

Homework 8

Due: 3 May 2005

Transmission Lines**1. Multiple Choice (16)**

a) The VSWR (standing wave ratio):

- a) is a measure of the match between the source impedance and line impedance
- b) may be equal to 1
- c) is a measure of the match between the load impedance and line impedance
- d) may be equal to 0

b) The impedance of a lossless transmission line depends on

- a) material properties
- b) voltage
- c) geometry
- d) current

c) If the total voltage as a function of position on a transmission line is:

$$V_L = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

- a) The load is an open circuit
- b) The transmission line is lossless
- c) $\Gamma_L = -1$
- d) $VSWR = \infty$ (infinity)

d) The characteristic impedance of typical cable TV transmission line is

- a) 93 Ω
- b) 50 Ω
- c) 75 Ω
- d) 300 Ω

2. For the following expressions state if the wave is a standing or traveling wave, explain. If it is a traveling wave give the direction. For either case give the wavelength. C_o is an arbitrary constant. (10 Pts).

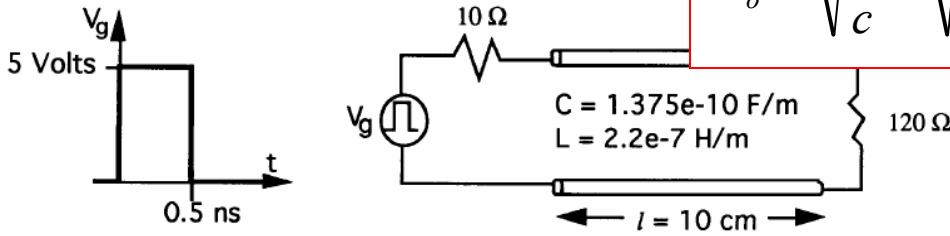
a) $C_o \sin(1000t + \pi/2) \cos(3000x + 4)$ *Standing because the arguments depend only on t or x , not both.*

b) $C_o \cos(4000t + 375y + 11)$ *Traveling because the argument depends on both t and y . The wave travels in the negative y direction because the sign of the y term is the same as the sign of the t term.*

3.

$$u = \frac{1}{\sqrt{lc}} = 1.82 \times 10^8$$

$$Z_o = \sqrt{\frac{l}{c}} = \sqrt{\frac{2.2e-7}{1.375e-10}} = 40$$



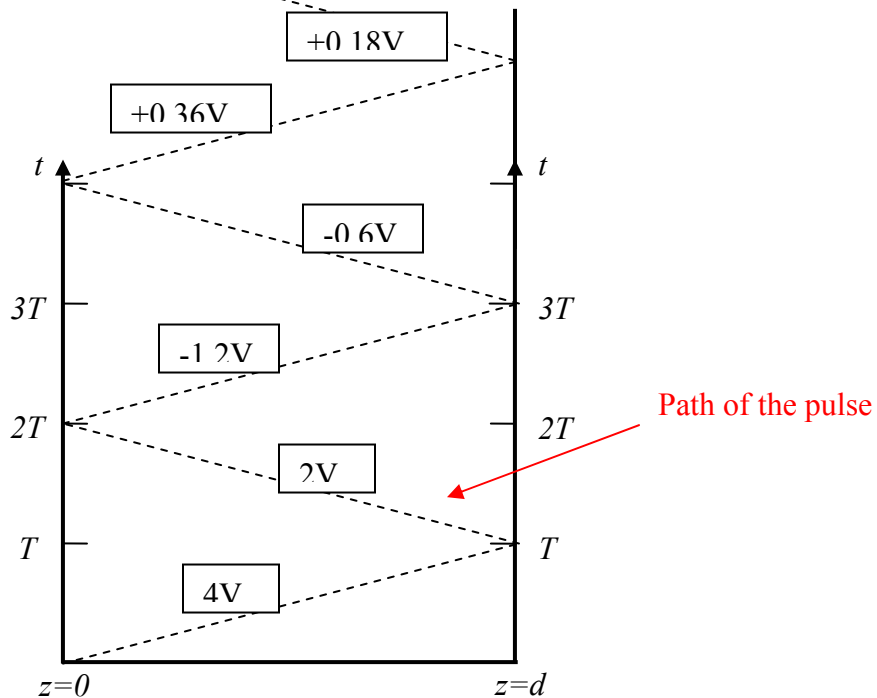
A component on a printed circuit board sends a 5 [V], 0.5 [ns] pulse down a 10 [cm] long transmission line. The circuit is shown in the figure. Assume that the line is lossless.

