{\rightarrow}$$

$$H = -\frac{\beta}{\mu_0 \omega} \left(E_m \sin \alpha x \cos(\omega t - \beta z)\right) x^{\rightarrow} - \frac{\alpha}{\mu_0 \omega} \left(E_m \cos \alpha x \sin(\omega t - \beta z)\right) z^{\rightarrow}$$

Also,

$$\nabla \times H^{\sim} = j\omega\, \epsilon E^{\sim}$$

$$\frac{j\,\beta^2}{\omega\mu_0} E_m \sin\alpha x + \frac{j\,\alpha^2}{\mu_0\omega} E_m \sin\alpha x\ e^{-j\beta z} = j\omega\epsilon(E_m \sin\alpha x\ e^{-j\,\beta z})$$

Therefore,

$$\alpha^2 + \beta^2 = \mu_0\epsilon_0\omega^2$$

3. Determine the polarization of the following waves

$E = 1\cos(\omega t + \beta z)\hat{a}_x + 1\sin(\omega t + \beta z)\hat{a}_y$

$E = 1\cos(\omega t + \beta z)\hat{a}_x - 1\sin(\omega t + \beta z)\hat{a}_y$

$E = 1\cos(\omega t + \beta z)\hat{a}_x - 2\sin(\omega t + \beta z - 45^0)\hat{a}_y$

Solution :

a) Since $|E_x| = |E_y|$, and the phase angle between the two components is 90 the polarization is circular
At z=0, ωt=0,
$$E = E_x$$
At z=0, ωt=π/2
$$E = E_y$$
Therefore the direction of rotation of phase is anti-clockwise.
Direction of propagation is along -z. So the polarization is left circular.

b) Since $|E_x| = |E_y|$, and the phase angle between the two components is 90 the polarization is circular
At z=0, ωt=0,
$$E = E_x$$
At z=0, ωt=π/2
$$E = -E_y$$
Therefore the direction of rotation of phase is clockwise.
Direction of propagation is along -z. So the polarization is right circular.

c) Since $|E_x| \neq |E_y|$, the polarization is elliptical.
At z=0, ωt=0,
$$E = E_x$$
At z=0, ωt=π/4
$$E = -E_y$$
Therefore the direction of rotation of phase is clockwise.
Direction of propagation is along -z. So the polarization is right elliptical.

4. A wave is traveling in a material with relative permittivity = 2 and a frequency of 3 GHz. Find the phase velocity, wave impedance and wavelength.

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{2}\sqrt{\mu_0\epsilon_0}} = \frac{3\times 10^8}{\sqrt{2}} = 2.12\times 10^8 m/s$$

$$\lambda = \frac{v_p}{f} = 2.12\times\frac{10^8}{3\times 10^9} = 7.07\times 10^{-2} m$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}} = 266.45\ \Omega$$