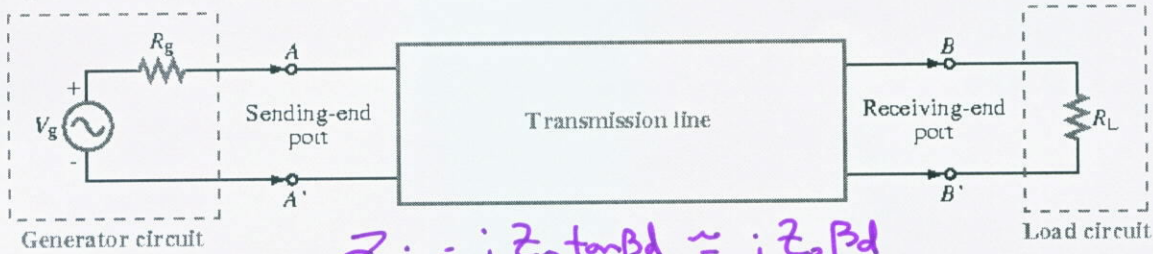


1. Short Questions (4 Points)

a. (0.4 Points) Transmission Lines



$Z_{in} = j Z_0 \tan \beta d \approx j Z_0 \beta d$

A 100 meter long air insulated, lossless transmission line has a short circuit at its load end. The characteristic impedance of the line is  $Z_0 = 100$  and the propagation speed is  $u = 2 \times 10^8 \text{ m/s}$ . At low frequencies (near  $\omega = 0$ ), it is possible to significantly simplify the expression for the input impedance of the line. Which of the following is the best approximation for  $Z_m$  if we simplify it as much as possible?

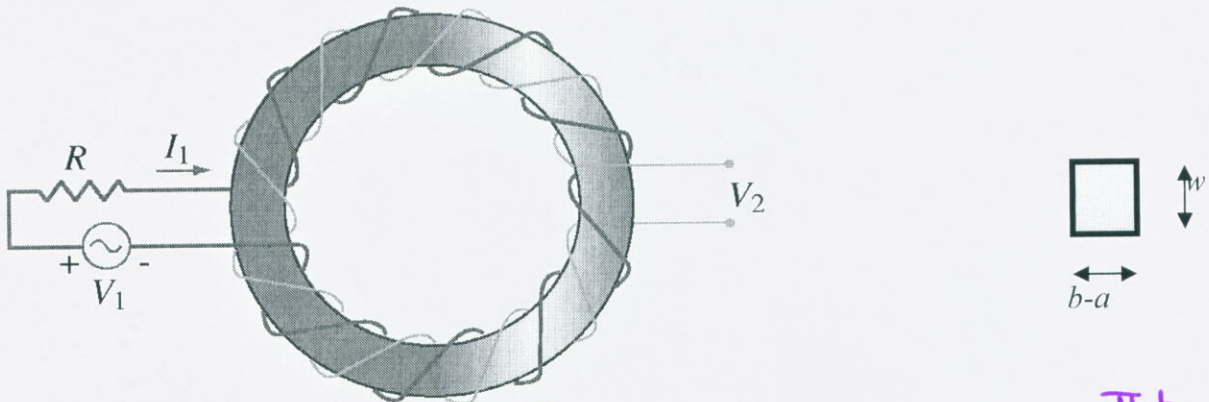
$Z_L = 0$   
 $\omega \approx 0$   
 $\Rightarrow \beta d$  small  
 $\Rightarrow \tan \beta d = \beta d$

$Z_m = Z_0 \frac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d}$    
  $Z_{in} = Z_0$    
  $Z_m = 0$    
  $Z_{in} = jZ_0 \frac{\omega}{u} d$    
  $Z_m = jZ_0 u$

