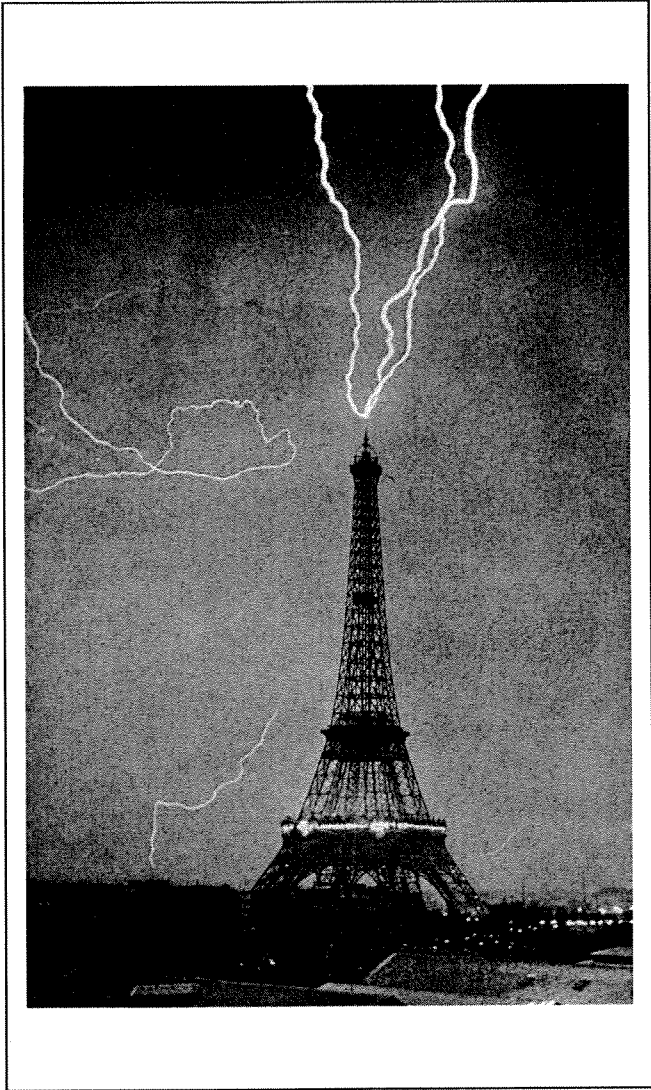


Name Solution

Section \_\_\_\_\_



1. (4 Pts) \_\_\_\_\_

2. (4 Pts) \_\_\_\_\_

3. (4 Pts) \_\_\_\_\_

4. (10 Pts) \_\_\_\_\_

5. (20 Pts) \_\_\_\_\_

6. (18 Pts) \_\_\_\_\_

7. (18 Pts) \_\_\_\_\_

8. (22 Pts) \_\_\_\_\_

**Total**

\_\_\_\_\_

Notes:

1. Please read over all questions before you begin your work. There may be some information in a later question that helps you with an earlier question.

2. For short answer questions, you should add some comments to justify your answer.

3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees & use 4 significant digits.

Some Comments and Helpful Info:

In this course, we use two types of notation for unit vectors. Keep in mind that

$$\hat{a}_x = \hat{x} \quad \hat{a}_y = \hat{y} \quad \hat{a}_z = \hat{z}$$

$$\hat{a}_r = \hat{r} \quad \hat{a}_\phi = \hat{\phi} \quad \hat{a}_\theta = \hat{\theta}$$

Also, sometimes  $R$  is used for spherical radius instead of  $r$ , so  $R$  is another term that gets used for more than one purpose. Pay attention to the context of the questions to minimize problems.



### 1. Solving for the Electric Field (4 Points)

If you determine the electric field in some region using Gauss' Law in integral form, which of the following is *always* useful for checking your answer?

- a. Applying Gauss' Law in Differential Form
- b. Asking the clerk at Radio Shack.
- c. Checking the answers in the back of the book
- d. Checking the form of the solution against the electric field due to a point charge.

