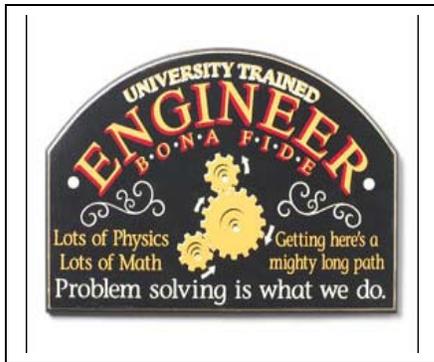


Quiz 2

**VECTOR CALCULUS  
ELECTROSTATICS  
CURRENT & RESISTANCE**

Name \_\_\_\_\_

Section \_\_\_\_\_



1. (4 Pts) \_\_\_\_\_

2. (4 Pts) \_\_\_\_\_

3. (4 Pts) \_\_\_\_\_

4. (8 Pts) \_\_\_\_\_

5. (8 Pts) \_\_\_\_\_

6. (8 Pts) \_\_\_\_\_

7. (8 Pts) \_\_\_\_\_

8. (10 Pts) \_\_\_\_\_

9. (24 Pts) \_\_\_\_\_

10. (12 Pts) \_\_\_\_\_

11. (10 Pts) \_\_\_\_\_

12. (Ex Cred) \_\_\_\_\_

**Total (100 Pts)** \_\_\_\_\_

Notes:

1. In the multiple choice questions, each question may have more than one correct answer; circle all correct answers.
2. For multiple choice questions, you may add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees.
4. Draw pictures for each problem to be sure that you understand the problem statement.

Some Comments and Helpful Info:

In this test, we use two types of notation for unit vectors. Keep in mind that

$$\hat{a}_x = \hat{x} \quad \hat{a}_y = \hat{y} \quad \hat{a}_z = \hat{z} \quad \hat{a}_r = \hat{r} \quad \hat{a}_\phi = \hat{\phi} \quad \hat{a}_\theta = \hat{\theta}$$

Also, sometimes  $R$  is used for spherical radius instead of  $r$ , so  $R$  is another term that gets used for more than one purpose. Pay attention to the context of the questions to minimize problems.

The general form of Ohm's Law is given by  $\vec{J} = \sigma \vec{E}$ . The resistance of a material with

conductivity  $\sigma$  is given by  $R = \frac{l}{\sigma A}$ .

## Quiz 2

**1. Solving for the Electric Field (4 Points)**

If you determine the electric field in some region using Gauss' Law in integral form, which of the following is *always* useful for checking your answer?

- a. Applying Gauss' Law in Differential Form
- b. Looking it up on Google
- c. Checking the answers in the back of the book
- d. Checking the form of the solution against the electric field due to a point charge.
- e. Asking the clerk at Stewarts.

**2. Equipotentials and Electric Field Lines (4 Points)**

Electric Field lines and equipotential surfaces are

- a. Always parallel to one another
- b. Never parallel to one another
- c. Always perpendicular to one another
- d. Never perpendicular to one another

**3. Laplace's Equation (4 Points)**

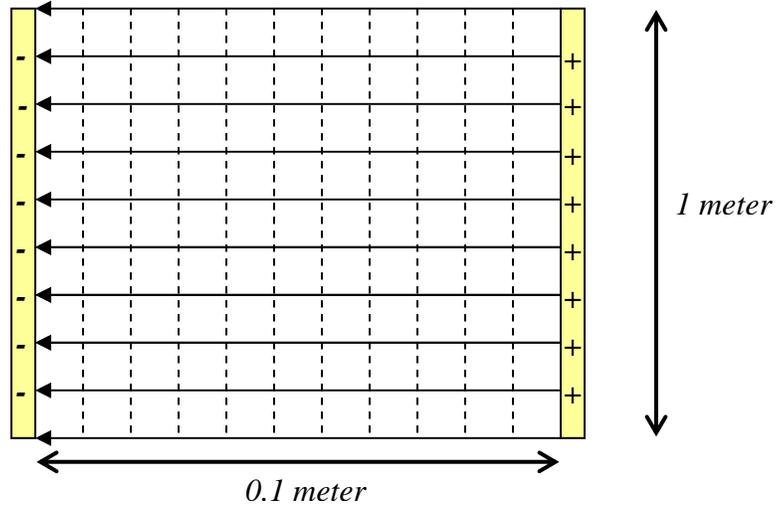
Which of the following statements are correct?

