

Quiz 2

**VECTOR CALCULUS  
ELECTROSTATICS  
CURRENT & RESISTANCE**



Name \_\_\_\_\_

Section \_\_\_\_\_

**Multiple Choice**

1. (5 Pts) \_\_\_\_\_

2. (6 Pts) \_\_\_\_\_

3. (16 Pts)  
or \_\_\_\_\_

3. (16 Pts) \_\_\_\_\_

4. (6 Pts) \_\_\_\_\_

5. (8 Pts) \_\_\_\_\_

**Regular Questions**

6. (12 Pts) \_\_\_\_\_

7. (12 Pts) \_\_\_\_\_

8. (20 Pts) \_\_\_\_\_

9. (15 Pts) \_\_\_\_\_

10. (Ex Cred) \_\_\_\_\_

**Total (100 Pts)**

\_\_\_\_\_

Notes:

1. In the multiple choice questions, each question may have more than one correct answer; circle all of them.
2. For multiple choice questions, you may add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees.

Some Comments and Helpful Info:

In this test, we use two types of notation for unit vectors. Keep in mind that

$$\hat{a}_x = \hat{x} \quad \hat{a}_y = \hat{y} \quad \hat{a}_z = \hat{z} \quad \hat{a}_r = \hat{r} \quad \hat{a}_\phi = \hat{\phi} \quad \hat{a}_\theta = \hat{\theta}$$

Be sure to show your work for the multiple choice questions.

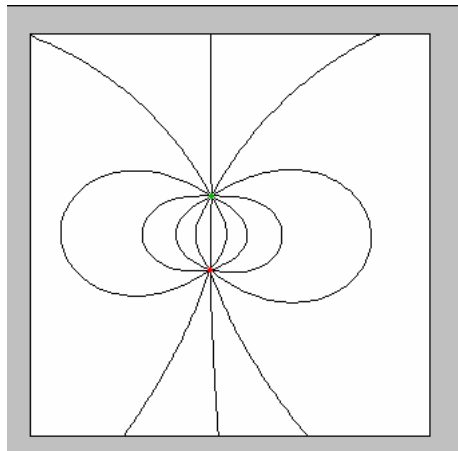
Draw pictures for each problem to be sure that you understand the problem statement.

Please read the note explaining how the extra credit problem will be graded, since there is a penalty for an incorrect answer.

## Quiz 2

**MULTIPLE CHOICE AND SHORT ANSWER QUESTIONS****1. Miscellaneous Questions (5 points)**

- a) Unlike charges attract, like charges repel.  
 i) True  
 ii) False
- b) The force between two charged particles decays as the square of the distance between the particles.  
 i) True  
 ii) False
- c) Volts are the units of the electric field.  
 i) True  
 ii) False



- d) The electric field lines shown above are for two identical positive charges.  
 i) True  
 ii) False
- e) There are free positive and negative charges in the air we breathe.  
 i) True  
 ii) False

**2. Gauss' Law (6 points)**

Gauss' Law is given in integral form as  $\oint \vec{D} \cdot d\vec{S} = \int \rho_v dv = Q_{encl}$ .

- a) Assuming that  $\rho_v = \rho_v(r)$ , where  $r$  is cylindrical radius, simplify the electric flux density vector  $\vec{D} = D_r(r, \phi, z)\hat{r} + D_\phi(r, \phi, z)\hat{\phi} + D_z(r, \phi, z)\hat{z} = ?$
- i)  $\vec{D} = D_\phi(r, \phi, z)\hat{\phi}$   
 ii)  $\vec{D} = D_\phi(r)\hat{\phi}$   
 iii)  $\vec{D} = D_r(r, \phi, z)\hat{r}$   
 iv)  $\vec{D} = D_r(r)\hat{r}$   
 v)  $\vec{D} = D_z(r, \phi, z)\hat{z}$   
 vi)  $\vec{D} = D_z(r)\hat{z}$

## Quiz 2

- b) Again, assuming that  $\rho_v = \rho_v(r)$ , what Gaussian surface will work well if we are to use Gauss' Law to find  $\vec{D}$ ? Also, what  $d\vec{S}$  goes with the surface?
- An open cylinder at radius  $r = \text{const}$  with  $d\vec{S} = \hat{r}rd\phi dz$
  - A closed cylinder consisting of a surface at radius  $r = \text{const}$  with  $d\vec{S} = \hat{r}rd\phi dz$ , length  $l$  and end caps at  $z = 0$  and  $z = l$  with  $d\vec{S} = \pm \hat{z}dr rd\phi$
  - A closed cylinder consisting of a surface at radius  $r = \text{const}$  with  $d\vec{S} = \hat{\phi}rd\phi dz$ , length  $l$  and end caps at  $z = 0$  and  $z = l$  with  $d\vec{S} = \pm \hat{z}dr rd\phi$
  - A closed cylinder consisting of a surface at radius  $r = \text{const}$  with  $d\vec{S} = \hat{r}rd\phi dz$ , length  $l$  and end caps at  $z = 0$  and  $z = l$  with  $d\vec{S} = \pm \hat{\phi}dr rd\phi$
- c) Given your solution to questions a) and b), evaluate the left hand side of Gauss' Law  $\oint \vec{D} \cdot d\vec{S}$
- $\oint \vec{D} \cdot d\vec{S} = D_r(r)2\pi r l$
  - $\oint \vec{D} \cdot d\vec{S} = D_r(r)\pi r^2 l$
  - $\oint \vec{D} \cdot d\vec{S} = D_r(r)\pi r^2$
  - $\oint \vec{D} \cdot d\vec{S} = 0$

## Quiz 2

**3. Capacitance, Voltage, and Electric Field (16 points) – Option 1**

A spherical capacitor consists of an inner conductor of radius  $a$  and an outer conductor of inner radius  $b$  and outer radius  $c$ . The total charge on the inner conductor is  $Q_o$  and the total charge on the outer conductor is  $-Q_o$ . The space between the conductors is empty (air).