a) Draw a reflection (bouncing) diagram illustrating the circuit behavior. Include all appropriate numbers. Model the behavior until after the signal gets to the load for the third time (i.e. 3 reflections off the load and 2 off the generator). (12 pts)

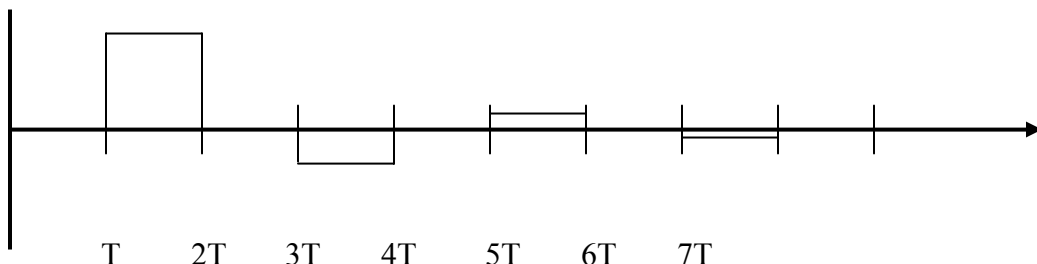
$$\Gamma_s = \frac{10 - 40}{10 + 40} = -0.6 \quad \Gamma_L = \frac{120 - 40}{120 + 40} = 0.5 \quad T = \frac{0.10}{1.82 \times 10^8} = 5.5 \times 10^{-10}$$

Voltage divider:

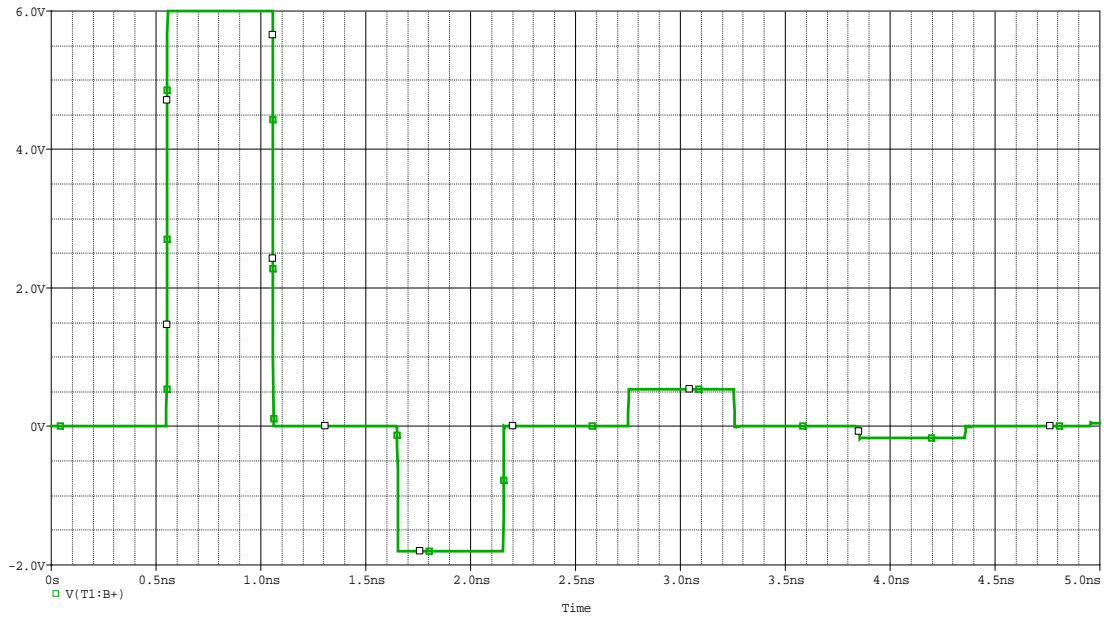
$$V_{IN} = 5 \frac{40}{40 + 10} = 4$$



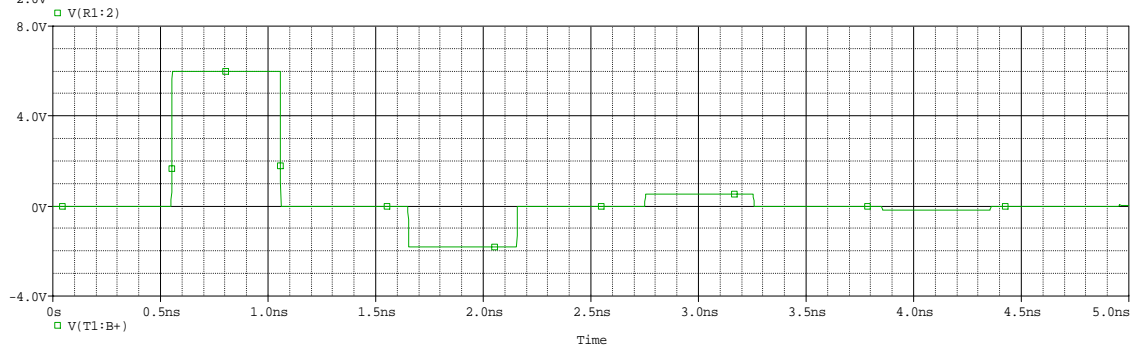
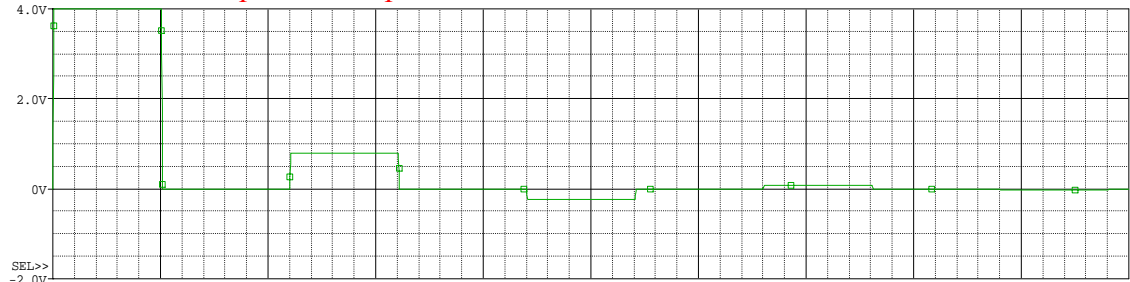
b) Plot the voltages at the load and the input as a function of time. Be sure to include all numerical values on the axis. (12 pts) *At the output end,*



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PSpice is used to plot this out just for neatness. It can always be used to check results for homework. At the input and output ends



Electrostatics**4 . Multiple Choice (16)**

a) The capacitance of two fixed conductors with free space between them:

- a) increases if the voltage difference between the conductors decreases
- b) increases if the magnitude of the charge on the conductors increases
- c) increases if a dielectric material is placed between the conductors
- d) increases if the stored electric energy W_E increases

b) Which of the following are always continuous across a material boundary?