- a. The value of a function that is a solution to Laplace's equation is equal to the average of the values at its nearest neighboring points.
- b. The solution to Laplace's equation in some region in space achieves its maximum and minimum values on the boundary of the region.
- c. Poisson's Equation becomes Laplace's Equation in any region with no volume charge.
- d. Laplace's Equation is a second order partial differential equation.

## Quiz 2

**4. Energy and Capacitance (8 Points)**

For an air insulated parallel plate capacitor, the field lines and equipotentials can be drawn as shown below. Assume that the capacitor has a depth of 1 meter into the page.



a. What is the capacitance of this parallel plate structure?

b. If the voltage on the right plate is  $100\text{ V}$  and the left plate is grounded, how much energy is stored in one of the small cells shown? Each cell is  $1\text{ meter}$  deep.

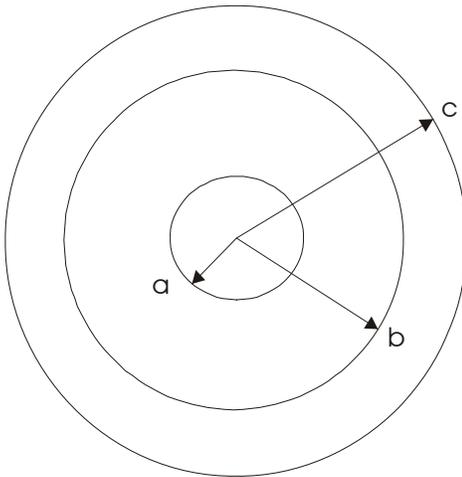
## Quiz 2

**5. Gauss' Law (8 points)**

The electric field for a particular charge distribution in free space is given by

$$\vec{E}(r, \theta, \phi) = \hat{a}_r \frac{\rho_o}{\epsilon_o} \left[ \frac{r}{3} - \frac{r^2}{4b} \right] \text{ for } r \leq b \text{ and } \vec{E}(r, \theta, \phi) = 0 \text{ for } r > b. \text{ Find the total charge}$$

enclosed for the two surfaces shown in the figure below: spherical surfaces at  $r = a$  and at  $r = c$ . *Be sure that you explain your method, since there is more than one way to do this problem. If you have time, you might try using more than one method to check your answer. Hint: Begin by writing Gauss' Law in integral form.*



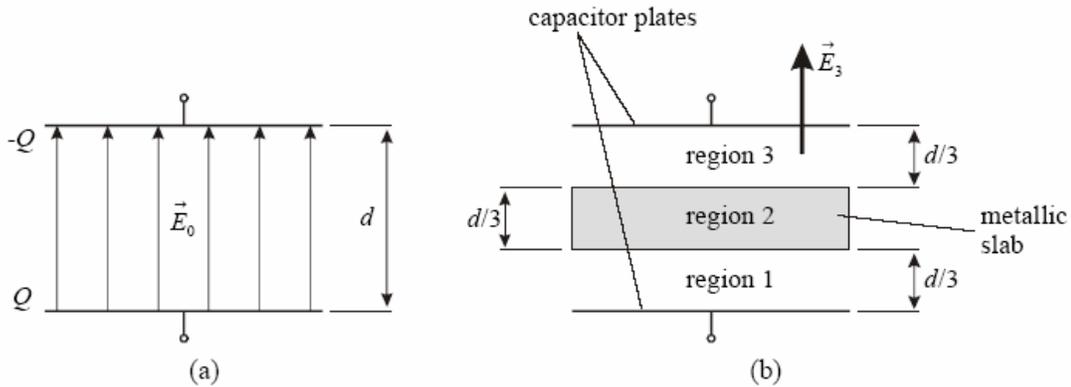
**6. Resistance and Conductivity (8 points)**

A 100 meter long steel pipe with inner diameter of 10 cm and outer diameter of 11 cm. The conductivity of the steel is  $10^6$  S/m. The pipe characteristics considered here are typical for pipes used in oil wells. What is the resistance of the pipe? *Be careful of units.*

