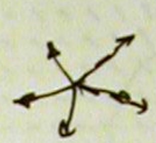
(1) Circle the most appropriate answer (25 pts.)

a) Concerning the figures below which show 2 vector fields, the arrows point in the direction of the field and the length of the arrow is proportional to the magnitude of the field. Which can be an electric and which a magnetic field. Give a short explanation.

see next page for figures

magnetic field

has curl. flux lines are closed paths



Electric field have divergence No curl.

b) Maxwell stress is anxiety during a Fields and Waves exam.

True

False

c) A Helmholtz coil is used to -

Produce a very high field

Produce a very uniform field ✓

Produce a field with high gradient

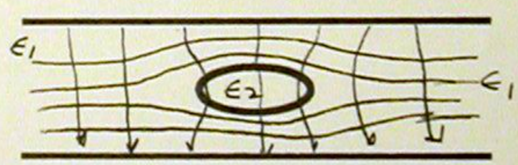
Produce a region of no field

d) For the capacitor shown there is a different material in region 1 and 2. The plot shows the equipotential and flux lines. Which is true?

$\varepsilon_1 > \varepsilon_2$

$\varepsilon_1 < \varepsilon_2$ ✓

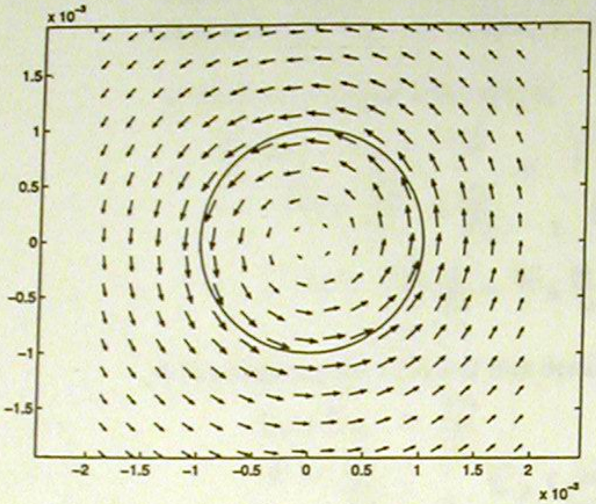
$\varepsilon_1 = \varepsilon_2$

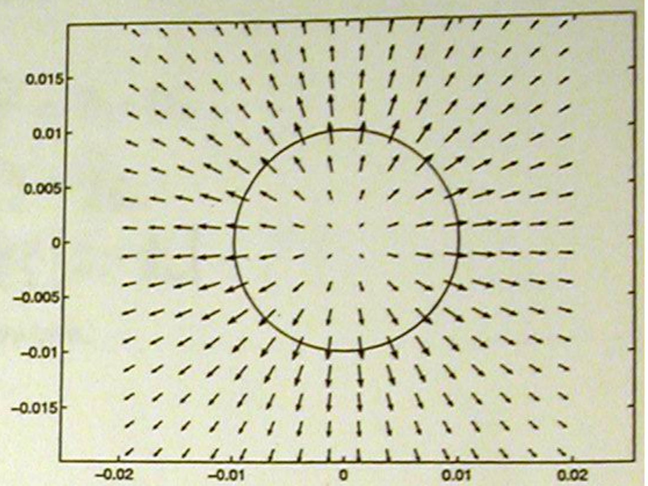


e) Why can we use a scalar potential in electrostatic problems but not (usually) in magnetostatic problems. $\nabla \times E = 0 \Rightarrow E = -\nabla V$ since $\nabla \times \nabla V = 0$

$\nabla \times H = J$ so since curl of grad is always zero. we can not use scalar potential.

Figures for problem 1a.





(2) (25 pts) A parallel plate capacitor (one meter deep) is shown in the figure below. There are 2 dielectric regions, ε_1 and ε_2, each of height d/2. For a charge Q on the top plate and –Q on the bottom plate do the following:

a) Find the potential difference V_0

$$\text{Area} = l \times 1 = l, \quad \rho_s = \frac{Q}{l} = D_1 = D_2$$

$$E_1 = \frac{D_1}{\epsilon_1} = \frac{Q}{l\epsilon_1}, \quad E_2 = \frac{D_2}{\epsilon_2} = \frac{Q}{l\epsilon_2}$$

$$V_0 = E_1 \frac{d}{2} + E_2 \frac{d}{2} = \frac{Qd}{l\,2}\left[\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2}\right]$$

b) Find the electric field and flux density everywhere.

