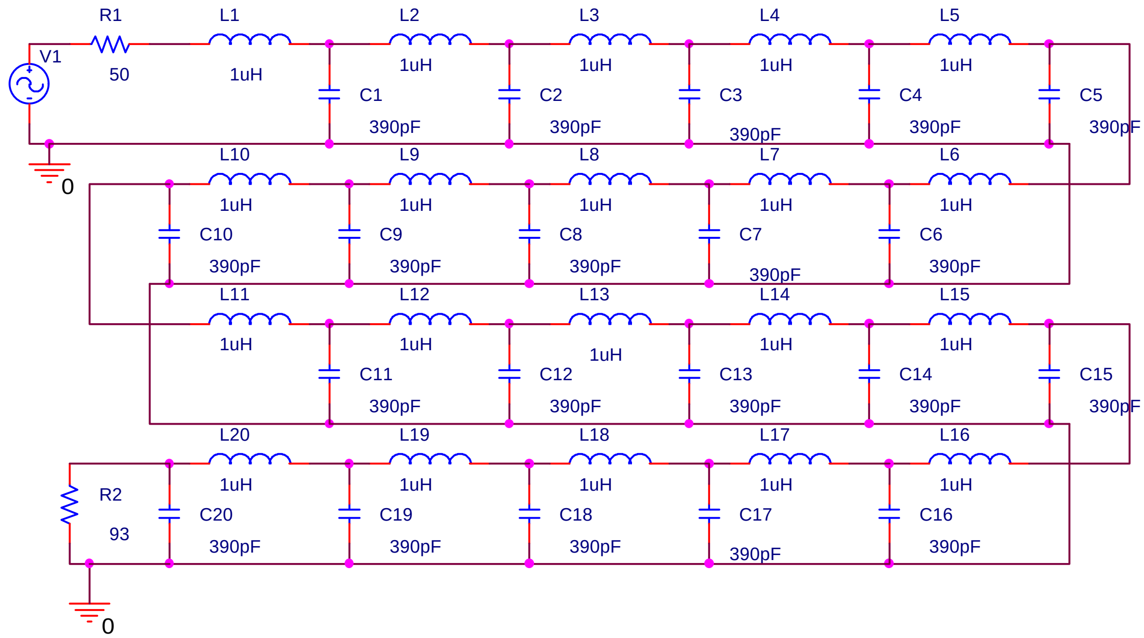
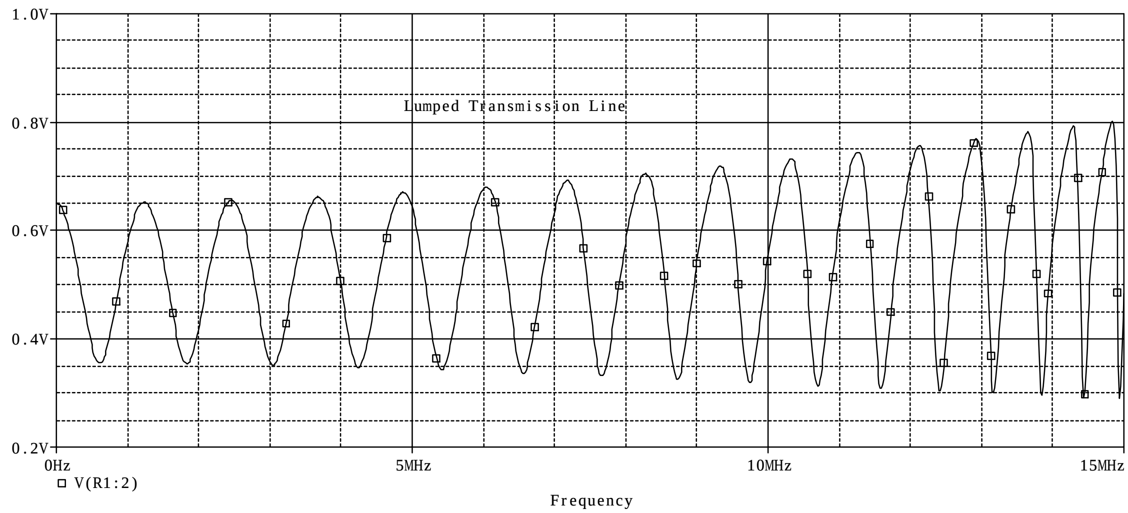
Lumped Model of Transmission Line