$Z_0 = \sqrt{\frac{L}{C}}$      $\beta = \frac{\omega}{u}$

b. (0.4 Points) Magnetic Circuits

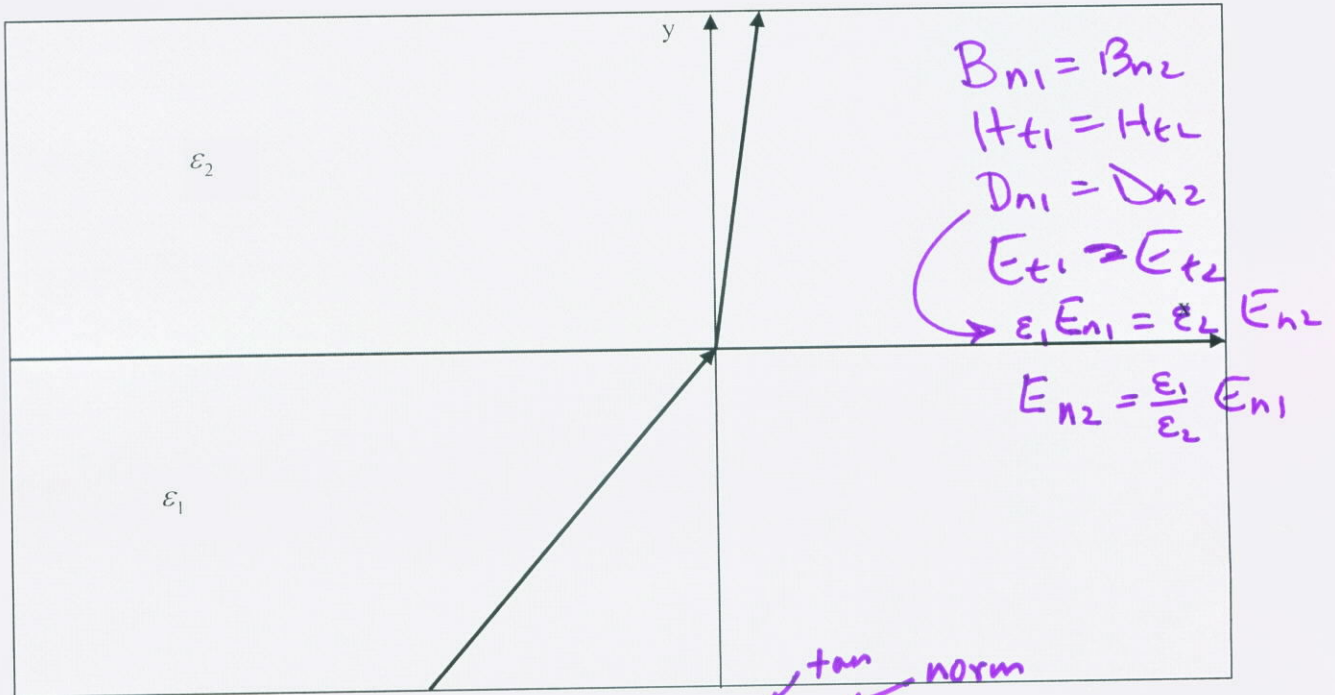
An  $N$  turn coil is wound on a square cross section torus of permeability  $\mu$ . If the inner radius of the torus is  $a$ , the outer radius is  $b$ , and the depth of the coil is  $w$ , what is the reluctance of the magnetic circuit? You can assume that  $b > a \gg (b - a)$  and  $b > a \gg w$



$R = \frac{l}{\mu A} = \frac{2\pi \frac{a+b}{2}}{\mu w (b-a)} \approx \frac{\pi a}{\mu w (b-a)} \approx \frac{\pi b}{\mu w (b-a)}$

any expression is OK

c. (0.4 Points) Electric Fields



Region 1 is air. The electric field in region 1 is  $\vec{E}_1 = E_0(\hat{a}_x + \hat{a}_y)$ . The permittivity in region 2 is  $\epsilon_2 = 6\epsilon_0$ . What is the electric field in region 2  $\vec{E}_2 = ?$ .

$$\vec{E}_2 = E_0 \left( \hat{x} + \frac{1}{6} \hat{y} \right)$$

Figure is wrong

d. (0.4 Points) Finite Difference Solution to Laplace's Equation

The array shown below is part of a spreadsheet, finite difference solution for a capacitor with a heart-shaped electrode. (The shape of the electrode has nothing to do with this problem.) There are no electrodes or other material boundaries in the region shown. Because only two significant digits are displayed, it appears that the values in some cells do not obey the correct relationship to their neighbors. Identify one cell that is correct and one that is not by circling and labeling them.