a) The electric field inside the inner conductor ( $r < a$ ) is

i)  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r} \hat{r}$

ii)  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r^2} \hat{r}$

iii)  $\vec{E} = 0$

iv)  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r^2} \hat{r}$

v)  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r} \hat{r}$

b) The field in the region between the conductors ( $a < r < b$ ) is

i)  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r} \hat{r}$

ii)  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r^2} \hat{r}$

iii)  $\vec{E} = 0$

iv)  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r^2} \hat{r}$

v)  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r} \hat{r}$

c) The field in the outer conductor ( $b < r < c$ ) is

i)  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r} \hat{r}$

ii)  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r^2} \hat{r}$

iii)  $\vec{E} = 0$

iv)  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r^2} \hat{r}$

v)  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r} \hat{r}$

## Quiz 2

d) The field in the region outside the outer conductor ( $c < r$ ) is

i)  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r} \hat{r}$

ii)  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r^2} \hat{r}$

iii)  $\vec{E} = 0$

iv)  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r^2} \hat{r}$

v)  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r} \hat{r}$

e) The voltage between the inner and outer conductors  $V(a) - V(b) = -\int_b^a \vec{E} \cdot d\vec{l}$  is

i)  $V(a) - V(b) = \frac{Q_o}{2\pi\epsilon_o} \ln \frac{b}{a}$

ii)  $V(a) - V(b) = \frac{Q_o}{2\pi\epsilon_o} \left( \frac{1}{a} - \frac{1}{b} \right)$

iii)  $V(a) - V(b) = 0$

iv)  $V(a) - V(b) = \frac{Q_o}{4\pi\epsilon_o} \left( \frac{1}{a} - \frac{1}{b} \right)$

v)  $V(a) - V(b) = \frac{Q_o}{4\pi\epsilon_o} \ln \frac{b}{a}$

f) The capacitance of the spherical capacitor is given by

i)  $\frac{2\pi\epsilon_o}{\ln \frac{b}{a}}$

ii)  $\frac{2\pi\epsilon_o}{\left( \frac{1}{a} - \frac{1}{b} \right)}$

iii)  $\infty$

iv)  $\frac{4\pi\epsilon_o}{\left( \frac{1}{a} - \frac{1}{b} \right)}$

v)  $\frac{4\pi\epsilon_o}{\ln \frac{b}{a}}$

## Quiz 2

Assume that the inner conductor is replaced by a uniform volume charge distribution but the total charge is still equal to  $Q_o$ .

g) What is the volume charge density in this case?

i)  $\rho_V = \frac{Q_o}{4\pi a^2}$

ii)  $\rho_V = \frac{Q_o}{\frac{4}{3}\pi a^3}$

iii)  $\rho_V = Q_o 4\pi a^2$

iv)  $\rho_V = Q_o \frac{4}{3}\pi a^3$

h) For the region outside the inner conductor ( $a < r$ ), the electric field will remain the same as for the solid inner conductor, since the charge enclosed remains the same. However inside the charge, the answer will be different. What is the electric field inside the charge ( $0 < r < a$ )?

i)  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o a^2} \hat{r}$

ii)  $\vec{E} = \frac{Q_o r}{4\pi\epsilon_o a^3} \hat{r}$

iii)  $\vec{E} = \frac{Q_o r^2}{4\pi\epsilon_o a^4} \hat{r}$

iv)  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r^2} \hat{r}$

## Quiz 2

**3. Charge, Voltage and Electric Field (16 points) – Option 2**

A coaxial cable consists of an inner conductor of radius  $a$  and an outer conductor of inner radius  $b$  and outer radius  $c$ . The total charge per unit length on the inner conductor is  $Q_o$  and the total charge per unit length on the outer conductor is  $-Q_o$ . The space between the conductors is empty (air). For the following questions, keep in mind that the units of  $Q_o$  are Coulombs per meter, not just Coulombs when you check your units.

a. The electric field inside the inner conductor ( $r < a$ ) is

i.  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r} \hat{r}$

ii.  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r^2} \hat{r}$

iii.  $\vec{E} = 0$

iv.  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r^2} \hat{r}$

v.  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r} \hat{r}$

b. The field in the region between the conductors ( $a < r < b$ ) is

i.  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r} \hat{r}$

ii.  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r^2} \hat{r}$

iii.  $\vec{E} = 0$

iv.  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r^2} \hat{r}$

v.  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r} \hat{r}$

c. The field in the outer conductor ( $b < r < c$ ) is

i.  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r} \hat{r}$

ii.  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r^2} \hat{r}$

iii.  $\vec{E} = 0$

iv.  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r^2} \hat{r}$

v.  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r} \hat{r}$

## Quiz 2

d. The field in the region outside the outer conductor ( $c < r$ ) is

i.  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r} \hat{r}$

ii.  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r^2} \hat{r}$

iii.  $\vec{E} = 0$

iv.  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r^2} \hat{r}$

v.  $\vec{E} = \frac{Q_o}{4\pi\epsilon_o r} \hat{r}$

e. The voltage between the inner and outer conductors

$$V(a) - V(b) = -\int_b^a \vec{E} \cdot d\vec{l} \text{ is}$$

i.  $V(a) - V(b) = \frac{Q_o}{2\pi\epsilon_o} \ln \frac{b}{a}$

ii.  $V(a) - V(b) = \frac{Q_o}{2\pi\epsilon_o} \left( \frac{1}{a} - \frac{1}{b} \right)$

iii.  $V(a) - V(b) = 0$

iv.  $V(a) - V(b) = \frac{Q_o}{4\pi\epsilon_o} \left( \frac{1}{a} - \frac{1}{b} \right)$

v.  $V(a) - V(b) = \frac{Q_o}{4\pi\epsilon_o} \ln \frac{b}{a}$

f. The capacitance per unit length of the coaxial cable is given by

i.  $\frac{2\pi\epsilon_o}{\ln \frac{b}{a}}$

ii.  $\frac{2\pi\epsilon_o}{\left( \frac{1}{a} - \frac{1}{b} \right)}$

iii.  $\infty$

iv.  $\frac{4\pi\epsilon_o}{\left( \frac{1}{a} - \frac{1}{b} \right)}$

v.  $\frac{4\pi\epsilon_o}{\ln \frac{b}{a}}$



## Quiz 2

Assume that the inner conductor is replaced by a uniform volume charge distribution but the total charge per unit length is still equal to  $Q_o$ .