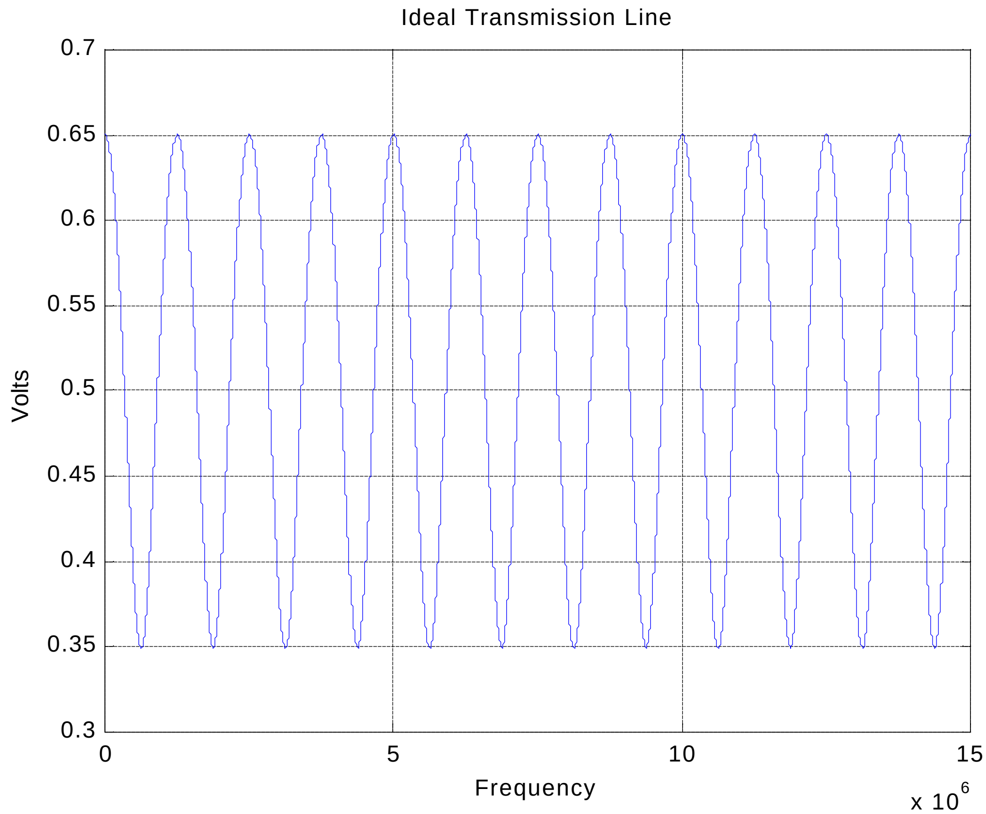
In the Instrumentation Studio, we use both a coil of coax (around 80-100 meters long) and an equivalent lumped model of the coil of coax to study transmission lines. The lumped circuit is configured in the following manner:




The physical circuit has one BNC connector at the input end and a BNC connector and two banana plug connector posts at the output end. This circuit can be modeled using PSpice to determine its frequency response. The input voltage as a function of frequency looks like:

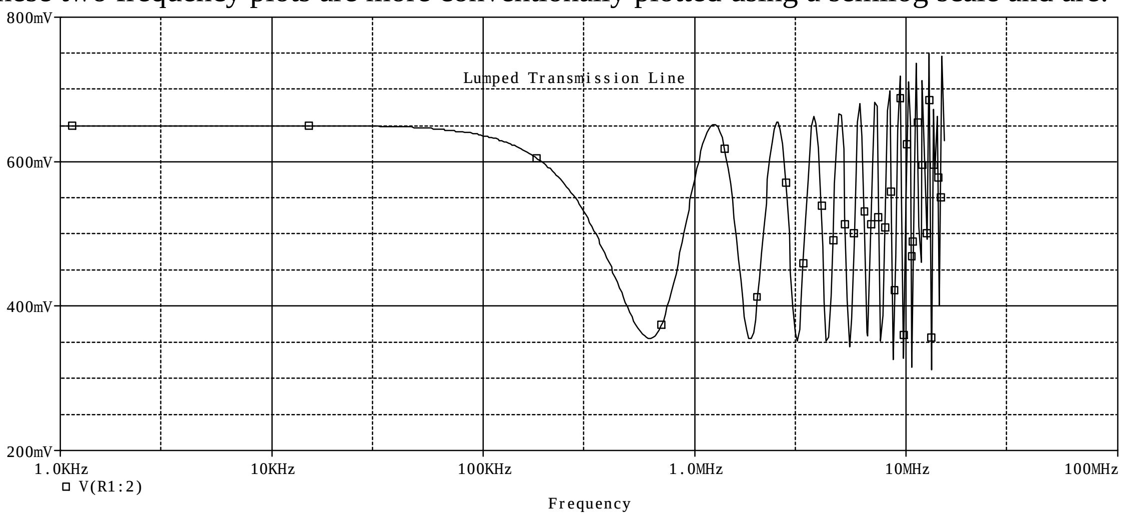



where the frequency has been plotted on a linear scale to show the details better. The input voltage can also easily be calculated and plotted using ideal formulas and Matlab. In this case the frequency response is more regular:

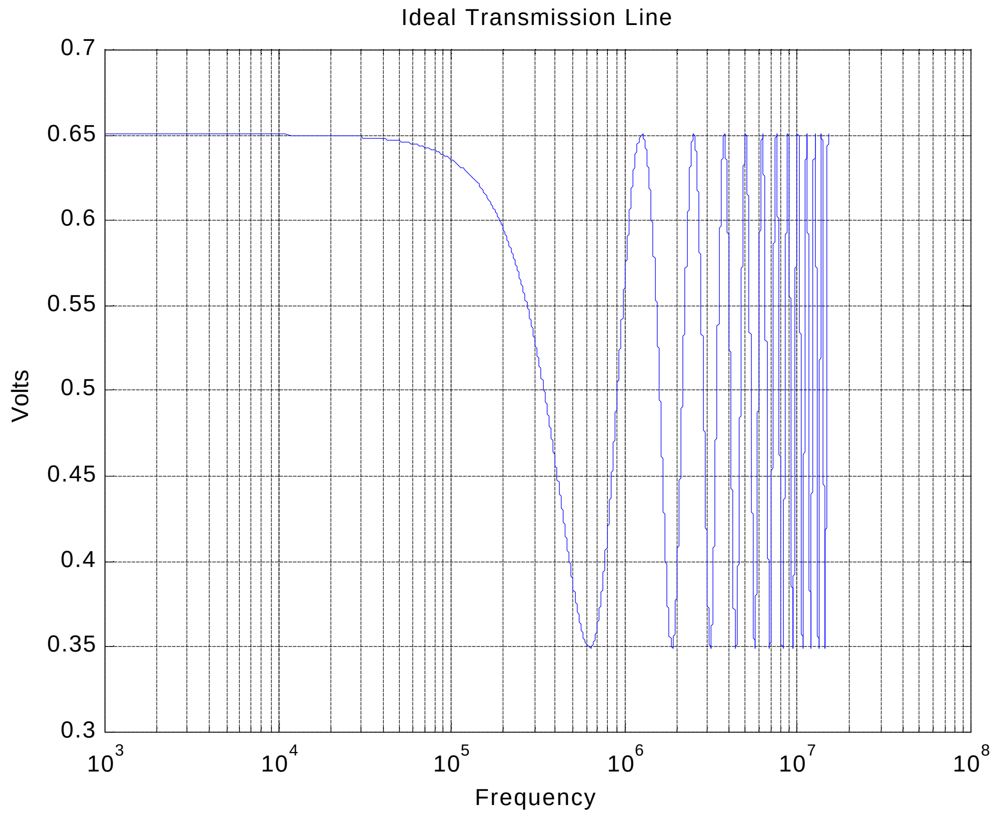



Note that the response is similar at lower frequencies, but begins to diverge significantly at higher frequencies. Also, no resistance has been included in either model.

These two frequency plots are more conventionally plotted using a semilog scale and are:

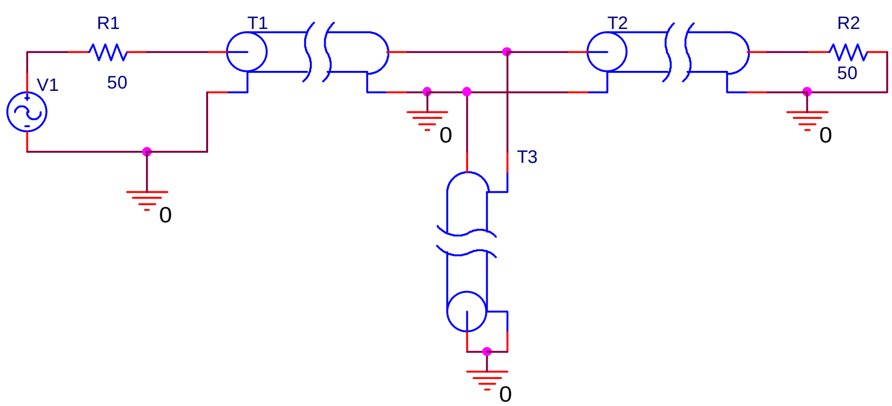



and

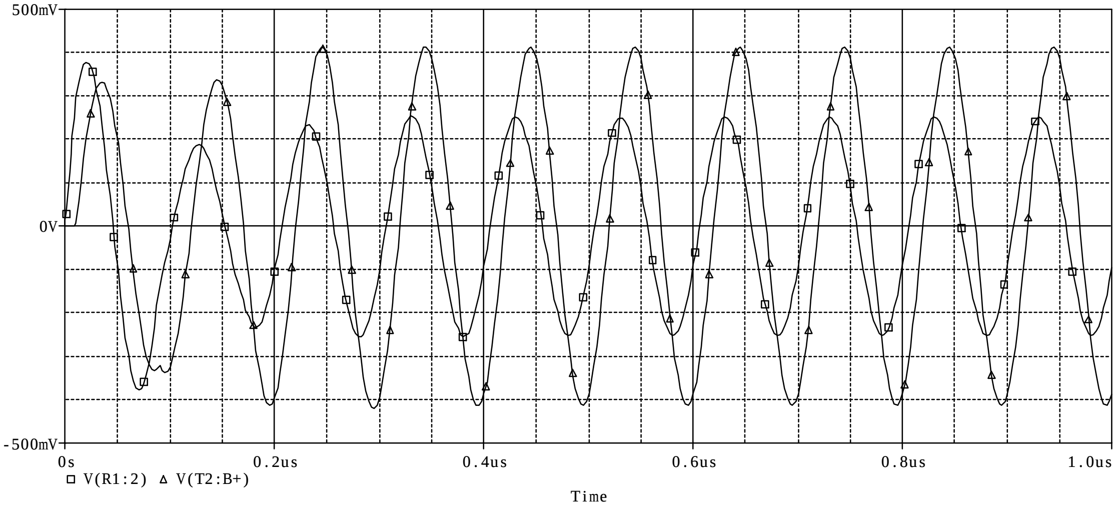



respectively.