47	59	69	79	89	99
41	51	61	70	80	89
36	45	53	62	70	79
31	38	45	53	60	68
25	32	38	44	50	56
20	25	30	35	39	44

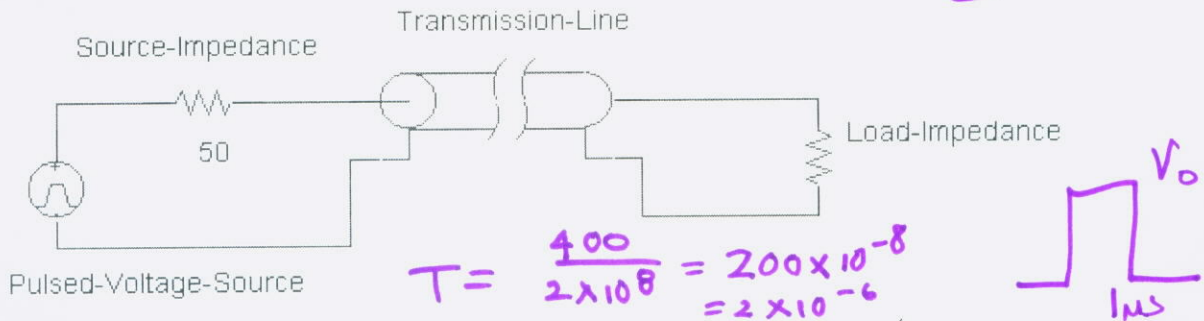
Most others do not work

$$44 = \frac{50 + 53 + 38 + 35}{4}$$

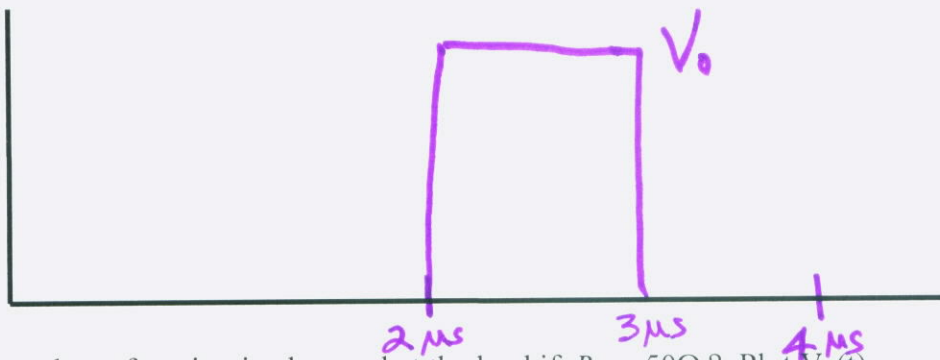
This works

e. (0.4 Points) Transmission Lines

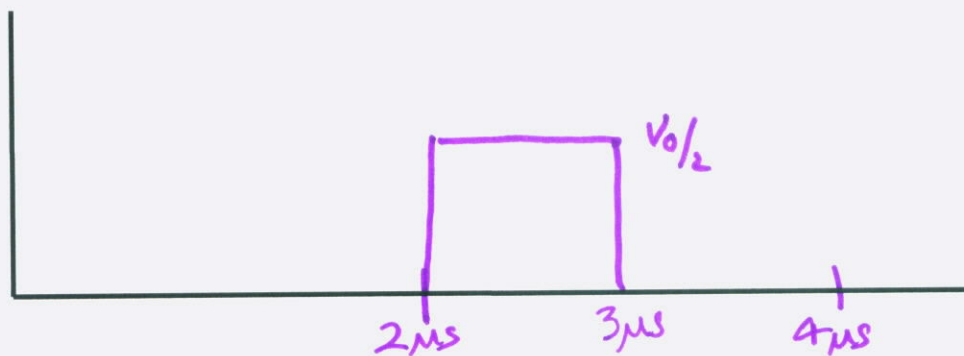
$$V_{in} = \frac{V_0}{2}$$



A 400 meter long transmission line with  $Z_o = 50 \Omega$  and  $u = 2 \times 10^8 \text{ m/s}$  connected a pulsed voltage source to a resistive load. A single pulse (magnitude =  $V_0$ , duration =  $1 \mu s$ ) is emitted from the source. The source impedance is  $50 \Omega$ . If the load resistance  $R_L = 1 M\Omega$ , what voltage is observed at the load? Plot  $V_L(t)$ .

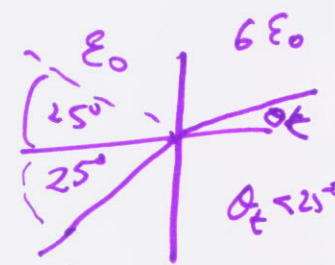


What voltage function is observed at the load if  $R_L = 50 \Omega$ ? Plot  $V_L(t)$ .



f. (0.4 Points) Electromagnetic Waves (Circle the correct answers.)

- The distance between two adjacent maxima in a standing wave pattern is  $\lambda/4$ ,  $\lambda/2$ ,  $\lambda$ ,  $2\lambda$
- A uniform plane wave is incident obliquely at  $25^\circ$  on the surface of a large piece of glass ( $\epsilon_r = 6$ ). The angle of incidence is (equal to, greater than, less than) the angle of reflection. The angle of transmission is (equal to, greater than, less than) the angle of incidence.



g. (0.4 Points) Transmission Lines (Circle the correct answer or answers)