g. What is the volume charge density in this case?

i.  $\rho_V = \frac{Q_o}{2\pi a}$

ii.  $\rho_V = \frac{Q_o}{\pi a^2}$

iii.  $\rho_V = Q_o 2\pi a$

iv.  $\rho_V = Q_o \pi a^2$

h. For the region outside the inner conductor ( $a < r$ ), the electric field will remain the same as for the solid inner conductor, since the charge enclosed remains the same. However inside the charge, the answer will be different. What is the electric field inside the charge ( $0 < r < a$ )?

i.  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o a} \hat{r}$

ii.  $\vec{E} = \frac{Q_o r}{2\pi\epsilon_o a^2} \hat{r}$

iii.  $\vec{E} = \frac{Q_o r^2}{2\pi\epsilon_o a} \hat{r}$

iv.  $\vec{E} = \frac{Q_o}{2\pi\epsilon_o r^2} \hat{r}$

## Quiz 2

**4. Resistance and Conductivity (6 points)**

a) A typical value for the conductivity of a good conductor (e.g. copper, silver, gold, aluminum, platinum, etc.) that we might make wire out of is:

i)  $\sigma = 200 \text{ S/m}$

ii)  $\sigma = 0.003 \text{ S/m}$

iii)  $\sigma = 2 \times 10^4 \text{ S/m}$

iv)  $\sigma = 6 \times 10^7 \text{ S/m}$

v)  $\sigma = 10^{-6} \text{ S/m}$

b) The resistance of a cylindrical conductor with a uniform circular cross section of radius  $a$ , a uniform conductivity  $\sigma_o \text{ S/m}$ , and a length  $d$ , is given by:

i)  $R = \frac{\sigma_o d}{\pi a}$

ii)  $R = \frac{\sigma_o d}{\pi a^2}$

iii)  $R = \frac{d}{\sigma_o \pi a^2}$

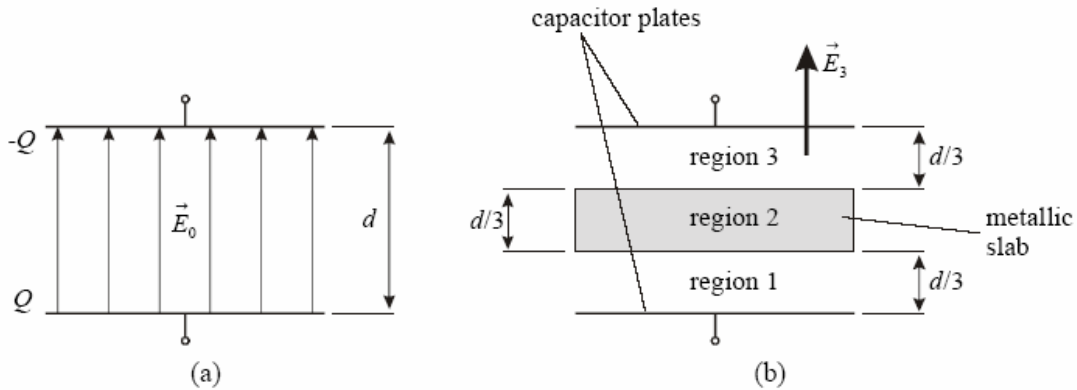
iv)  $R = \frac{d}{\sigma_o 2\pi a}$

v)  $R = \frac{\sigma_o \pi a^2}{d}$

## Quiz 2

**5. Capacitance (8 points)**

An air-filled parallel-plate capacitor is charged and its terminals are left open. The charges of the lower and upper plates are  $Q$  and  $-Q$ , respectively, the fringing effects can be neglected, and the electric field intensity vector between the plates is  $\vec{E}_0$  [Figure (a)]. An uncharged metallic slab is then inserted between the plates, without touching the plates by hands or any other conducting body [Figure (b)].



a) The electric field intensity vector in region 3 between the slab and the upper plate in the new stage is