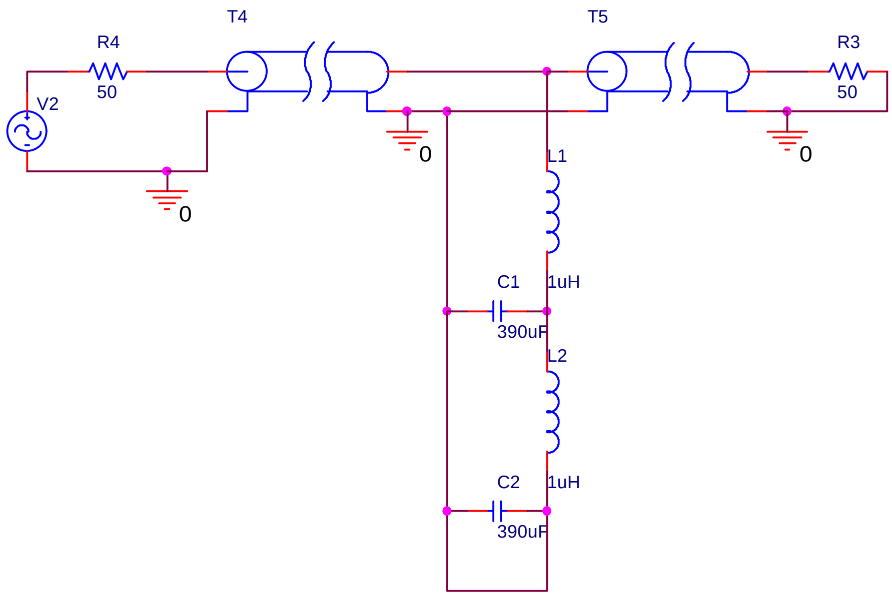
In Project 2 (Spring 2000) we use the lumped line as the line we short and connect as a stub into the line that runs from the function generator to the matched load at the scope. It turns out that this line does not work the way we would like it to work because of the way it is constructed. A simple simulation using PSpice shows why this is the case. First, we can simulate ideal, lossless transmission lines with some selected length in the following configuration:



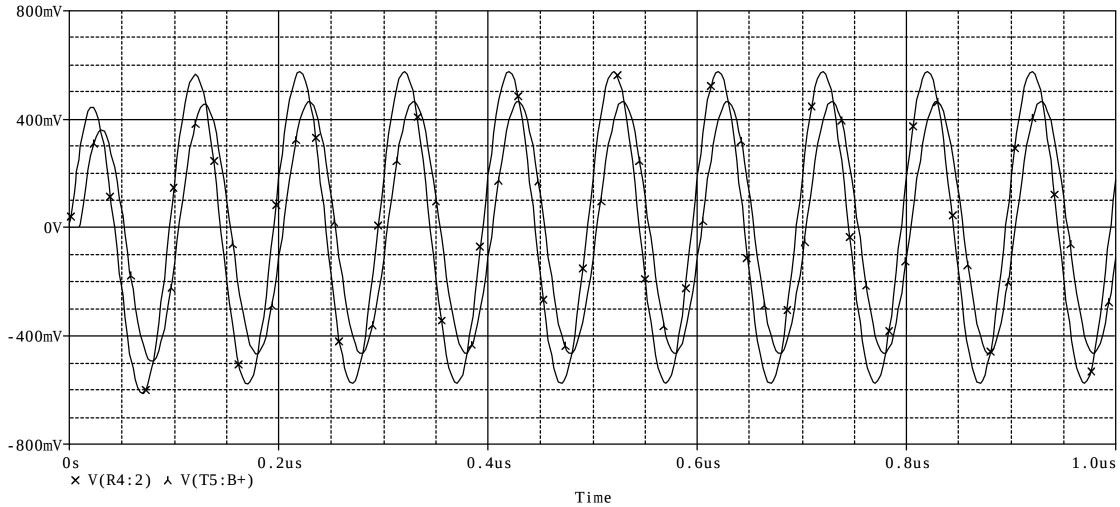

With a voltage source frequency selected to be 10MHz, the voltages at the input and output look like