A standard RG/58 cable is built with a new experimental insulator that has a dielectric constant 9 times that of solid polyethylene. What are the characteristic impedance and propagation velocity for this modified cable?

$Z_o = 50\Omega$  and  $u=2 \times 10^8$  m/s

$Z_o = 75\Omega$  and  $u=1.5 \times 10^8$  m/s

$Z_o = 16.7\Omega$  and  $u=2 \times 10^8$  m/s

$Z_o = 16.7\Omega$  and  $u=6.7 \times 10^7$  m/s

$Z_o = 25\Omega$  and  $u=1 \times 10^8$  m/s

Both Smaller by 3

$Z_o = \sqrt{\frac{L}{C}} \sim \frac{1}{\sqrt{\epsilon}}$

$C \sim \epsilon$

$u = \frac{1}{\sqrt{LC}} \propto \frac{1}{\sqrt{\epsilon}}$

h. (0.4 Points) Capacitance (Circle the correct answer or answers)

Find the capacitance per unit length of a coaxial cable insulated with Polyethylene. The inner radius of the cable is 1mm and the outer radius is 4mm.

$9.0 \times 10^{-11}$  F/m

$3.33 \times 10^{-13}$  F/m

$7.5 \times 10^{-13}$  F/m

$1.48 \times 10^{-13}$  F/m

$4.0 \times 10^{-11}$  F/m

$\frac{2\pi\epsilon}{\ln\frac{b}{a}}$

i. (0.4 Points) Force and Energy

If the magnetic energy density on one side of a boundary between two magnetic, non-conducting materials is higher than the magnetic energy density on the other side of such a boundary,

- there will be a force on the boundary toward the region with the higher energy density.
- there will be a force on the boundary toward the region with the lower energy density.
- there is not enough information to tell which direction the force will be.