## Quiz 2

**7. Capacitance (8 points)**

An air-filled parallel-plate capacitor is charged and its terminals are left open. The charges of the lower and upper plates are  $Q$  and  $-Q$ , respectively, the fringing effects can be neglected, and the electric field intensity vector between the plates is  $\vec{E}_0$  [Figure (a)]. An uncharged metallic slab is then inserted between the plates, without touching the plates by hands or any other conducting body [Figure (b)].



a) The electric field intensity vector in region 3 between the slab and the upper plate in the new stage is

i)  $\vec{E}_3 = 0$

ii)  $\vec{E}_3 = -\vec{E}_0$

iii)  $\vec{E}_3 = \vec{E}_0/3$

iv)  $\vec{E}_3 = 3\vec{E}_0/2$

v)  $\vec{E}_3 = \vec{E}_0$

b) If the capacitance in figure (a) is  $C_a$ , the capacitance of figure (b) is

i)  $C_b = 0$

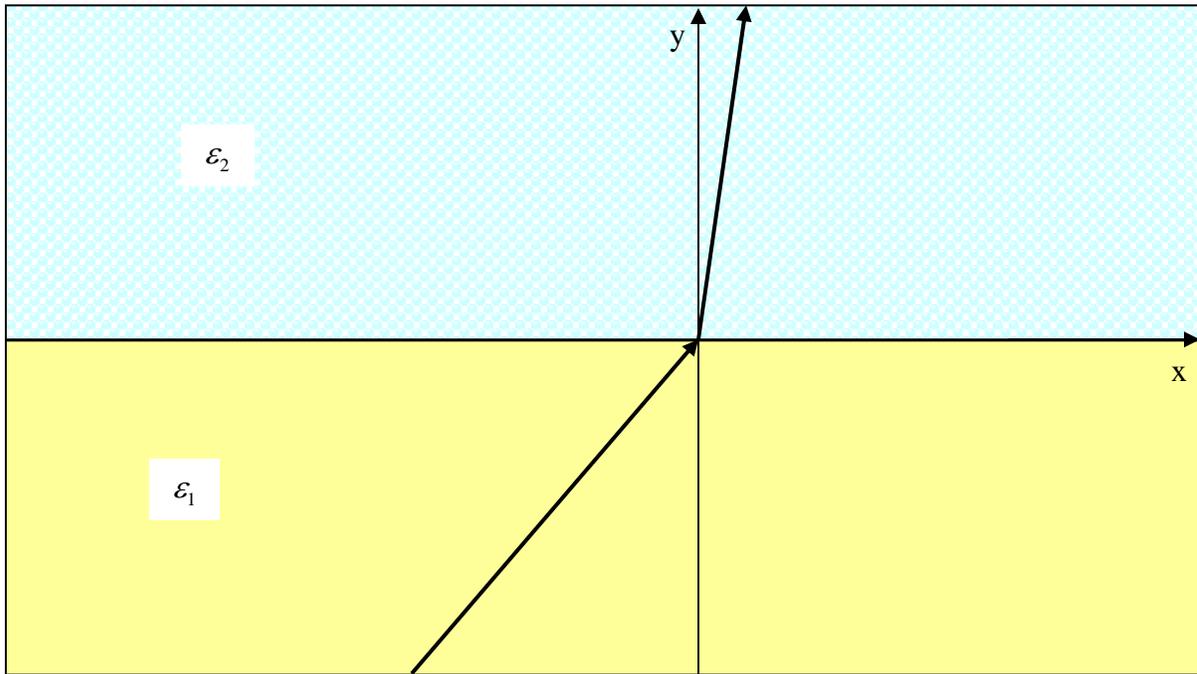
ii)  $C_b = C_a$

iii)  $C_b = 3C_a$

iv)  $C_b = \left(\frac{2}{3}\right)C_a$

v)  $C_b = \left(\frac{3}{2}\right)C_a$

## Quiz 2

**8. Boundary Conditions (10 points)**

The electric field in region 1 is  $\vec{E}_1 = E_o(\hat{a}_x + \hat{a}_y)$ . The electric field in region 2 is  $\vec{E}_2 = E_o(\hat{a}_x + 3\hat{a}_y)$ .