- a) Tangential electric field, \vec{E}_t
- b) Normal electric field, \vec{E}_n
- c) Tangential electric flux, \vec{D}_t
- d) Voltage, V

c) Which of the following differential operations result in a scalar (\vec{A} is an arbitrary vector, f is an arbitrary scalar)

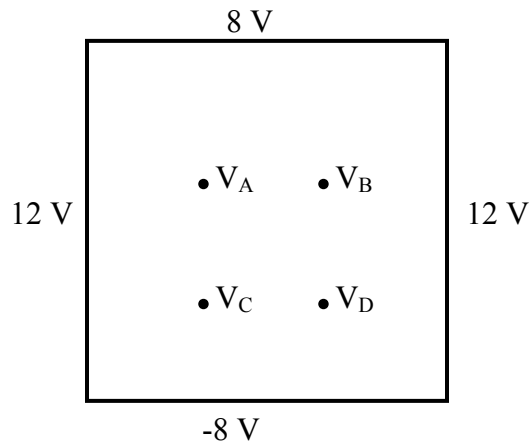
- a) the divergence $\nabla \cdot \vec{A}$
- b) the gradient ∇f
- c) the curl $\nabla \times \vec{A}$
- d) the Δ
- e) the Laplacian, $\nabla^2 f$

d) The field inside a cube with sides of length a is $\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y$. The stored energy in this region is:

- a) $\frac{1}{2} \epsilon (E_x + E_y) a^3$
- b) $\frac{1}{2} \epsilon (E_x + E_y)^2 a^3$
- c) $\frac{1}{2} \epsilon (E_x^2 + E_y^2) a^3$
- d) 0

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5. Use the finite difference approximation to estimate the voltages at the four points shown in the figure. (13 pts)



The voltage at each point the first time through is equal to:

$$V_A = (12 + 8 + 0 + 0) / 4 = 5$$

$$V_B = (12 + 8 + 5 + 0) / 4 = 6.25$$

$$V_C = (12 - 8 + 5 + 0) / 4 = 2.25$$

$$V_D = (12 - 8 + 2.25 + 6.25) / 4 = 12.5 / 4 = 3.125$$

The voltage at each point the second time through is equal to:

$$V_A = (12 + 8 + 6.25 + 2.25) / 4 = 7.125$$

$$V_B = (12 + 8 + 7.125 + 3.125) / 4 = 7.5625$$

$$V_C = (12 - 8 + 7.125 + 3.125) / 4 = 3.5625$$

$$V_D = (12 - 8 + 7.5625 + 3.5625) / 4 = 12.5 / 4 = 3.7813$$

The voltage at each point the n th time through is equal to:

$$V_A = 8$$

$$V_B = 8$$

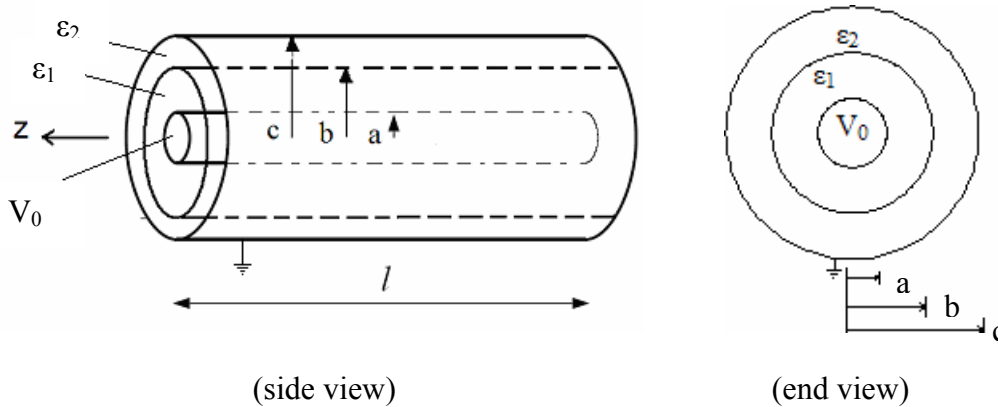
$$V_C = 4$$

$$V_D = 4$$

The voltage can also be guessed and checked. This same result came from a spreadsheet.