Force can either be attractive or repulsive

j. (0.4 Points) Laplace's Equation

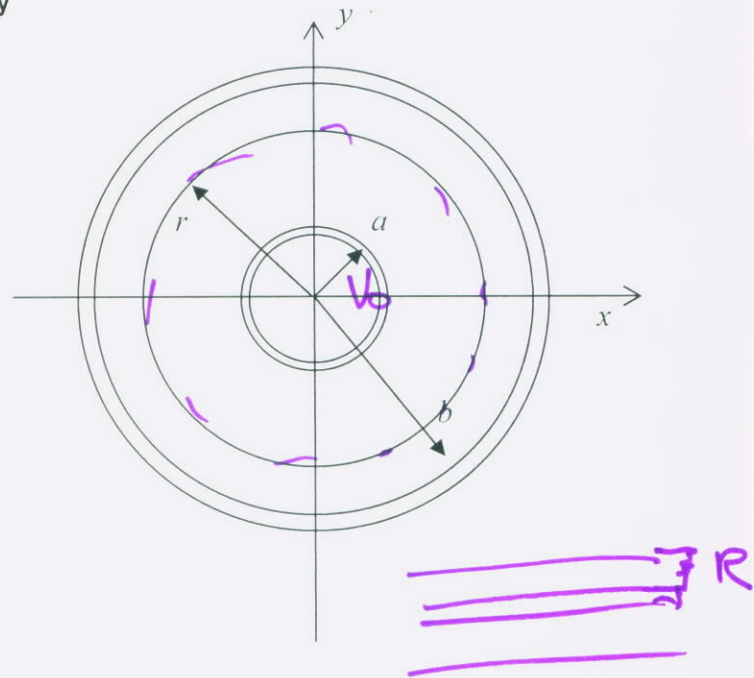
Which of the following statements are correct?

- The value of a function that is a solution to Laplace's equation is equal to the average of the values at its nearest neighboring points.
- The solution to Laplace's equation in some region in space achieves its maximum and minimum values on the boundary of the region.
- Poisson's Equation becomes Laplace's Equation in any region with no volume charge.
- Laplace's Equation is a second order partial differential equation.
- Coulomb's Law for the electric scalar potential is not a solution to Laplace's equation.

Both are acceptable. Away from charge it is OK at charge it is sol'n to Poisson's Eq.

2. Electric Fields (4 Points)

An air insulated coaxial cable is constructed as shown at the right with a hollow center conductor of radius  $a$  and a hollow outer conductor of radius  $b$ . Both conductors are very thin (assume they are constructed using a very thin foil). The voltage on the inner conductor is  $V_0$  and the outer conductor is grounded. The load end of the cable is connected to a resistor  $R$ . In this problem, we are interested in finding the value of the vector field at a radius  $r$  in the region between  $a$  and  $b$ . A dashed line is shown at the radius of interest.



a. Write down the appropriate Maxwell's Equation to find the field at  $r$ .

$$\oint \vec{D} \cdot d\vec{S} = Q_{enc}$$

$$\text{or } \oint \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_0} \leftarrow \text{air}$$

b. Using the equation from part a, determine the electric flux density  $\vec{D}(r, \phi, z)$  in the region between the conductors ( $a < r < b$ )

$$D_r(r) 2\pi r l = \rho_{sa} 2\pi a l$$

$$D_r(r) = \frac{\rho_{sa} a}{r}$$

$\rho_{sa} = \text{surf. charge dens. at } r=a$

c. Determine the electric field  $\vec{E}(r, \phi, z)$  in the same region.

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\vec{E} = \frac{\rho_{sa} a}{\epsilon_0 r}$$

d. Determine the surface charge density on the conductors. Indicate clearly where your answers are valid.

$$V_0 = -\int_b^a \frac{\rho_{sa} a}{\epsilon_0 r} dr = \frac{\rho_{sa} a}{\epsilon_0} \ln \frac{b}{a}$$
$$\Rightarrow \rho_{sa} = \frac{\epsilon_0 V_0}{a \ln b/a}$$