### 2. Equipotentials and Electric Field Lines (4 Points)

Electric Field lines and equipotential surfaces are

- a. Always parallel to one another
- b. Never parallel to one another
- c. Always perpendicular to one another
- d. Never perpendicular to one another

### 3. Laplace's Equation (4 Points)

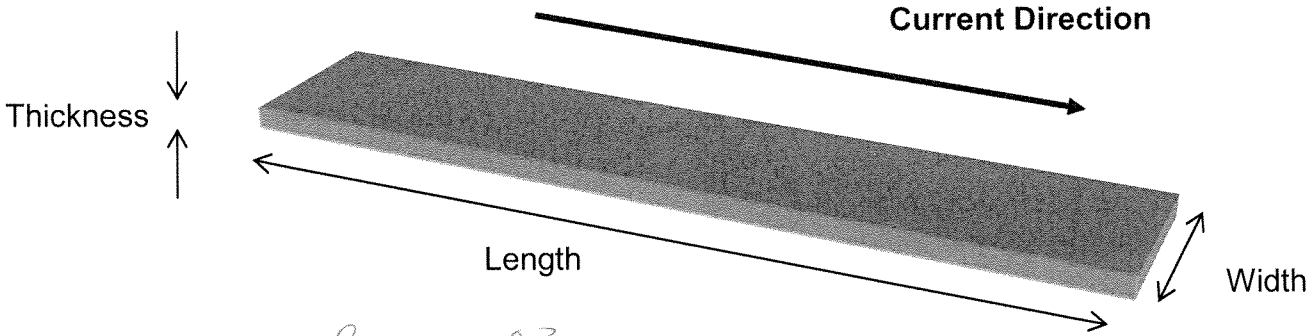
Which of the following statements are correct?

- a. The value of a function that is a solution to Laplace's equation is equal to the average of the values at its nearest neighboring points.
- b. The solution to Laplace's equation in some region in space is not unique unless a voltage is specified at some location.
- c. Poisson's Equation becomes Laplace's Equation in any region with no volume charge.
- d. The voltages in the charge free regions between two conductors will always have values between those specified on the conductors.



4. Resistance and Conductivity (10 Points)

a. Determine the resistance of a thin trace of gold (conductivity  $\sigma = 4.2 \times 10^7 \text{ S/m}$ ), 3mm wide, 0.1mm thick, and 3cm long. Be careful of units.



$$R = \frac{l}{\sigma A} = \frac{.03}{(4.2 \times 10^7)(.0001)(.003)}$$

$$= 2.4 \text{ m}\Omega$$

b. Assume that you have a fixed amount of gold (volume is given), but that you want to deposit it on a surface so that its resistance is double the value you just calculated. Determine a new length, width and thickness that keeps the volume the same but doubles the resistance. Verify that your new volume is the same for both cases.

$$R = \frac{l}{\sigma A} = \frac{l^2}{\sigma (Al)} = \text{twice}$$

$$\Rightarrow l = \sqrt{2}(.03) = 4.24 \text{ cm}$$

let  $w = .003$  be the same.