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6. A coaxial cable is constructed using two insulating materials. There is a voltage V_0 on the inner conductor and the outer conductor is grounded. The charge per unit length on the inner conductor is $\rho_l [C/m]$.



a) Determine the electric field \vec{E} in the region $a \leq r \leq c$ (7pts)

The electric flux density is $D_r = \frac{\rho_l}{2\pi r}$ so the electric field is $E_r = \frac{D_r}{\epsilon} = \frac{\rho_l}{2\pi \epsilon r}$ where $\epsilon = \epsilon_1$ or $\epsilon = \epsilon_2$ for $a < r < b$ or $b < r < c$, respectively.

b) Determine the voltage (potential), V at all points in the same region, $a \leq r \leq c$ (7 pts)

The voltage is $V(a) = -\int_c^a E_r dr = -\int_c^b \frac{\rho_l}{2\pi \epsilon_2 r} dr - \int_b^a \frac{\rho_l}{2\pi \epsilon_1 r} dr = \frac{\rho_l}{2\pi} \left(\frac{1}{\epsilon_1} \ln \frac{b}{a} + \frac{1}{\epsilon_2} \ln \frac{c}{b} \right)$

$V(r) = -\int_c^r E_r dr = -\int_c^b \frac{\rho_l}{2\pi \epsilon_2 r} dr - \int_b^r \frac{\rho_l}{2\pi \epsilon_1 r} dr = \frac{\rho_l}{2\pi} \left(\frac{1}{\epsilon_1} \ln \frac{b}{r} + \frac{1}{\epsilon_2} \ln \frac{c}{b} \right)$ for $a < r < b$

$V(r) = -\int_c^r E_r dr = -\int_c^r \frac{\rho_l}{2\pi \epsilon_2 r} dr = \frac{\rho_l}{2\pi} \left(\frac{1}{\epsilon_2} \ln \frac{c}{r} \right)$ for $b < r < c$

c) Determine the capacitance, C per unit length of this cable? (7 pts)

$V(a) = V_o = \frac{\rho_l}{2\pi} \left(\frac{1}{\epsilon_1} \ln \frac{b}{a} + \frac{1}{\epsilon_2} \ln \frac{c}{b} \right)$ so that the capacitance per unit length is

$\frac{C}{l} = \left(\frac{\rho_l}{2\pi} \left(\frac{1}{\epsilon_1} \ln \frac{b}{a} + \frac{1}{\epsilon_2} \ln \frac{c}{b} \right) \right)^{-1} = \frac{2\pi}{\frac{1}{\epsilon_1} \ln \frac{b}{a} + \frac{1}{\epsilon_2} \ln \frac{c}{b}}$ which is exactly the same result one

would get by taking the series combination of the two cylindrical capacitors that represent each region of insulator.

Magnetic Fields**7. Multiple Choice (16)**

a) Which of the following is true regarding the magnetic properties of materials:

- a) relative permeability can be less than 1
- b) relative permeability can be greater than 10000
- c) plastic makes an excellent core for a transformer
- d) it is possible to make frogs levitate

b) For Beakman's Motor:

- a) The field due to the current in the coil may be approximated as a solenoid type of field
- b) The battery internal resistance wasn't negligible in the circuit analysis
- c) The movie brought tears to my eyes; I can't wait for the sequel
- d) The oscilloscope impedance is approximately the same magnitude as the resistive components in the circuit (wire, paper clips, etc.)

c) For a current carrying wire with a uniform current density :

- a) $\nabla \cdot \vec{B} = 0$ inside the wire
- b) $\nabla \cdot \vec{B} = 0$ outside the wire
- c) $\nabla \times \vec{H} = 0$ inside the wire
- d) $\nabla \times \vec{H} = 0$ outside the wire

d) Hysteresis:

- a) describes the nonlinear relationship between \vec{H} and \vec{B}
- b) describes the nonlinear relationship between \vec{E} and \vec{D}
- c) applies to ferromagnetic materials
- d) applies to dielectric materials

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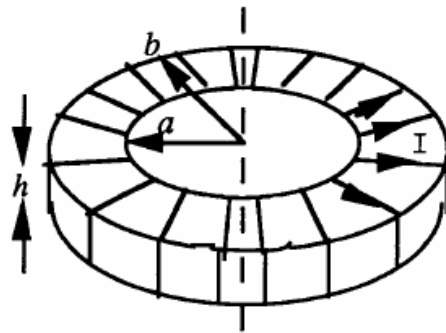
8. A circular conductor of radius $r_0 = 1$ [cm] has an internal field $\vec{H} = \frac{10^4}{r} \left(\frac{1}{a^2} \sin ar - \frac{r}{a} \cos ar \right) \hat{\phi}$ [A/m] where $a = \frac{\pi}{2r_0}$. Find the total current in the conductor. (10 pts)

The total current can be determined from the line integral of H around the boundary of the conductor $I_{encl} = \oint \vec{H} \cdot d\vec{l}$ First evaluate H at the boundary $r_0 = 1$.

$$\vec{H} = \frac{10^4}{r} \left(\frac{1}{a^2} \sin ar - \frac{r}{a} \cos ar \right) \hat{\phi} = \frac{10^4}{1} \left(\frac{1}{1^2} \sin \frac{\pi}{2} - \frac{1}{1} \cos \frac{\pi}{2} \right) \hat{\phi} = 10^4 \hat{\phi}$$

$$I_{encl} = \oint \vec{H} \cdot d\vec{l} = 2\pi 10^4$$

9. A toroidal magnet is constructed using a rectangular cross section toroidal core around which N turns of wire carrying a current I are wound. The permeability of the core is $\mu \gg \mu_0$.



- a) Find the magnetic field, \vec{B} everywhere? (6 pts)

The magnetic field outside of the core is zero. Inside the core, we can use Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B_\phi = \mu NI \text{ so that } B_\phi = \frac{\mu NI}{2\pi r}.$$

- b) Determine the magnetic flux, ψ linked by the wires? (6 pts)

$$\text{The magnetic flux } \psi = \int \vec{B} \cdot d\vec{S} = h \frac{\mu NI}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu NI}{2\pi} \ln \frac{b}{a} h$$

- c) Determine the total magnetic field energy, W_m stored in the core material (6 pts)

$$\text{The magnetic field energy is } W_m = \frac{1}{2} \int \vec{B} \cdot \vec{H} dv = \frac{1}{2} 2\pi h \int_a^b \frac{\mu NI}{2\pi r} \frac{NI}{2\pi r} r dr = \frac{1}{2} \frac{\mu N^2 I^2}{2\pi} \ln \frac{b}{a} h$$

- d) Determine the inductance of the toroid, L from your answer to (c) (6 pts)

$$\text{The inductance determined from the energy } W_m = \frac{1}{2} LI^2 = \frac{1}{2} \frac{\mu N^2 I^2}{2\pi} \ln \frac{b}{a} h \text{ is}$$

$$L = \frac{\mu N^2}{2\pi} \ln \frac{b}{a} h$$

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EM Waves

10. Multiple Choice (16)

a) A transverse EM wave in free space

- a) is traveling with the speed of light ($c=3 \times 10^8$ m/s)
- b) has a magnetic field which is parallel to the direction of propagation
- c) has an intrinsic impedance of 377 Ohms
- d) has electric and magnetic fields perpendicular to each other

b) Complex permittivity

- a) should be considered when conductivity is finite in a dielectric
- b) is independent of frequency
- c) has only a real part if the medium is lossless
- d) is a function of skin depth

c) A plane wave in free space is normally incident on a lossy dielectric material