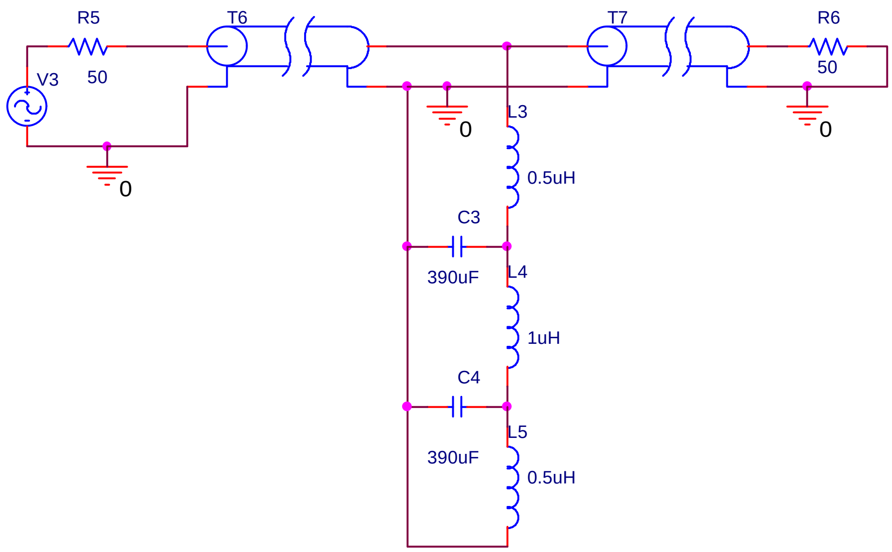

where the smaller voltage is the output signal.  Disregard the variations at the beginning, since the signal does not reach steady-state until about 0.2us.  Assume that we decide to use two sections of the lumped line to represent an 8 meter cable.  We have a circuit that looks like



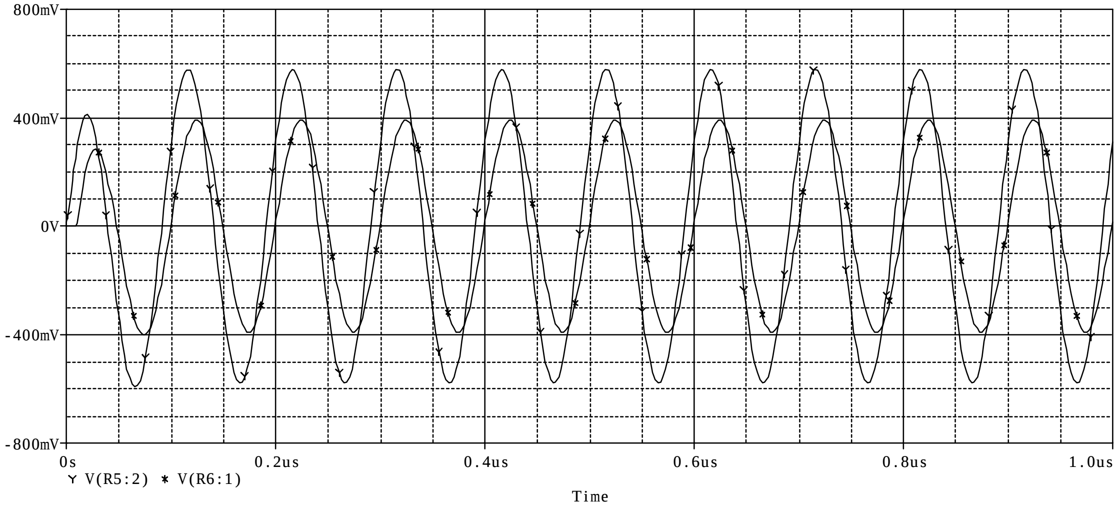

Note that the short at the end of the lumped line shorts out the capacitor and thus the lumped line is missing a component.  The lumped representation for each section of the line must contain both an inductor and a capacitor, but the second section here contains only an inductor.  Thus, the line looks like a 4 meter long transmission line with an inductor as a load rather than an 8 meter long cable with a short at the end. The voltages observed at the input and output do not look like the real line.  Rather they look like