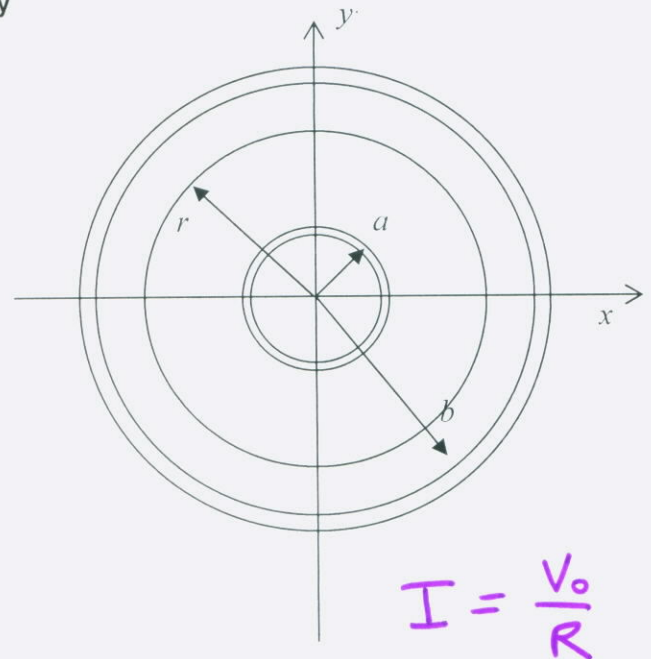
e. Determine the capacitance per unit length of this cable. Indicate which method you have used.

$$Q \text{ per unit length} = \rho_{sa} 2\pi a \text{ l}$$

$$Q/l = \frac{2\pi \epsilon_0 V_0}{\ln b/a} \Rightarrow C/l = \frac{2\pi \epsilon_0}{\ln b/a}$$

**2. Magnetic Fields (4 Points)**

An air insulated coaxial cable is constructed as shown at the right with a hollow center conductor of radius  $a$  and a hollow outer conductor of radius  $b$ . Both conductors are very thin (assume they are constructed using a very thin foil). The voltage on the inner conductor is  $V_0$  and the outer conductor is grounded. The load end of the cable is connected to a resistor  $R$ . In this problem, we are interested in finding the value of the vector field at a radius  $r$  in the region between  $a$  and  $b$ . A dashed line is shown at the radius of interest.



- a. Write down the appropriate Maxwell's Equation to find the field at  $r$ .

$$\oint \vec{H} \cdot d\vec{\ell} = I_{\text{enc}}.$$

- b. Using the equation from part a, determine the magnetic field intensity  $\vec{H}(r, \phi, z)$  in the region between the conductors ( $a < r < b$ )

$$H_{\phi} 2\pi r = I \quad H_{\phi} = \frac{I}{2\pi r}$$

- c. Determine the magnetic flux density  $\vec{B}(r, \phi, z)$  in the same region.

$$B_{\phi} = \mu_0 H_{\phi} = \frac{\mu_0 I}{2\pi r}$$

d. Determine the surface current density in the conductors (the conductors are so thin that they can be considered to carry surface currents). Indicate clearly where your answers are valid.

$$I = J_{sa} 2\pi a \Rightarrow J_{sa} = \frac{I}{2\pi a} \quad r=a$$

$$J_{sb} = \frac{I}{2\pi b} \quad \text{one is } + \text{ \& } \text{one is } -$$

e. Determine the inductance per unit length of this cable. Indicate which method you have used.