i)  $\vec{E}_3 = 0$

ii)  $\vec{E}_3 = -\vec{E}_0$

iii)  $\vec{E}_3 = \vec{E}_0/3$

iv)  $\vec{E}_3 = 3\vec{E}_0/2$

v)  $\vec{E}_3 = \vec{E}_0$

b) If the capacitance in figure (a) is  $C_a$ , the capacitance of figure (b) is

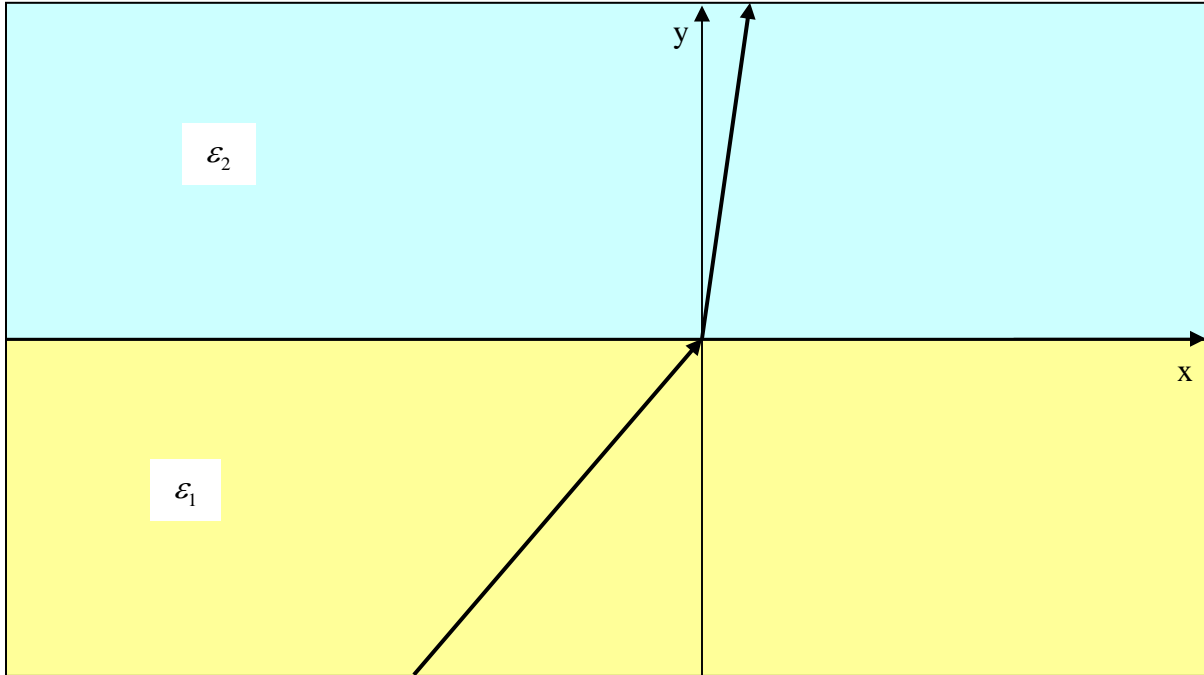
i)  $C_b = 0$

ii)  $C_b = C_a$

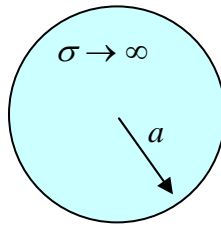
iii)  $C_b = 3C_a$

iv)  $C_b = \left(\frac{2}{3}\right)C_a$

v)  $C_b = \left(\frac{3}{2}\right)C_a$

**REGULAR QUESTIONS****6. Boundary Conditions (12 points)**

The electric field in region 1 is  $\vec{E}_1 = E_o(\hat{a}_x + \hat{a}_y)$ . The electric field in region 2 is  $\vec{E}_2 = E_o(\hat{a}_x + 5\hat{a}_y)$ . Assuming that one of these regions is free space, what is the dielectric constant  $\epsilon$  of the other region? Identify which region is free space (air), region 1 or region 2.

**7. Simple Problem (12 points)**

A solid cylindrical perfect conductor with a radius of  $a$  is shown above. The surrounding medium is free space. There is a total charge  $Q_o$  per meter distributed on this conductor.

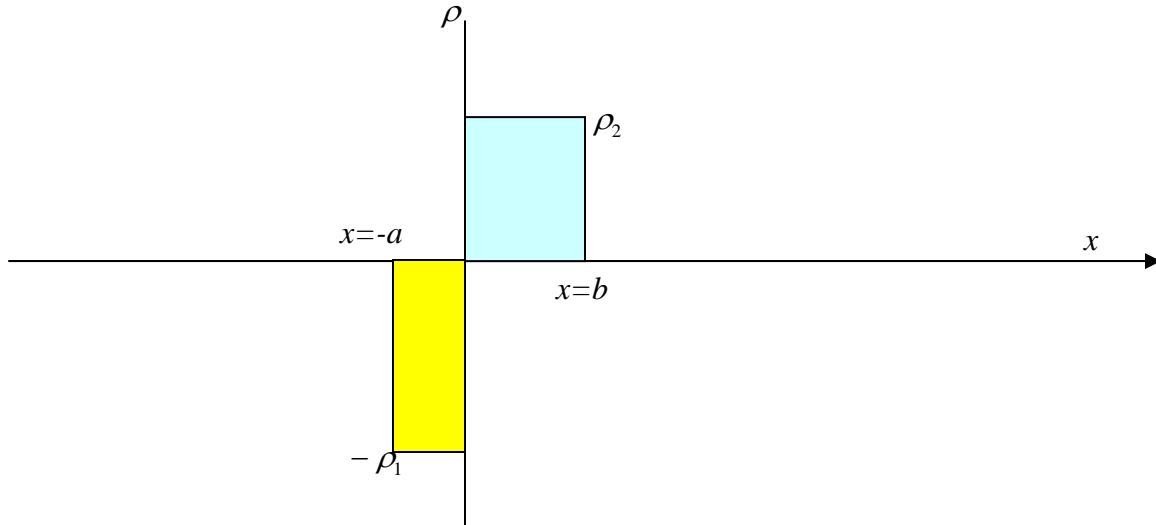
What is the charge density? (6)

What is the electric field  $\vec{E}$  everywhere (for all values of radius)? (6)