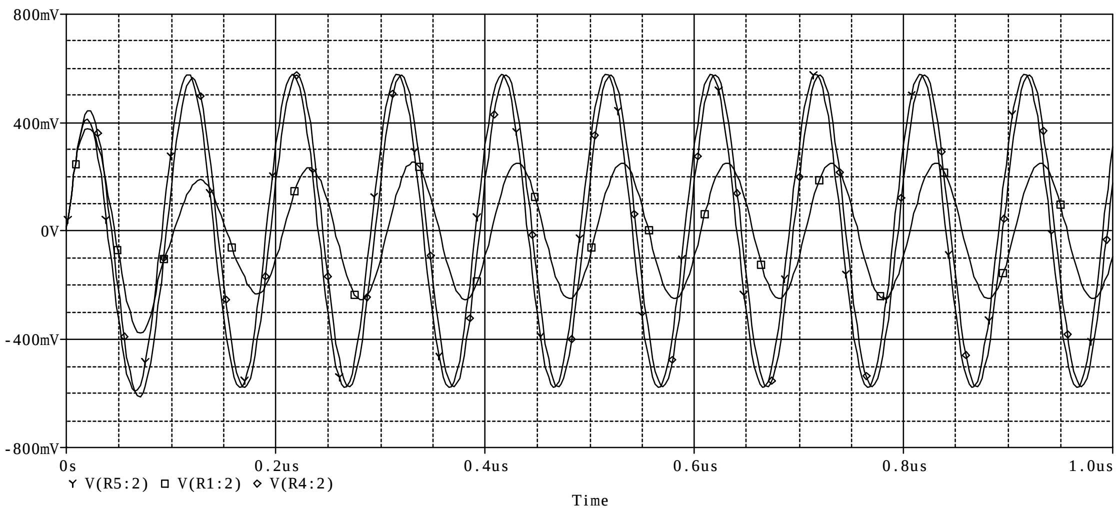

We can fix this problem by constructing our lumped transmission line a little differently. Each section should be represented by a TEE configuration with an inductor on either side of a capacitor so that the 8 meter long line now looks like