$$\Phi_m = \int_0^l dz \int_a^b \frac{\mu_0 I}{2\pi r} dr$$

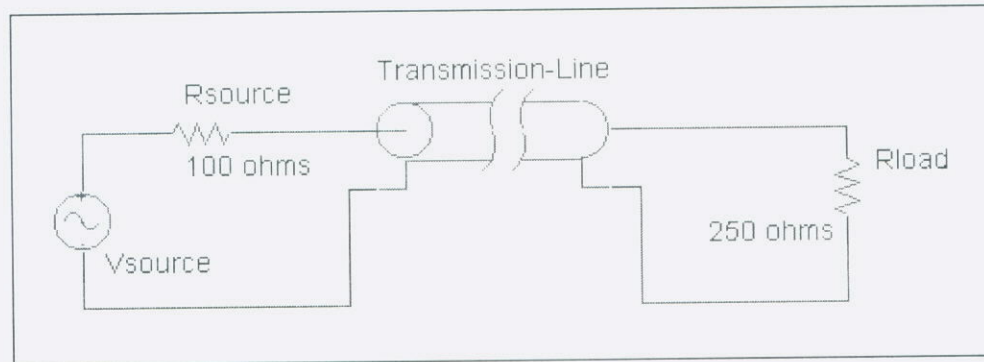
$$= \frac{\mu_0 I}{2\pi} \ln \frac{b}{a} = L I$$

$$L = \frac{\mu_0 \ln \frac{b}{a}}{2\pi}$$



## 4. Transmission Lines (4 Points)

A lossless transmission line is connected to a resistive load as shown. The frequency of the voltage source is  $10\text{ MHz}$  and its amplitude is  $1\text{ V}$ . The characteristic impedance of the line is  $50\Omega$ , the length of the line is  $40\text{ meters}$ , the phase velocity is  $\frac{4}{3} \times 10^8\text{ m/s}$ .



a. Determine the wavelength  $\lambda$  in the transmission line.

$$\lambda = \frac{u}{f} = \frac{\frac{4}{3} \times 10^8}{10^7} = \frac{4}{3} \times 10 = \frac{40}{3}$$

b. Determine the input impedance of the line.

$$40\text{ meters} = 3\lambda \Rightarrow Z_{\text{in}} = Z_L = 250\Omega$$

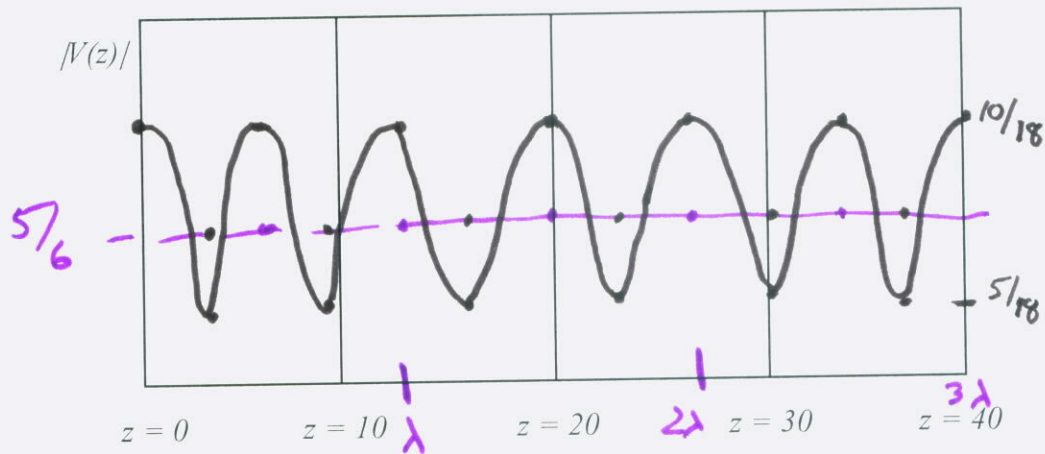
c. Determine the magnitude of the voltage at the input and the load.

$$V_{\text{in}} = 1 \frac{250}{250+50} = \frac{5}{6} = V_{\text{out}} \text{ must be the same.}$$

d. Determine the average power delivered to the load.

$$P_{out} = \frac{1}{2} \frac{V_{out}^2}{Z_L} = \frac{1}{2} \frac{(5/6)^2}{250}$$