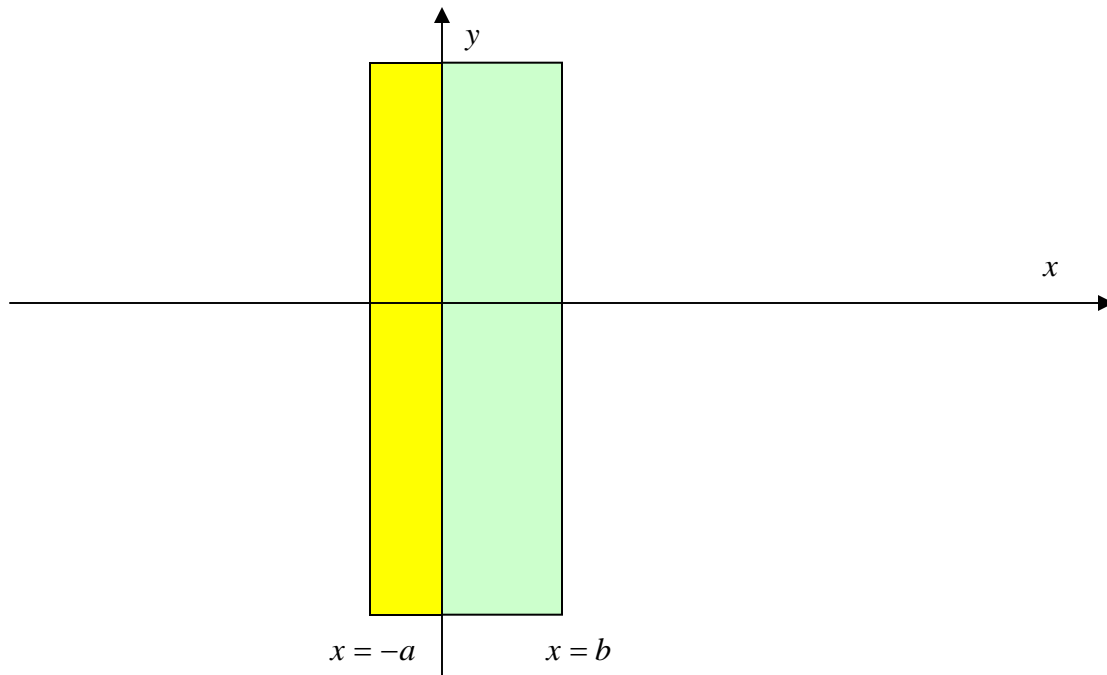
## Quiz 2

**8. Gauss' Law (20 points)**

In a planar region, there is a uniform negative volume charge density  $\rho_v = -\rho_1$  for  $-a \leq x \leq 0$  and a uniform positive volume charge density  $\rho_v = \rho_2$  for  $0 \leq x \leq b$



The total charge is zero. This configuration looks like the following in space.



Draw the figure you would use to apply Gauss' Law. Position your shape exactly where you intend to apply the integral equation. Describe why you chose that shape and location (i.e. how the field behaves on each surface, etc.). (5)

## Quiz 2

Determine the static electric field  $\vec{E}$  everywhere. (10)

Determine the voltage difference between  $x = -a$  and  $x = b$ . (5)

Quiz 2

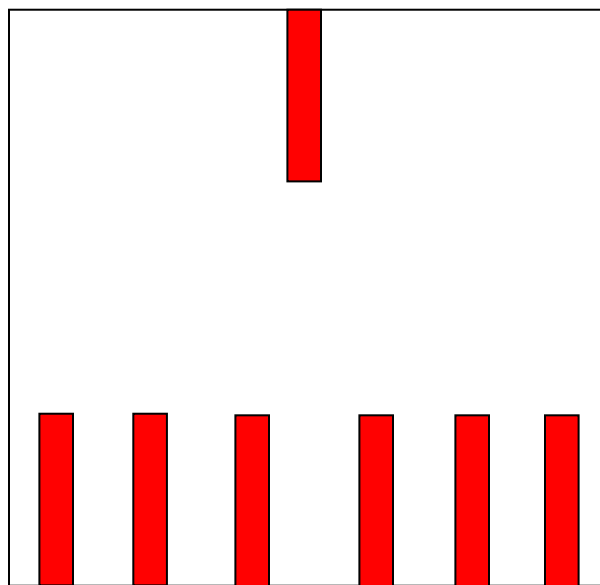
**9. Finite Difference – Capacitance and Resistance (15 points)**

A set of electrodes consisting of two parallel plates with one or more very thin fins attached was analyzed using a spreadsheet resulting in the numbers given below. Remember that this configuration is two-dimensional. That is, it extends indefinitely into the page. Thus, for capacitance or resistance calculations, we are interested in per unit length values or the values for a 1 meter depth.