$$\Rightarrow \text{thickness} = \frac{.0001}{\sqrt{2}} = .0707 \text{ mm}$$

thickness could also be the same but then width



5. Capacitance from Numerical Calculations (20 Points)

AG42	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	
2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.3	9.3	9.3	9.3	9.3	9.3	9.3	9.2	9.1	9.0	8.9	8.9	9.0	9.1	9.2	9.2	9.2	
3	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.5	8.5	8.5	8.5	8.6	8.6	8.6	8.6	8.5	8.4	8.3	8.1	7.8	7.8	8.1	8.3	8.5	8.5	8.5
4	7.7	7.7	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.7	7.7	7.7	7.8	7.8	7.8	7.9	7.9	7.9	7.9	7.8	7.7	7.6	7.2							
5	6.9	6.9	6.9	6.8	6.8	6.7	6.7	6.7	6.8	6.8	6.9	6.9	7.0	7.0	7.1	7.1	7.2	7.2	7.2	7.2	7.1	7.0	6.7								
6	6.2	6.2	6.1	6.1	6.0	5.9	5.8	5.9	5.9	6.0	6.1	6.2	6.3	6.3	6.4	6.4	6.5	6.5	6.5	6.6	6.6	6.6	6.5								
7	5.4	5.4	5.4	5.3	5.2	5.0	4.9	4.9	5.0	5.2	5.3	5.4	5.5	5.6	5.6	5.7	5.8	5.8	5.9	6.0	6.0	6.1	6.2	6.2							
8	4.8	4.8	4.7	4.6	4.4	4.2	3.9	3.9	4.2	4.4	4.6	4.7	4.8	4.9	4.9	5.0	5.1	5.1	5.2	5.3	5.5	5.6	5.8	6.0							
9	4.1	4.1	4.1	4.0	3.7	3.3			3.3	3.7	3.9	4.0	4.1	4.2	4.2	4.3	4.3	4.4	4.6	4.7	4.9	5.1	5.4	5.8							
10	3.6	3.6	3.5	3.4	3.2	2.9			2.9	3.2	3.3	3.4	3.4	3.5	3.5	3.5	3.6	3.7	3.9	4.0	4.2	4.5	4.8	5.4							
11	3.0	3.0	3.0	3.0	2.9	2.7			2.6	2.7	2.8	2.8	2.8	2.8	2.8	2.8	2.8	3.0	3.1	3.3	3.5	3.8	4.1	4.5	4.9	5.0	4.7	4.5	4.4	4.3	4.3
12	2.6	2.6	2.5	2.5	2.5	2.5			2.4	2.4	2.3	2.2	2.1	2.1	2.0	2.0	2.0	2.2	2.4	2.6	2.8	3.1	3.3	3.6	3.8	3.9	3.8	3.7	3.7	3.6	3.6
13	2.1	2.1	2.1	2.1	2.2	2.3			2.2	2.0	1.8	1.6	1.5	1.3	1.2	1.1	1.1	1.3	1.6	1.8	2.1	2.3	2.6	2.8	2.9	3.0	3.0	3.0	2.9	2.9	2.9
14	1.6	1.6	1.6	1.7	1.8	2.1			2.0	1.6	1.3	1.1	0.8	0.6	0.3	0.1	0.1	0.4	0.8	1.1	1.3	1.6	1.8	2.0	2.1	2.2	2.2	2.2	2.2	2.2	2.2
15	1.0	1.0	1.1	1.2	1.3	1.7			1.6	1.1	0.7	0.5	0.2	-0.1	-0.6			-0.5	0.0	0.4	0.6	0.9	1.1	1.2	1.3	1.4	1.4	1.5	1.5	1.5	1.5
16	0.4	0.4	0.5	0.6	0.7	0.9	1.2	1.1	0.8	0.4	0.1	-0.1	-0.4	-0.7				-0.7	-0.3	0.0	0.2	0.4	0.5	0.6	0.6	0.7	0.7	0.7	0.7	0.7	
17	-0.2	-0.2	-0.2	-0.1	0.0	0.1	0.2	0.1	-0.1	-0.3	-0.5	-0.7	-0.9							-0.8	-0.5	-0.4	-0.3	-0.2	-0.1	-0.1	-0.1	0.0	0.0	0.0	0.0
18	-0.9	-0.9	-0.8	-0.8	-0.8	-0.7	-0.7	-0.7	-0.9	-1.0	-1.2	-1.3									-1.0	-0.9	-0.9	-0.9	-0.8	-0.8	-0.8	-0.8	-0.7	-0.7	-0.7
19	-1.6	-1.6	-1.5	-1.5	-1.5	-1.5	-1.5	-1.6	-1.7	-1.9												-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5
20	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2	-2.3	-2.3	-2.4	-2.5	-2.6	-2.3	-2.3	-2.4	-2.5	-2.5	-2.4	-2.2	-2.2	-2.1	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2
21	-2.9	-2.9	-2.9	-2.9	-2.9	-3.0	-3.0	-3.0	-3.1	-3.2	-3.2	-3.2	-3.3	-3.6	-3.9	-3.9	-3.5	-3.2	-3.1	-3.0	-2.9	-2.9	-2.9	-2.9	-2.9	-2.9	-2.9	-2.9	-2.9	-2.9	-2.9
22	-3.7	-3.7	-3.7	-3.7	-3.7	-3.7	-3.7	-3.7	-3.7	-3.8	-3.8	-3.9	-4.0	-4.2	-4.6					-4.6	-4.1	-3.9	-3.7	-3.7	-3.6	-3.6	-3.6	-3.6	-3.6	-3.6	-3.6
23	-4.4	-4.4	-4.4	-4.4	-4.4	-4.4	-4.4	-4.4	-4.4	-4.4	-4.5	-4.5	-4.6	-4.8	-5.1					-5.1	-4.8	-4.5	-4.4	-4.4	-4.3	-4.3	-4.3	-4.3	-4.3	-4.3	-4.3
24	-5.1	-5.1	-5.1	-5.1	-5.1	-5.1	-5.1	-5.1	-5.1	-5.1	-5.1	-5.2	-5.2	-5.3	-5.4					-5.4	-5.3	-5.1	-5.1	-5.0	-5.0	-5.0	-5.0	-5.0	-5.0	-5.0	-5.0
25	-5.8	-5.8	-5.8	-5.6	-5.8	-5.8	-5.8	-5.8	-5.8	-5.8	-5.8	-5.8	-5.7	-5.7	-5.7					-5.7	-5.7	-5.7	-5.7	-5.7	-5.7	-5.7	-5.7	-5.7	-5.7	-5.7	-5.7
26	-6.5	-6.5	-6.5	-6.5	-6.5	-6.5	-6.5	-6.5	-6.4	-6.4	-6.4	-6.3	-6.3	-6.1	-5.9					-5.9	-6.1	-6.2	-6.3	-6.3	-6.4	-6.4	-6.4	-6.4	-6.4	-6.4	-6.4
27	-7.2	-7.2	-7.2	-7.2	-7.2	-7.2	-7.2	-7.1	-7.1	-7.1	-7.0	-6.8	-6.6	-6.2						-6.2	-6.6	-6.8	-6.9	-7.0	-7.1	-7.1	-7.1	-7.1	-7.1	-7.1	-7.1
28	-7.9	-7.9	-7.9	-7.9	-7.9	-7.9	-7.9	-7.8	-7.8	-7.7	-7.6	-7.5	-7.2	-6.7						-6.7	-7.2	-7.5	-7.6	-7.7	-7.8	-7.8	-7.8	-7.8	-7.9	-7.9	-7.9
29	-8.6	-8.6	-8.6	-8.6	-8.6	-8.6	-8.6	-8.5	-8.5	-8.5	-8.4	-8.3	-8.1	-7.8	-7.4	-7.4	-7.8	-8.1	-8.3	-8.4	-8.5	-8.5	-8.5	-8.5	-8.5	-8.6	-8.6	-8.6	-8.6	-8.6	-8.6
30	-9.3	-9.3	-9.3	-9.3	-9.3	-9.3	-9.3	-9.3	-9.3	-9.2	-9.2	-9.1	-9.0	-8.9	-8.8	-8.8	-8.9	-9.0	-9.1	-9.2	-9.2	-9.2	-9.2	-9.3	-9.3	-9.3	-9.3	-9.3	-9.3	-9.3	-9.3
31	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	
32	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	