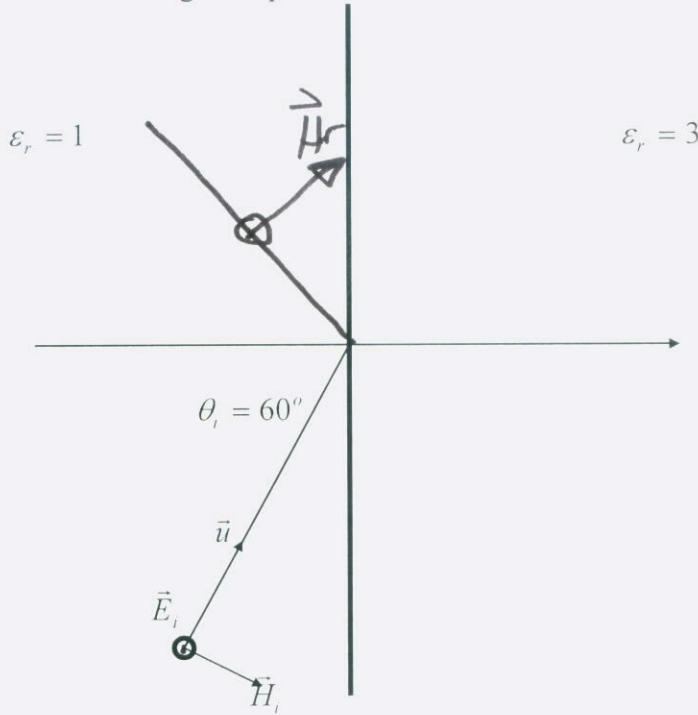
e. Draw the standing wave pattern for the voltage. Be sure you label your plot with numerical values.



$$\Gamma_{Load} = \frac{250 - 50}{250 + 50} = + \frac{200}{300} = + \frac{2}{3}$$

5. Electromagnetic Waves (4 Points)

A perpendicularly polarized wave is incident upon a dielectric interface at an angle of  $60^\circ$ . The amplitude of the incident wave is  $1 \text{ V/m}$ . The angular frequency  $\omega = 6\pi \times 10^9$  radians. You can leave your answers in terms of parameters, but all parameters must be evaluated in terms of the given quantities.



$$\begin{aligned}
 n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\
 1 \sin 60^\circ &= \sqrt{3} \sin \theta_2 \\
 \sin \theta_2 &= \frac{\sin 60^\circ}{\sqrt{3}} \\
 &= \frac{\sqrt{3}}{\sqrt{3} \cdot 2} = \frac{1}{2} \\
 \theta_2 &= 30^\circ
 \end{aligned}$$

a. What is the transmitted wave angle?

$30^\circ$

b. What is the reflection coefficient  $\Gamma$ ?

$$\begin{aligned}
 \Gamma_{\perp} &= \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2} \\
 &= -1.53
 \end{aligned}$$

c. Write the incident and reflected electric field vectors in phasor form.

$$\begin{aligned}\vec{E}_i &= \hat{y} 1 e^{-j\vec{k}_i \cdot \vec{r}} & \vec{k}_i &= \hat{x} \cos \theta_i + \hat{z} \sin \theta_i & \theta_i &= 60^\circ \\ \vec{H}_i &= \left( \frac{1}{\eta_0} e^{-j\vec{k}_i \cdot \vec{r}} \right) (-\hat{x} \cos \theta_i + \hat{z} \sin \theta_i) \\ \vec{E}_r &= \hat{y} \Gamma_{\perp} e^{-j\vec{k}_r \cdot \vec{r}} & \vec{k}_r &= -\hat{x} \cos \theta_i + \hat{z} \sin \theta_i \\ \vec{H}_r &= \left( \frac{\Gamma_{\perp}}{\eta_0} e^{-j\vec{k}_r \cdot \vec{r}} \right) (+\hat{x} \cos \theta_i + \hat{z} \sin \theta_i)\end{aligned}$$

d. Write the corresponding magnetic field vectors in phasor form.

See above.

e. What fraction of the incident power is reflected?

$$\frac{\Gamma_{\perp}^2}{2\eta_0} \text{ vs } \frac{1}{2\eta_0} \quad \text{ratio} = \Gamma_{\perp}^2 = 28^\circ$$