- a) at the location of the boundary, the electric field in both regions must be the same
- b) at the location of the boundary, the magnetic field in both regions must be the same
- c) the magnitude of the reflection coefficient will be greater than 1
- d) a current density will be induced in the bulk of the material

d) Polarization

- a) describes the state (e.g. circular, linear, etc) of the wave
- b) describes the locus of the tip of \vec{E}
- c) depends on the phase difference between two orthogonal components of the \vec{E}
- d) depends on the ratio of the magnitudes of two orthogonal components of the \vec{E}

11. A plane wave is normally incident on distilled water ($\sigma \approx 0$, $\epsilon_r = 80$). Find the reflection and transmission coefficients. (16 pts)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \eta_1 = \eta_0 = 120\pi \quad \eta_2 = \frac{1}{\sqrt{80}} \eta_0 = \frac{120\pi}{\sqrt{80}} = \frac{120\pi}{8.94} = 13.4\pi = 42$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\Gamma = \frac{1 - \sqrt{80}}{1 + \sqrt{80}} = -0.8 \quad \tau = \frac{2}{1 + \sqrt{80}} = 0.2$$

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12. The electric field phasor of a uniform plane wave traveling in a lossless medium with an intrinsic impedance of $188.5 [\Omega]$ is given by $\tilde{E} = 10e^{-j4\pi y} \hat{a}_z [mV/m]$. Determine

a) The associated magnetic field phasor \tilde{H} (6 pts)

The magnitude of the magnetic field phasor $\tilde{H} = \frac{\tilde{E}}{\eta} = \frac{10e^{-j4\pi y}}{188.5}$

The wave propagates in the y-direction, which must be the direction of the cross product of E and H. E is in the z-direction which means that H is in the x-direction.

Thus, $\tilde{H} = \frac{10e^{-j4\pi y}}{188.5} \hat{a}_x$

b) The instantaneous expression for \vec{E} if the medium is non-magnetic ($\mu = \mu_0$) (6 pts)

$\vec{E}(y,t) = \hat{a}_z 10 \cos(\omega t - \beta y)$ *From the given expression, $\beta = 4\pi = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r}$. We*

also know that $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\frac{1}{\epsilon_r}} = 188.5$ which gives us $\epsilon_r = \left(\frac{377}{188.5}\right)^2 = 4$. We then have

that $4\pi = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r} = \frac{\omega 2}{3 \times 10^8}$ so that $\omega = (2\pi) 3 \times 10^8 = 6\pi \times 10^8$

c) The instantaneous expression for \vec{H} if the medium is non-magnetic ($\mu = \mu_0$) (6 pts)

$\vec{H}(y,t) = \hat{a}_x \frac{10}{188.5} \cos(\omega t - \beta y) = \hat{a}_x 0.53 \cos(\omega t - \beta y)$

EXTRA CREDIT (10 pts)

For a single uniform plane wave in a lossless medium, show that the average Poynting vector \vec{S} , equals the product of the average energy density times the phase velocity.

$$\vec{S}_{ave} = \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*) = \frac{1}{2} \frac{E_m e^{-j\beta z} E_m e^{+j\beta z}}{\eta} = \frac{1}{2} \frac{E_m^2}{\eta}$$

$u = \frac{1}{\sqrt{\mu\epsilon}}$ *and the energy density is* $w_m = \frac{1}{4} \text{Re}(\vec{D} \cdot \vec{E}^*) + \frac{1}{4} \text{Re}(\vec{B} \cdot \vec{H}^*) = \frac{1}{2} \epsilon E_m^2$ *so the*

product is equal to $uw_m = \frac{1}{\sqrt{\mu\epsilon}} \frac{1}{2} \epsilon E_m^2 = \frac{1}{\sqrt{\frac{\mu}{\epsilon}}} \frac{1}{2} E_m^2 = \frac{1}{2} \frac{E_m^2}{\eta}$