Top Row:

9.2 9.2 9.2 9.2 9.2 9.2 9.2 9.2 9.2 9.2 9.2 9.2 9.3 9.3 9.3

Bottom Row:

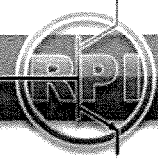
9.3 9.3 9.3 9.3 9.3 9.3 9.3 9.3 9.3 9.3 9.2 9.2 9.1 9.0 8.9 8.8

8.8 8.9 9.0 9.1 9.2 9.2 9.2 9.3 9.3 9.3 9.3 9.3 9.3 9.3 9.3

average voltage = 9.2  
 Voltage diff = .8  
 $E_{ave} = \frac{.8}{101} = 80$   
 $D_{me} = 4.80 \epsilon_0 = 320 \epsilon_0 = 13$

$$C = \frac{96 \epsilon_0}{20} = 4.8 \epsilon_0$$

$$Q = 96 \epsilon_0 \text{ units/m}$$



For the structure above, there is a dielectric with  $\epsilon_r = 4$  in the region between the plates, except for the four conducting blobs scattered randomly about the region. These conductors are not connected to anything so they will float to the average voltage of their neighboring region. Having these blobs in the region is what might happen when some imperfections find their way into the insulating material during a messy manufacturing process. For the calculations below, also assume that the size of each cell is  $1\text{cm}$  by  $1\text{cm}$ .

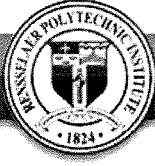
a. From the information provided in the plot and in the paragraph above, determine the capacitance per unit length of this section of a parallel plate structure. Note that the information in some of the cells has been written in a more readable form below the figure.