| AG27 | fx  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|
|      | A   | B   | C   | D   | E   | F   | G   | H   | I   | J   | K   | L   | M   | N   | O   | P   | Q   | R   | S   | T   | U   | V   | W   | X   | Y   | Z   | AA  | AB  | AC  | AD  | AE  |   |
| 1    | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |   |
| 2    | 96  | 96  | 96  | 97  | 97  | 97  | 97  | 97  | 97  | 98  | 98  | 98  | 99  | 99  | 100 | 100 | 100 | 99  | 99  | 98  | 98  | 98  | 97  | 97  | 97  | 97  | 97  | 97  | 96  | 96  | 96  |   |
| 3    | 93  | 93  | 93  | 93  | 93  | 93  | 94  | 94  | 94  | 95  | 95  | 96  | 97  | 97  | 98  | 99  | 100 | 99  | 98  | 97  | 97  | 96  | 95  | 95  | 94  | 94  | 93  | 93  | 93  | 93  | 93  |   |
| 4    | 89  | 89  | 89  | 89  | 90  | 90  | 91  | 91  | 92  | 93  | 94  | 95  | 96  | 97  | 99  | 100 | 99  | 97  | 96  | 95  | 94  | 93  | 92  | 91  | 91  | 90  | 90  | 89  | 89  | 89  | 89  |   |
| 5    | 85  | 85  | 86  | 86  | 86  | 87  | 87  | 88  | 89  | 90  | 91  | 93  | 94  | 96  | 98  | 100 | 98  | 96  | 94  | 93  | 91  | 90  | 89  | 88  | 87  | 87  | 86  | 86  | 86  | 85  | 85  |   |
| 6    | 82  | 82  | 82  | 82  | 83  | 84  | 85  | 86  | 87  | 89  | 91  | 93  | 95  | 97  | 100 | 97  | 95  | 93  | 91  | 89  | 87  | 86  | 85  | 84  | 83  | 82  | 82  | 82  | 82  | 82  | 82  |   |
| 7    | 78  | 78  | 78  | 78  | 79  | 79  | 80  | 81  | 82  | 84  | 86  | 88  | 91  | 94  | 97  | 100 | 97  | 94  | 91  | 88  | 86  | 84  | 82  | 81  | 80  | 79  | 79  | 78  | 78  | 78  | 78  |   |
| 8    | 73  | 73  | 74  | 74  | 75  | 75  | 76  | 77  | 79  | 81  | 83  | 85  | 88  | 92  | 96  | 100 | 96  | 92  | 88  | 85  | 83  | 81  | 79  | 77  | 76  | 75  | 74  | 74  | 73  | 73  | 73  |   |
| 9    | 69  | 69  | 69  | 70  | 71  | 71  | 72  | 73  | 75  | 77  | 79  | 82  | 86  | 90  | 95  | 100 | 95  | 90  | 86  | 82  | 79  | 77  | 75  | 73  | 72  | 71  | 70  | 70  | 69  | 69  | 69  |   |
| 10   | 65  | 65  | 65  | 65  | 66  | 67  | 68  | 69  | 71  | 73  | 75  | 79  | 82  | 87  | 93  | 100 | 93  | 87  | 82  | 79  | 75  | 73  | 71  | 69  | 68  | 67  | 66  | 65  | 65  | 65  | 65  |   |
| 11   | 60  | 60  | 60  | 60  | 61  | 62  | 63  | 64  | 66  | 68  | 71  | 74  | 78  | 83  | 91  | 100 | 91  | 83  | 78  | 74  | 71  | 68  | 66  | 64  | 63  | 62  | 61  | 60  | 60  | 60  | 60  |   |
| 12   | 55  | 55  | 55  | 56  | 56  | 57  | 58  | 59  | 61  | 63  | 66  | 69  | 73  | 78  | 86  | 100 | 86  | 78  | 73  | 69  | 66  | 63  | 61  | 59  | 58  | 57  | 56  | 56  | 55  | 55  | 55  |   |
| 13   | 50  | 50  | 50  | 50  | 51  | 52  | 53  | 54  | 56  | 58  | 60  | 63  | 66  | 70  | 74  | 78  | 74  | 70  | 66  | 63  | 60  | 58  | 56  | 54  | 53  | 52  | 51  | 50  | 50  | 50  | 50  |   |
| 14   | 45  | 45  | 45  | 45  | 46  | 46  | 47  | 49  | 50  | 52  | 53  | 56  | 58  | 61  | 64  | 65  | 64  | 61  | 58  | 56  | 53  | 52  | 50  | 49  | 47  | 46  | 46  | 45  | 45  | 45  | 45  |   |
| 15   | 39  | 39  | 39  | 40  | 40  | 41  | 42  | 43  | 44  | 45  | 47  | 49  | 51  | 52  | 54  | 55  | 54  | 52  | 51  | 49  | 47  | 45  | 44  | 43  | 42  | 41  | 40  | 40  | 39  | 39  | 39  |   |
| 16   | 33  | 33  | 34  | 34  | 34  | 35  | 36  | 37  | 38  | 39  | 40  | 42  | 43  | 44  | 45  | 46  | 45  | 44  | 43  | 42  | 40  | 39  | 38  | 37  | 36  | 35  | 34  | 34  | 34  | 33  | 33  |   |
| 17   | 28  | 28  | 28  | 28  | 29  | 29  | 30  | 31  | 32  | 33  | 34  | 35  | 36  | 37  | 37  | 37  | 36  | 35  | 34  | 33  | 32  | 31  | 30  | 30  | 29  | 29  | 28  | 28  | 28  | 28  | 28  |   |
| 18   | 22  | 22  | 22  | 22  | 23  | 23  | 24  | 24  | 25  | 26  | 26  | 27  | 28  | 29  | 29  | 30  | 29  | 29  | 28  | 27  | 26  | 26  | 25  | 24  | 24  | 23  | 23  | 22  | 22  | 22  | 22  |   |
| 19   | 16  | 16  | 16  | 16  | 17  | 17  | 17  | 17  | 18  | 19  | 20  | 20  | 20  | 21  | 22  | 22  | 22  | 21  | 20  | 20  | 20  | 19  | 18  | 17  | 17  | 17  | 17  | 16  | 16  | 16  | 16  |   |
| 20   | 9.3 | 9.3 | 8.8 | 10  | 11  | 12  | 11  | 10  | 12  | 13  | 13  | 13  | 12  | 14  | 15  | 16  | 15  | 14  | 12  | 13  | 13  | 13  | 12  | 10  | 11  | 12  | 11  | 10  | 8.8 | 9.3 | 9.3 |   |
| 21   | 3.6 | 3.6 | 0   | 4.9 | 6.7 | 6.8 | 5.1 | 0   | 5.5 | 7.6 | 7.8 | 5.8 | 0   | 6.7 | 9.5 | 10  | 9.5 | 6.7 | 0   | 5.8 | 7.8 | 7.6 | 5.5 | 0   | 5.1 | 6.8 | 6.7 | 4.9 | 0   | 3.6 | 3.6 |   |
| 22   | 1.4 | 1.4 | 0   | 2.5 | 3.8 | 3.8 | 2.6 | 0   | 2.8 | 4.3 | 4.3 | 2.9 | 0   | 3.6 | 5.7 | 6.4 | 5.7 | 3.6 | 0   | 2.9 | 4.3 | 4.3 | 2.8 | 0   | 2.6 | 3.8 | 3.8 | 2.5 | 0   | 1.4 | 1.4 |   |
| 23   | 0.5 | 0.5 | 0   | 1.3 | 2.1 | 2.1 | 1.3 | 0   | 1.5 | 2.4 | 2.4 | 1.5 | 0   | 2.1 | 3.5 | 3.9 | 3.5 | 2.1 | 0   | 1.5 | 2.4 | 2.4 | 1.5 | 0   | 1.3 | 2.1 | 2.1 | 1.3 | 0   | 0.5 | 0.5 |   |
| 24   | 0.2 | 0.2 | 0   | 0.7 | 1.1 | 1.1 | 0.7 | 0   | 0.8 | 1.3 | 1.3 | 0.8 | 0   | 1.2 | 2.1 | 2.4 | 2.1 | 1.2 | 0   | 0.8 | 1.3 | 1.3 | 0.8 | 0   | 0.7 | 1.1 | 1.1 | 0.7 | 0   | 0.2 | 0.2 |   |
| 25   | 0.1 | 0.1 | 0   | 0.4 | 0.6 | 0.6 | 0.4 | 0   | 0.4 | 0.7 | 0.7 | 0.4 | 0   | 0.7 | 1.2 | 1.4 | 1.2 | 0.7 | 0   | 0.4 | 0.7 | 0.7 | 0.4 | 0   | 0.4 | 0.6 | 0.6 | 0.4 | 0   | 0.1 | 0.1 |   |
| 26   | 0   | 0   | 0   | 0.2 | 0.3 | 0.3 | 0.2 | 0   | 0.2 | 0.4 | 0.4 | 0.2 | 0   | 0.4 | 0.7 | 0.9 | 0.7 | 0.4 | 0   | 0.2 | 0.4 | 0.4 | 0.2 | 0   | 0.2 | 0.3 | 0.3 | 0.2 | 0   | 0   | 0   | 0 |
| 27   | 0   | 0   | 0   | 0.1 | 0.2 | 0.2 | 0.1 | 0   | 0.1 | 0.2 | 0.2 | 0.1 | 0   | 0.3 | 0.4 | 0.5 | 0.4 | 0.3 | 0   | 0.1 | 0.2 | 0.2 | 0.1 | 0   | 0.1 | 0.2 | 0.2 | 0.1 | 0   | 0   | 0   | 0 |
| 28   | 0   | 0   | 0   | 0.1 | 0.1 | 0.1 | 0.1 | 0   | 0.1 | 0.1 | 0.1 | 0.1 | 0   | 0.1 | 0.3 | 0.3 | 0.3 | 0.1 | 0   | 0.1 | 0.1 | 0.1 | 0.1 | 0   | 0.1 | 0.1 | 0.1 | 0.1 | 0   | 0   | 0   | 0 |
| 29   | 0   | 0   | 0   | 0   | 0.1 | 0.1 | 0   | 0   | 0   | 0.1 | 0.1 | 0   | 0   | 0.1 | 0.1 | 0.2 | 0.1 | 0.1 | 0   | 0   | 0.1 | 0.1 | 0   | 0   | 0   | 0.1 | 0.1 | 0   | 0   | 0   | 0   | 0 |
| 30   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0.1 | 0.1 | 0.1 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0 |
| 31   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0 |