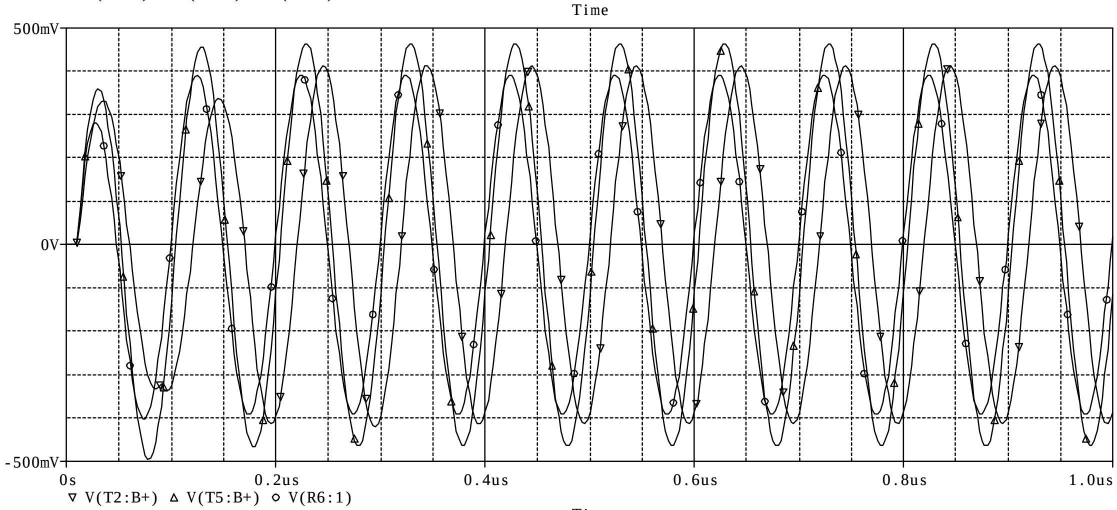



Notice that the short circuit now does not eliminate any of the components. Unfortunately, there is no good way to build a lumped model of a transmission line that works exactly right for both a short and an open circuit load. With our new model, the last inductor would be ignored if the load was an open circuit. For the new model the voltages at the input and output now look much more like the ideal line case. The voltages are still not exactly the same.



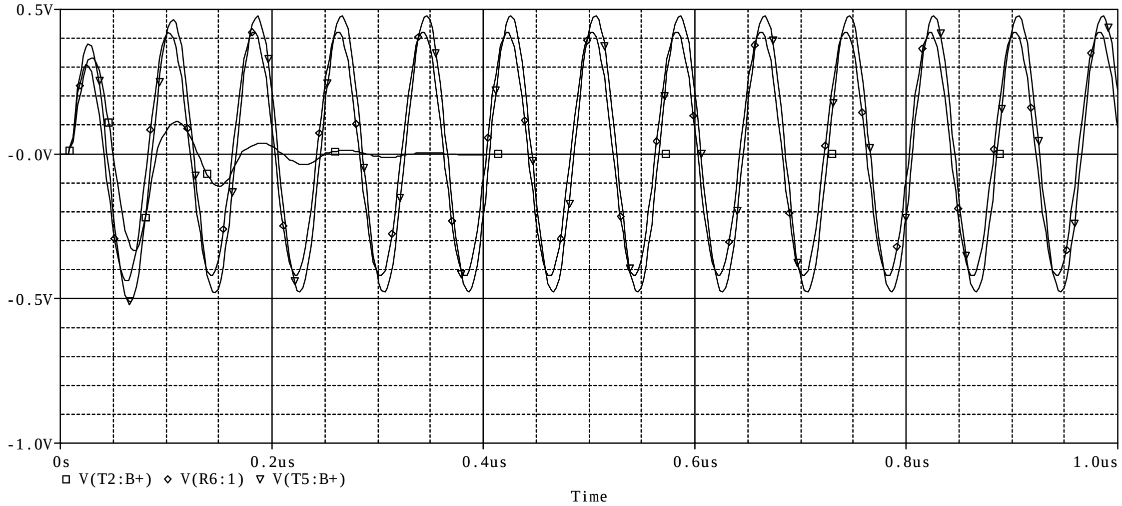

To make the comparison easier to see, we can plot just the input and the output in the three cases and note that the output signals are the most similar for the first and third cases:




However, the input signals are the most similar for the second and third cases. This is probably due to the inadequacies of a lumped transmission line model with only two sections in it. It is unrealistic to assume that only two sections can represent the full transmission line with any real accuracy.  The frequency of 10 MHz was chosen arbitrarily.  Let us select a frequency that should be blocked.  For blocking we need a frequency for which the stub is a half wavelength long.  If we assume a propagation speed 2/3 of the speed of light, then 8 meters of cable will be a half wavelength at

$f = \frac{u}{l} = \frac{u}{2d} = \frac{2 \times 10^8}{2 \cdot 8} = 12.5 MHz$.  The output voltages for this frequency show that neither lumped line works perfectly, but that the last model is a little better.  Note that the ideal line does exactly what it is supposed to do.




When using the lumped line, most groups found that the frequency for blocking was a bit lower than the ideal model would predict. Let us try such a lower frequency.