$$C = 4.8 \epsilon_0$$

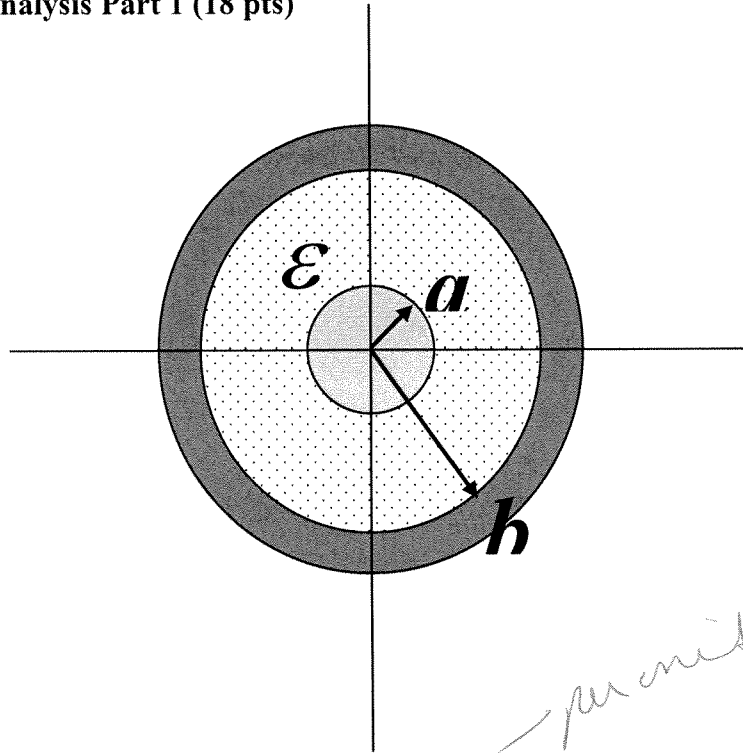
b. Assume now that the insulator was made without the conducting imperfections in it (as we hope would be done under usual circumstances). Otherwise, everything else is the same. Determine analytically the capacitance per unit length. Also, find the Electric Field  $\vec{E}$  in the region between the plates and the surface charge density  $\rho_s$  on the plates. Compare your answer to part a and discuss why it is larger or smaller.

$$C = \frac{4\epsilon_0 A}{d} = 4\epsilon_0 \frac{13}{13} = 4\epsilon_0$$

With the metal blobs, it should be larger because the field region is reduced.



6. Capacitance Analysis Part 1 (18 pts)



A cylindrical capacitor consists of two conducting electrodes. The inner solid cylindrical electrode has a radius  $a$  and a charge  $+Q$  per unit length on it. The outer hollow cylindrical electrode has an inner radius  $b$  and a charge  $-Q$  per unit length on it. The region between the two conductors is filled with an insulator with permittivity  $\epsilon$ .

a. What is the surface charge density on the inner and outer conductors? (4 pts)

$$\rho_{sa} = \frac{Q}{2\pi a} \quad \rho_{sb} = -\frac{Q}{2\pi b}$$

b. Find the electric flux density vector  $\vec{D}(\vec{r})$  for this configuration. (3 pts)

$$\vec{D} = \hat{r} D_r = \frac{Q}{2\pi r} \hat{r}$$



c. Find the corresponding electric field vector  $\vec{E}(\vec{r})$ . (3 pts)

$$E_r = \frac{Q}{2\pi\epsilon r}$$

d. Find the scalar potential function  $V(\vec{r})$  if the outer conductor is grounded. (4 pts)

$$V(r) - V(b) = -\int_b^r \frac{Q}{2\pi\epsilon r} dr$$

$$V(r) = \frac{Q}{2\pi\epsilon} \ln \frac{b}{r}$$

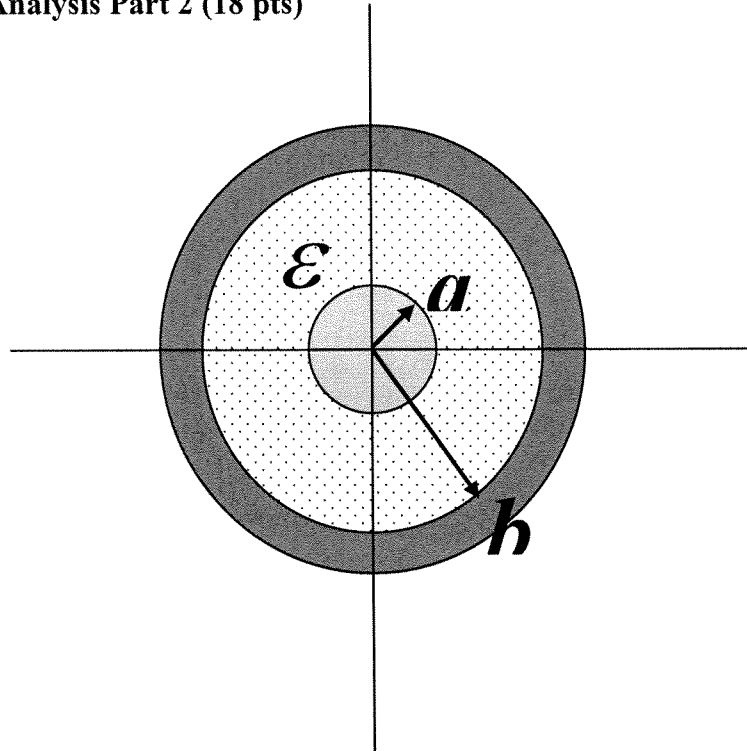
e. Using the information you have so far, determine the capacitance per unit length of this configuration. You may use either the voltage method or the energy method. Be sure you indicate which method you are using. (4pts)

$$\frac{Q}{2\pi\epsilon} \ln \frac{b}{a} = V(a)$$

$$C = \frac{2\pi\epsilon}{\ln(b/a)}$$



7. Capacitance Analysis Part 2 (18 pts)



For the same cylindrical capacitor addressed in the previous problem, the insulator has been replaced by a material whose permittivity is a function of radius