Since these numbers are hard to read, they are repeated on the next page for the left half of the configuration. Symmetry tells us that the numbers are a mirror image on the right half. Assume that the cell size in this case is quite small. Each cell is  $1 \times 10^{-6}$  meters in width and height. The insulator between the plates has  $\epsilon = 2\epsilon_0$ .

a. Carefully sketch 5 E field lines and 5 equipotentials on the diagram below. (5)





Quiz 2

|     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 96  | 96  | 96  | 97  | 97  | 97  | 97  | 97  | 97  | 98  | 98  | 98  | 99  | 99  | 100 | 100 |
| 93  | 93  | 93  | 93  | 93  | 93  | 94  | 94  | 95  | 95  | 96  | 97  | 97  | 98  | 99  | 100 |
| 89  | 89  | 89  | 89  | 90  | 90  | 91  | 91  | 92  | 93  | 94  | 95  | 96  | 97  | 99  | 100 |
| 85  | 85  | 86  | 86  | 86  | 87  | 87  | 88  | 89  | 90  | 91  | 93  | 94  | 96  | 98  | 100 |
| 82  | 82  | 82  | 82  | 82  | 83  | 84  | 85  | 86  | 87  | 89  | 91  | 93  | 95  | 97  | 100 |
| 78  | 78  | 78  | 78  | 79  | 79  | 80  | 81  | 82  | 84  | 86  | 88  | 91  | 94  | 97  | 100 |
| 73  | 73  | 74  | 74  | 75  | 75  | 76  | 77  | 79  | 81  | 83  | 85  | 88  | 92  | 96  | 100 |
| 69  | 69  | 69  | 70  | 70  | 71  | 72  | 73  | 75  | 77  | 79  | 82  | 86  | 90  | 95  | 100 |
| 65  | 65  | 65  | 65  | 66  | 67  | 68  | 69  | 71  | 73  | 75  | 79  | 82  | 87  | 93  | 100 |
| 60  | 60  | 60  | 60  | 61  | 62  | 63  | 64  | 66  | 68  | 71  | 74  | 78  | 83  | 91  | 100 |
| 55  | 55  | 55  | 56  | 56  | 57  | 58  | 59  | 61  | 63  | 66  | 69  | 73  | 78  | 86  | 100 |
| 50  | 50  | 50  | 50  | 51  | 52  | 53  | 54  | 56  | 58  | 60  | 63  | 66  | 70  | 74  | 78  |
| 45  | 45  | 45  | 45  | 46  | 46  | 47  | 49  | 50  | 52  | 53  | 56  | 58  | 61  | 64  | 65  |
| 39  | 39  | 39  | 40  | 40  | 41  | 42  | 43  | 44  | 45  | 47  | 49  | 51  | 52  | 54  | 55  |
| 33  | 33  | 34  | 34  | 34  | 35  | 36  | 37  | 38  | 39  | 40  | 42  | 43  | 44  | 45  | 46  |
| 28  | 28  | 28  | 28  | 29  | 29  | 30  | 30  | 31  | 32  | 33  | 34  | 35  | 36  | 37  | 37  |
| 22  | 22  | 22  | 22  | 23  | 23  | 24  | 24  | 25  | 26  | 26  | 27  | 28  | 29  | 29  | 30  |
| 16  | 16  | 16  | 16  | 17  | 17  | 17  | 17  | 18  | 19  | 20  | 20  | 20  | 21  | 22  | 22  |
| 9.3 | 9.3 | 8.8 | 10  | 11  | 12  | 11  | 10  | 12  | 13  | 13  | 13  | 12  | 14  | 15  | 16  |
| 3.6 | 3.6 | 0   | 4.9 | 6.7 | 6.8 | 5.1 | 0   | 5.5 | 7.6 | 7.8 | 5.8 | 0   | 6.7 | 9.5 | 10  |
| 1.4 | 1.4 | 0   | 2.5 | 3.8 | 3.8 | 2.6 | 0   | 2.8 | 4.3 | 4.3 | 2.9 | 0   | 3.6 | 5.7 | 6.4 |
| 0.5 | 0.5 | 0   | 1.3 | 2.1 | 2.1 | 1.3 | 0   | 1.5 | 2.4 | 2.4 | 1.5 | 0   | 2.1 | 3.5 | 3.9 |
| 0.2 | 0.2 | 0   | 0.7 | 1.1 | 1.1 | 0.7 | 0   | 0.8 | 1.3 | 1.3 | 0.8 | 0   | 1.2 | 2.1 | 2.4 |
| 0.1 | 0.1 | 0   | 0.4 | 0.6 | 0.6 | 0.4 | 0   | 0.4 | 0.7 | 0.7 | 0.4 | 0   | 0.7 | 1.2 | 1.4 |
| 0   | 0   | 0   | 0.2 | 0.3 | 0.3 | 0.2 | 0   | 0.2 | 0.4 | 0.4 | 0.2 | 0   | 0.4 | 0.7 | 0.9 |
| 0   | 0   | 0   | 0.1 | 0.2 | 0.2 | 0.1 | 0   | 0.1 | 0.2 | 0.2 | 0.1 | 0   | 0.3 | 0.4 | 0.5 |
| 0   | 0   | 0   | 0.1 | 0.1 | 0.1 | 0.1 | 0   | 0.1 | 0.1 | 0.1 | 0.1 | 0   | 0.1 | 0.3 | 0.3 |
| 0   | 0   | 0   | 0   | 0.1 | 0.1 | 0   | 0   | 0   | 0.1 | 0.1 | 0   | 0   | 0.1 | 0.1 | 0.2 |
| 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0.1 | 0.1 |
| 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

