Solution

ECSE 2100 – Fields and Waves I
Fall 2004
Homework #1

1. For the following wave expressions, indicate if the wave is standing or traveling. If the wave is traveling, find the direction of propagation and the velocity.

a) $\sin(377t+0.05x)$ traveling in $-x$, $v = \omega/\beta = 7540$ m/s

b) $\cos(10^5 t - 2\times10^{-1} z)$ traveling in $+z$, $v = \omega/\beta = 5\times10^5$ m/s

c) $\cos(120t)\sin(55x)$ standing

2. Find the phasor representation of the following expressions

a) $v(t) = 5\cos\left(\omega t - \frac{\pi}{4}\right)$ $\quad \tilde{V} = 5e^{-j\pi/4}$

b) $v(t) = 120\sin\left(\omega t + \frac{\pi}{4}\right)$ $\quad \tilde{V} = 120\cos\left(\pi/2 - (\omega t + \pi/4)\right) = 120\cos(\omega t - \pi/4)$

$\tilde{V} = 120 e^{-j\pi/4}$

c) $v(t) = 3\sin\left(\omega t + \frac{2\pi}{3}\right) + 2\cos\left(\omega t - \frac{\pi}{6}\right)$ $= 3\cos\left(\pi/2 - (\omega t + \frac{2\pi}{3})\right) + 2\cos(\omega t - \pi/6)$

$= 3\cos(\omega t + \pi/6) + 2\cos(\omega t - \pi/6)$

$\tilde{V} = 4.35 e^{j0.115}$

3. Find the time domain expression for the following phasors.

a) $\tilde{V} = 6 + j4V$ $\quad 7.2\, e^{j33.7^\circ} \Rightarrow v(t) = 7.2\cos(\omega t + 33.7^\circ)$

b) $\tilde{V} = 2e^{j\frac{3\pi}{4}}V$ $\quad , v(t) = \mathrm{Re}\left[2e^{j3\pi/4} e^{j\omega t}\right] = 2\cos(\omega t + 3\pi/4)$

4. A wave is described by $v(t,z) = 3e^{-\alpha z}\sin\left(2\pi\times10^9 t - 10\pi z\right)V$. Find the frequency, wavelength and velocity. At z = 2m the magnitude is measured as 1V. Find the attenuation constant.

5. In class we showed that any function, f(x-ct) is a solution of the wave equation. By the same process, show that f(x+ct) is also a solution.

4. $f = 10^9$, $\beta = 10\pi$, $\lambda = 2\pi/\beta = 0.2$m, $v = \frac{\omega}{\beta} = 2\times10^8$ m/s

$3e^{-\alpha 2} = 1 \Rightarrow \alpha = -\frac{1}{2}\ln 1/3 = 0.55$

5. Let $\xi = x + ct$, we have $\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$, for $f(\xi) = E$

$\frac{\partial E}{\partial x} = \frac{df}{d\xi}\frac{\partial \xi}{\partial x} = \frac{df}{d\xi}$; $\frac{\partial^2 E}{\partial x^2} = \frac{d}{d\xi}\left(\frac{df}{d\xi}\right)\frac{\partial \xi}{\partial x} = \frac{d^2 f}{d\xi^2}$

$\frac{\partial E}{\partial t} = \frac{df}{d\xi}\frac{\partial \xi}{\partial t} = c\frac{df}{d\xi}$; $\frac{\partial^2 E}{\partial t^2} = \frac{d}{d\xi}\left[c\frac{df}{d\xi}\right]\frac{\partial \xi}{\partial t} = c^2\frac{d^2 f}{d\xi^2}$

so $\frac{d^2 f}{d\xi^2} = \frac{1}{c^2}\left[c^2\frac{d^2 f}{d\xi^2}\right] \Rightarrow f(x+ct)$ is a solution