$\epsilon(r) = \epsilon \frac{a}{r}$  where  $\epsilon$  is the same constant we have used in the previous problem.

Repeat the same 5 steps for the new insulating material. *Hint: Some of the answers will be the same.*

a. What is the surface charge density on the inner and outer conductors? (3 pts)

Same  $\rho_{sa} = \frac{Q}{2\pi a}$   $\rho_{sb} = -\frac{Q}{2\pi b}$

b. Find the electric flux density vector  $\vec{D}(\vec{r})$  for this configuration. (3 pts)

Same  $D_r = \frac{Q}{2\pi r}$





c. Find the corresponding electric field vector  $\vec{E}(\vec{r})$ . (3 pts)

$$E = \frac{D}{\epsilon} = \frac{Q}{2\pi r \epsilon a} = \frac{Q}{2\pi a \epsilon}$$

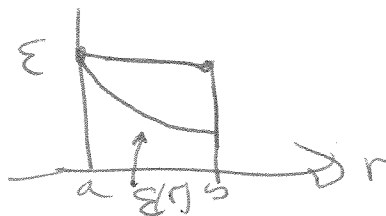
d. Find the scalar potential function  $V(\vec{r})$  (3 pts)

$$V(r) - V(b) = - \int_b^r \frac{Q}{2\pi a \epsilon} dr$$
$$V(r) = \frac{Q}{2\pi a \epsilon} (b - r)$$

e. Using the information you have so far, determine the capacitance per unit length of this configuration. You may use either the voltage method or the energy method. Be sure you indicate which method you are using. (3 pts)

$$C = \frac{2\pi a \epsilon}{b - a}$$

f. The new capacitance should be different than the old capacitance. Explain why one is larger than the other. (3 pts)

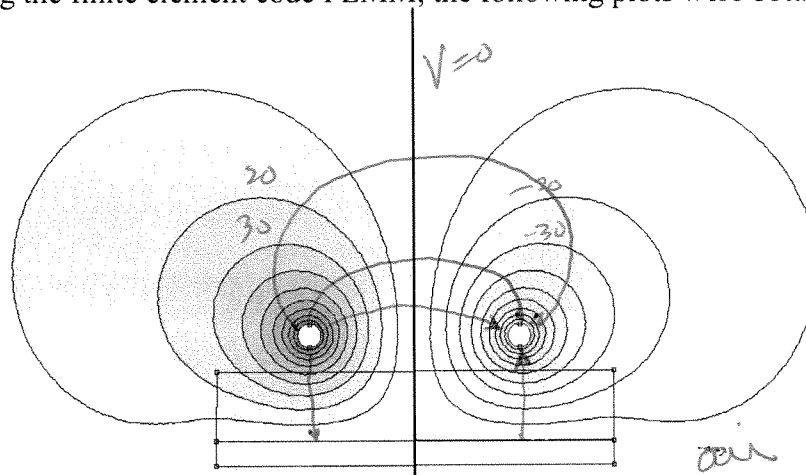


average  $E(r) < E$   
since  $r > a$   
 $\Rightarrow$  Capacitance is less than the first case.

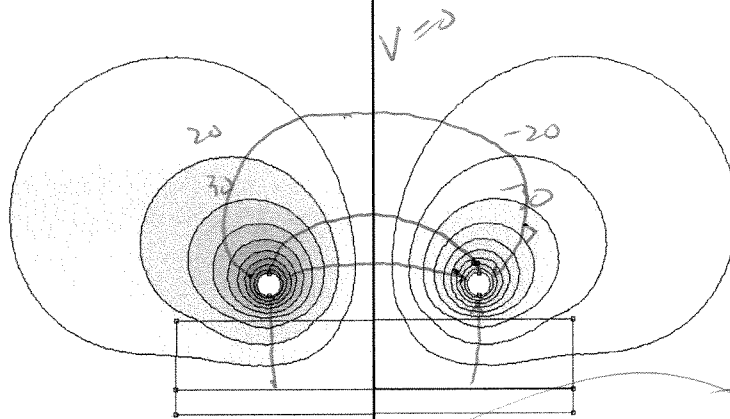


8. Conductors and Materials (22 Points)

A two wire transmission line exists near a conducting plate that supports a dielectric block. Using the finite element code FEMM, the following plots were obtained.