b. Using the information given, find the total charge per unit length on each plate. (5)

c. Find the capacitance per unit length. (3)

d. Determine the total electric flux  $\psi_e = \int \vec{D} \cdot d\vec{S}$  leaving the 100V electrode. (2)

e. Now assume that the insulator is quite lossy, having a conductivity of  $\sigma = 1 \times 10^{-3} \text{ S/m}$ . Determine the resistance or conductance per unit length. (Extra Credit 4 Pts)

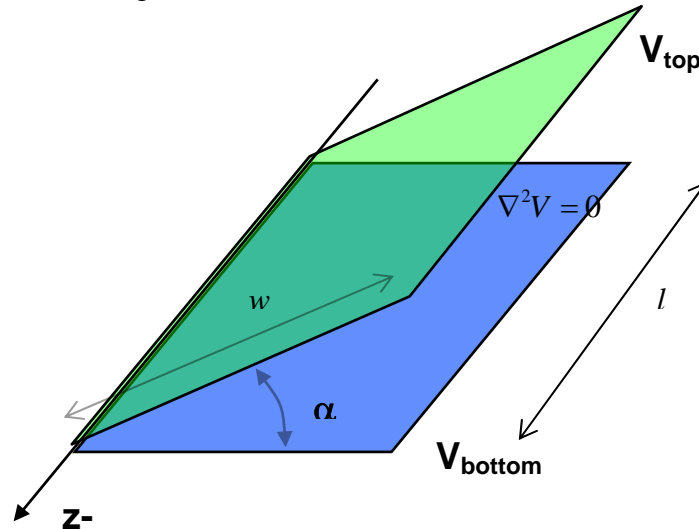
Note: There are many ways to do this problem. We learned one way to find the capacitance and applied it in a homework assignment. Since there are many choices, you may want to consider using other methods to check your results. Also, for the resistance or conductance calculation, recall that you should be able to use the result you obtained for capacitance to make this a much easier question to answer.

## Quiz 2

**10. Laplace's Equation, Capacitance (Extra Credit)**

*Note- For this problem, guessing is not permitted. If your answer to part a is incorrect, you will lose 5 pts for attempting this problem. Thus, you should be sure that you understand it.*

a. In this problem, we will address the electric potential structure between two electrodes that form a wedge with the upper electrode at a voltage  $V_o$  and lower electrode grounded. The plates are  $l$  meters long and  $w$  meters wide.



We assume that the angle between the two plates is small and the plates are very large.

Thus, we can analyze this by assuming that  $\frac{\partial}{\partial r} = \frac{\partial}{\partial z} = 0$ . Using the given information,

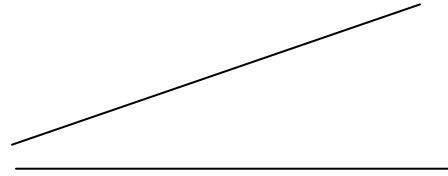
Laplace's Equation was solved for the voltage as a function of position. Unfortunately, the person who solved the problem mixed up the solution to three similar problems and now cannot recall which is the correct solution for this problem. The three solutions are

as follows:  $V(r, \phi, z) = V_o \frac{r\phi}{w\alpha}$      $V(r, \phi, z) = V_o \frac{\phi}{\alpha}$      $V(r, \phi, z) = V_o \frac{\phi^2}{\alpha}$

Circle the correct solution and then show that it does indeed satisfy Laplace's equation and the given boundary conditions. (5 Pts)

## Quiz 2

b. From your solution for  $V(\vec{r})$ , determine the electric field in the region between the plates  $\vec{E}(\vec{r})$ . Note that the electric field will become infinite if the plates almost touch. Thus, we assume that the plates do not go all the way to  $r = 0$ , but rather start at a small value of  $r = a$ . (5 Pts)



c. From your solution for  $\vec{E}(\vec{r})$  find the charge on one of the plates and the capacitance of this structure. (5 Pts)