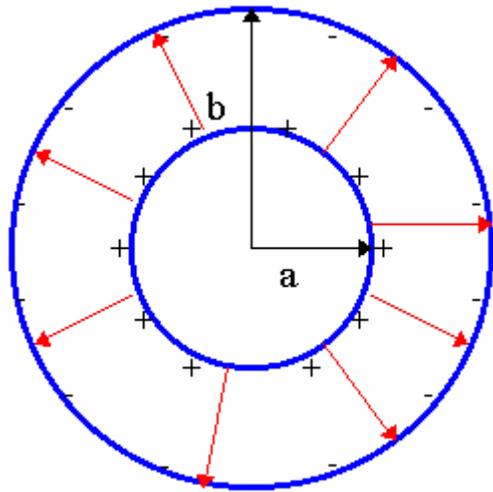
a. Assuming that one of these regions is free space, what is the dielectric constant  $\epsilon$  of the other region? (6 Points)

b. Identify which region is free space (air), region 1 or region 2. (4 Points)

## Quiz 2

**9. Gauss' Law, Materials and Capacitance (24 points)**

The diagram of a spherical capacitor shown below was found at a website for an electromagnetic theory course at another university. The inner conductor, radius =  $a$ , has a total positive charge on it  $+Q$  while the outer conductor, radius =  $b$ , has a total negative charge  $-Q$ . The plus (+) and minus (-) signs indicate where the charge is found. Also included in this figure are some typical, if badly drawn, electric field lines.



A spherical capacitor

Air insulated

Inner radius =  $a$

Outer radius =  $b$

Total charge  $Q$  on each conductor

a. What are the surface charge densities at  $r = a$  and  $r = b$ ,  $\rho_{sa}$  &  $\rho_{sb}$ ?

b. Given that this capacitor is air insulated ( $\epsilon = \epsilon_0$ ), determine the electric flux density  $D$  everywhere in space in terms of parameters like  $Q$ ,  $\epsilon_0$  and the dimensions of the capacitor. *It may not be necessary to use all of these terms.*

## Quiz 2

c. Determine the electric field  $E$  using your answer to part b.

d. Now assume that the air insulator has been replaced by a dielectric with a relative permittivity of  $\epsilon_r = 3$ . Write the new expressions for  $D$  and  $E$

You can complete this problem using either of the general methods for finding capacitance.

**Option 1:**

e. Assuming that the outer conductor is grounded, determine the potential on the inner conductor  $V(a)$  using your expression for  $E$  from part d.

f. Using the voltage method, find the capacitance of this structure with  $\epsilon_r = 3$ .

## Quiz 2

**Option 2:**

e. Find the energy stored in the region between  $r = a$  and  $r = b$  using your expression for  $D$  and  $E$  from part d.

f. Use the energy method to find the capacitance of this structure with  $\epsilon_r = 3$ .

Quiz 2

**10. Finite Difference Solution (12 Points)**

An Excel spreadsheet was used to determine the capacitance per unit length of the following transmission line configuration. The insulator is characterized by  $\epsilon_r = 2.25$

	AE30																									
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
3	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
4	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
5	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
6	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
7	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
8	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
9	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
10	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
11	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
12	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
13	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
14	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
15	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
16	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
17	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
18	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
19	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
20	0	###	###	###	###	###	###	###	###	10	10	10	10	10	10	10	10	###	###	###	###	###	###	###	###	0
21	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
22	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
23	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
24	0	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26																										
27																										

Note that the grounded outer conductor is a square trough, 24 mm by 24 mm and the inner conductor is a horizontal plate connected to a 10V DC source. From the spreadsheet, it is determined that the capacitance per unit length is approximately

$$C/l = 10\epsilon_0$$

## Quiz 2

a. Determine the charge per unit length for this transmission line.

b. Note that the voltages in the interior cells are not shown. (The number of significant digits is too large to fit in the cell, so Excel just shows the # sign.) From your knowledge of this method, what is the average voltage in the cells neighboring (just inside) the outer conductor.

## Quiz 2

**11. True or False (10 Points)**

*It has been our experience through many years with this course that there is one electromagnetic fact almost every student can recall from their basic Physics classes.*

True or false? There is no electric field inside of a perfect conductor.

**12. Extra Credit (5 Points)**

*It is always a frustration to study for a test and then discover that one of the problems you were sure would be on the test is not included. This is your chance to earn some extra points for your efforts. In the space below, write out a problem you think would be fair to ask on this test, but is not like anything already included. It is not necessary to solve the problem, only to state it clearly. Remember that your new problem must be different and not just a variation of something given above.*