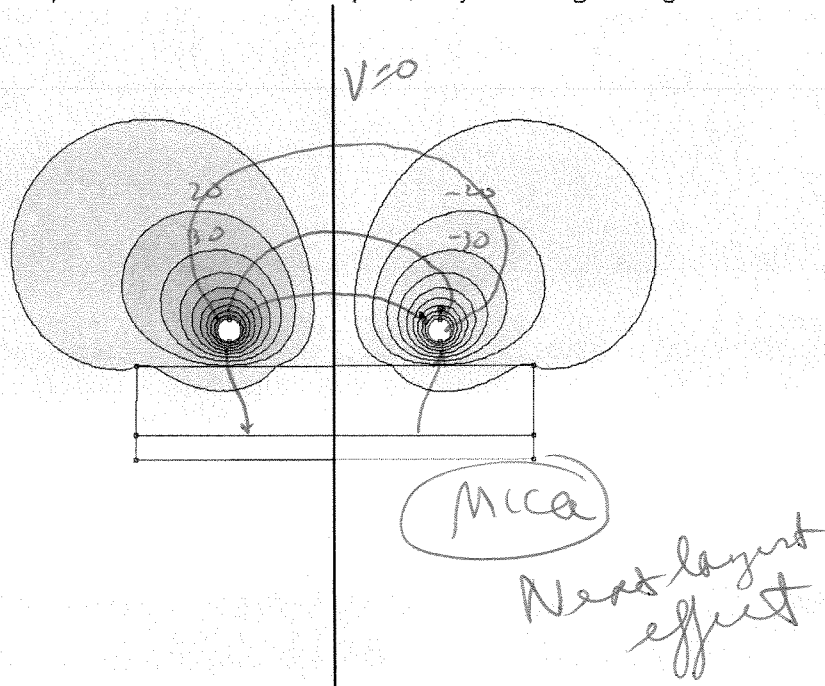
Plot A



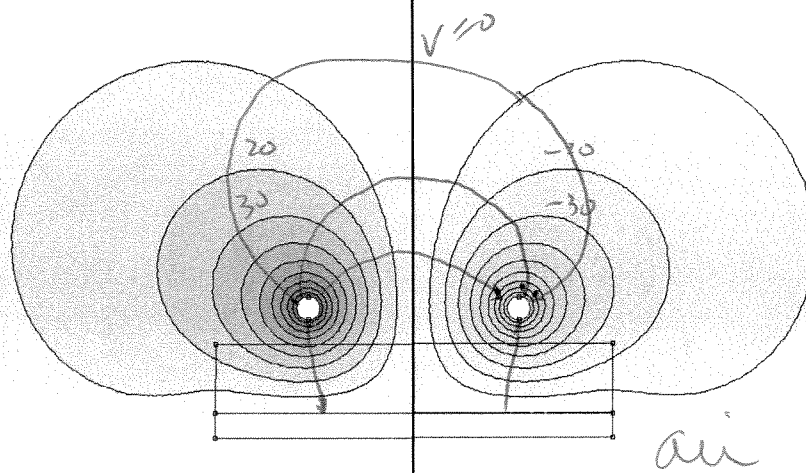
I should be more careful to be sure that

