Problem Solution #5

Problem 1

$\vec{E} = -\nabla V \quad \rightarrow \quad V = -\int \vec{E} \cdot d\vec{l} = Ed \quad \rightarrow \quad \vec{D} = \varepsilon \vec{E} \quad \rightarrow \quad D = \rho_s \quad \rightarrow$

$Q = \rho_s A \quad \rightarrow \quad C = \dfrac{Q}{V} \qquad\qquad C \approx 23.6\,pF$

Problem 2

First consider the Coaxial capacitor is filled with insulating material of permittivity ε

$\vec{E} = \dfrac{q}{2\pi\varepsilon r}\hat{r}$ (q is the total charge on the inner conductor per unit depth)

$V_0 = -\int_b^a \dfrac{q}{2\pi\varepsilon r}\hat{r}\cdot d\vec{r} = \dfrac{q}{2\pi\varepsilon} In(b/a)$ assume $V(r=b)=0$

$C = \dfrac{Q}{V} = \dfrac{q}{V_0} = 2\pi\varepsilon \big/ In(b/a) \qquad C = \dfrac{\varepsilon A}{d} \qquad c = \dfrac{A'}{A}C = \dfrac{\theta r l}{2\pi r l} = \dfrac{\theta}{2\pi}$ In this problem,

equivalently, there are 1/6 ($\dfrac{60^o}{360^0} = \dfrac{1}{6}$) capacitor ($\varepsilon_s$) and 5/6 capacitor ($\varepsilon_g$)

$$c = \frac{1}{6}c_s + \frac{5}{6}c_g = \frac{1}{6}[2\pi\varepsilon_s \big/ In(b/a)] + \frac{5}{6}[2\pi\varepsilon_g \big/ In(b/a)] = \boxed{\frac{\pi}{3In(b/a)}[\varepsilon_s + 5\varepsilon_g]}$$

$\vec{E} = \dfrac{q}{2\pi\varepsilon r}\hat{r} = \dfrac{2\pi\varepsilon V_0}{2\pi\varepsilon r In(b/a)}\hat{r} = \boxed{\dfrac{V_0}{In(b/a)r}\hat{r}}$ (a<r<b), $\vec{E} = 0$ (else)

$\vec{D} = \varepsilon\vec{E} \quad \vec{D}_s = \boxed{\dfrac{\varepsilon_s V_0}{In(b/a)r}\hat{r}}$ and $\vec{D}_g = \boxed{\dfrac{\varepsilon_g V_0}{In(b/a)r}\hat{r}}$ (a<r<b), zero (else)

$V = -\int_b^r \vec{E}\cdot d\vec{l} = -\int_b^r \dfrac{V_0}{rIn(b/a)}\hat{r}\cdot\hat{r}dl = \boxed{\dfrac{V_0}{In(b/a)} In(b/r)}$ (a<r<b),

$V = V_0 \quad (r \le a)$ and $V = 0 \quad (r = b)$

At the inner surface:

$E_{1t} = E_{2t} = 0 \quad D_{1t} = D_{2t} = 0 \quad E_{1n} = 0 \quad E_{2n} = \rho_{s2}/\varepsilon_2 \quad D_{1n} = 0 \quad D_{2n} = \rho_{s2}$

Satisfy the interface conditions.

At the outer surface:

$E_{3t} = E_{2t} = 0 \quad D_{3t} = D_{2t} = 0 \quad E_{3n} = 0 \quad E_{2n} = \rho_{s2}/\varepsilon_2 \quad D_{3n} = 0 \quad D_{2n} = \rho_{s2}$

Satisfy the interface conditions.