Plot B

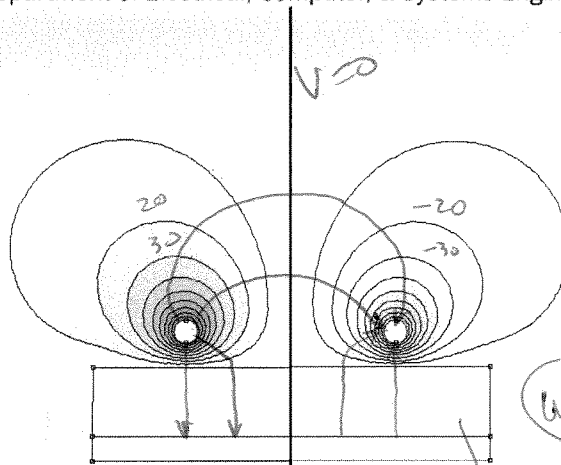
Polyethylene  
best effect



Plot C



Plot D

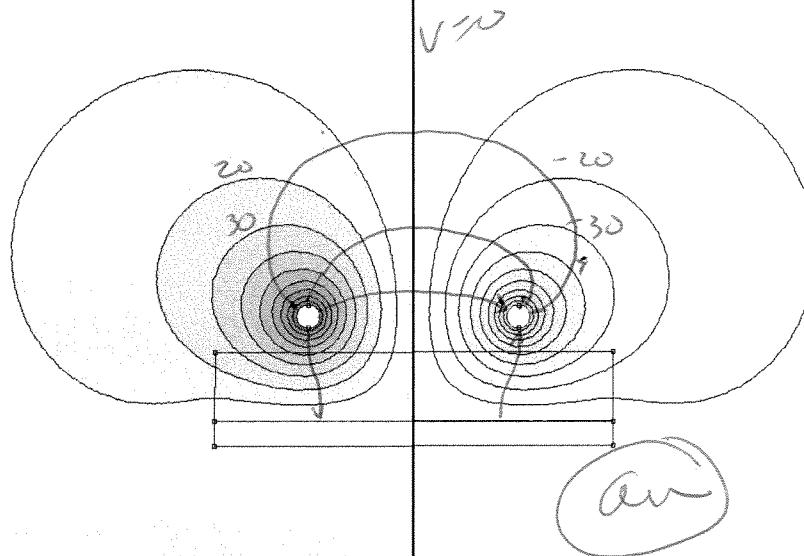


The key is to identify this one & plus the others are for

Water

high  $\epsilon$  excludes the equipotential region

Plot E



air

Plot F

For each of the plots, there are two cylindrical conductors, the left one at +100V and the right one at -100V. There is also a grounded conducting plate. On top of the plate is a dielectric block. The remainder of the space in the problem is air. On each page above, there are two plots, one for which the dielectric block has the same properties as air and



one for which the block has a dielectric constant greater than one. There are three dielectric materials analyzed – Polyethylene ( $\epsilon_r = 2.3$ ), Mica ( $\epsilon_r = 6$ ) and Water ( $\epsilon_r = 80.4$ ).

- a. (5 Points) For each plot, label the five equipotentials  $V=0, V = \pm 20V, V = \pm 30V$ .
- b. (5 Points) For each plot, carefully draw five electric field lines, making sure that you put arrows on them to indicate direction.
- c. (6 Points) For all six plots, identify the material of the dielectric block. Explain your answer.

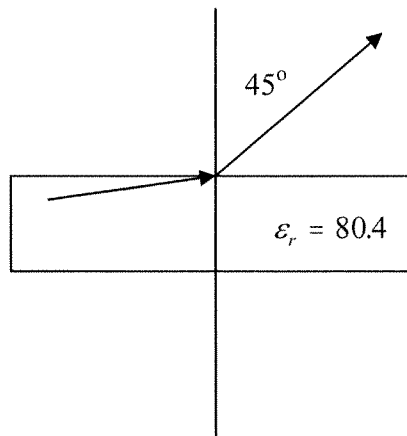
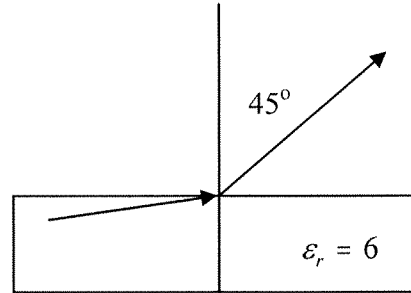
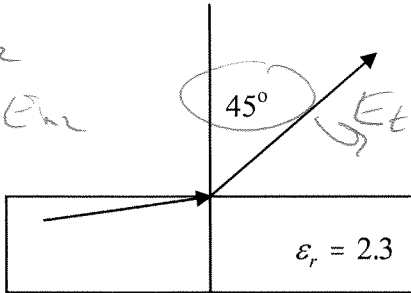
*see plots*

- d. (6 Points) Determine the unknown angle for each of the plots below, where the electric field vector is directed at  $45^\circ$  on the air side of the boundary. That is, find the angle in the dielectric material for the three cases.

$E_{t1} = E_{t2}$

$D_{n1} = D_{n2}$

$\epsilon_1 E_{n1} = \epsilon_2 E_{n2}$



$$\tan \theta = \frac{E_{t1}}{E_{n1}} = \frac{E_{t2}}{\frac{\epsilon_2}{\epsilon_1} E_{n2}} = \frac{\epsilon_1}{\epsilon_2}$$

$\arctan(6) = 80.5^\circ$

$\arctan(2.3) = 66.5^\circ$

$\arctan(80.4) = 89.3^\circ$