$$D_1 = D_2 = \frac{Q}{l}$$

$$E_1 = \frac{Q}{l\epsilon_1}, \quad E_2 = \frac{Q}{l\epsilon_2}$$

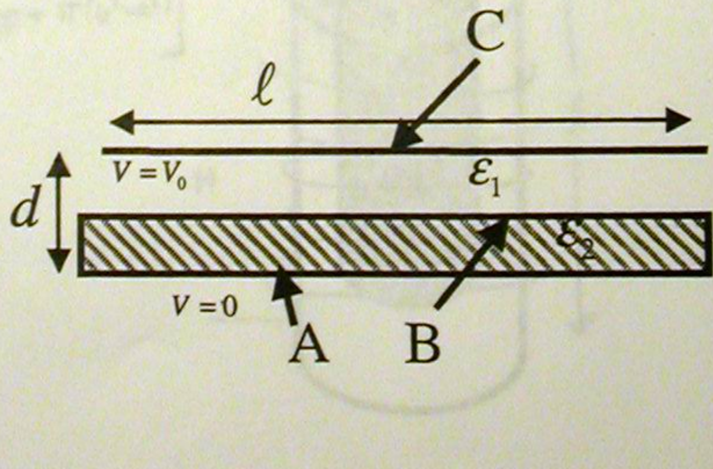
c) Comment on how the interface conditions are met at points A ,B and C.

at A + C $\rho_s = D_n = \frac{Q}{l}$ no tangential component

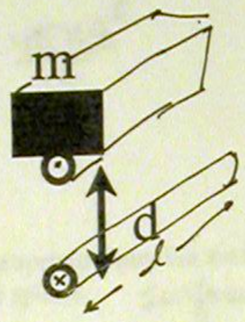
at B, $D_{n1} = D_{n2}$, there is no tangential component.

d) Find the capacitance per meter depth.

$$C = \frac{Q}{V} = \frac{\cancel{Q}\, l * 2}{\cancel{Q}\, d} \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}$$



3. (25) a) A mass **m** is attached to a circular conductor (assume no mass for the conductor) as shown below. The return conductor is located a distance d below the upper conductor. To achieve a stable position the forces must balance. Find the current necessary to achieve this.



$F = i\, l \times B$

FORCE Down = mg

$B = \frac{\mu_0 I}{2\pi d}$

$mg = \frac{\mu_0 l}{2\pi d} I^2$

$I = \sqrt{\frac{2\pi d\, mg}{\mu_0 l}}$

b) The solenoid with circular cross section below has a radius $\boldsymbol{b}$. There is a cylinder of magnetic material inside of radius $\boldsymbol{a}$ and the material has relative permeability μ_r. There are $\boldsymbol{N}$ turns uniformly wound around the outside and the length is ℓ..

c) Find the flux density in the air and in the magnetic material for a current I.

$H = NI/l \qquad B_{air} = \frac{\mu_0 NI}{l}, \quad B_{STEEL} = \frac{\mu_0 \mu_r NI}{l}$

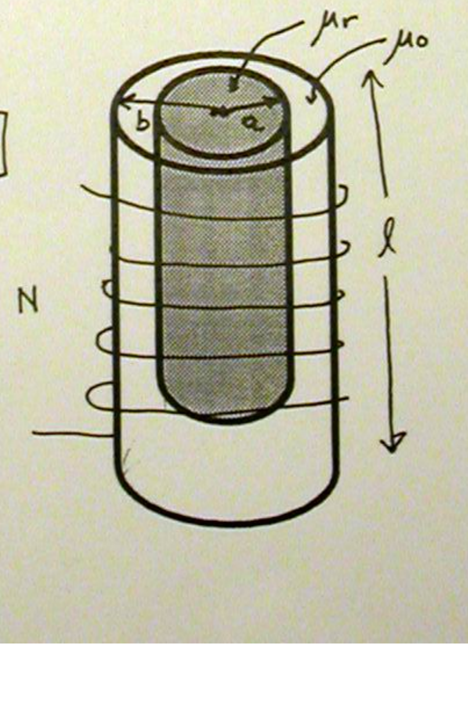
d) Find the flux in the air and in the steel.

$\Psi_{STEEL} = B_{STEEL} \cdot A_{STEEL} = \frac{\mu_0 \mu_r NI}{l} \pi a^2, \quad \Psi_{air} = \frac{\mu_0 NI}{l} \pi (b^2 - a^2)$

e) Find the inductance.

$\lambda = N(\Psi_{STEEL} + \Psi_{AIR})$

$L = \frac{\lambda}{I} = \frac{N^2}{l} \mu_0 \left[\pi a^2 \mu_r + \pi (b^2 - a^2) \right]$

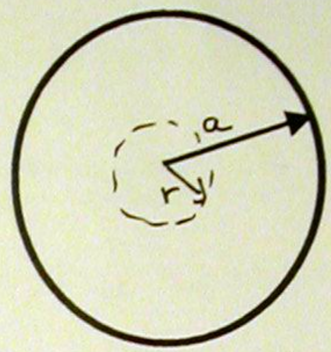


4. (25) The sphere shown below with radius a has a charge density of $\rho_v = kr^2$ The charge density is zero outside the sphere.

a) Find the total charge in the sphere.

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho_v \, r^2 \sin\theta \, dr \, d\varphi \, d\theta$$

$$= 4\pi \int_0^a K r^4 dr = \frac{4\pi K a^5}{5}$$



b) Choose an appropriate Gaussian surface and use Gauss's law to find the electric flux density inside the sphere. Surface is sphere centered at $r=0$.

$$\oint D \cdot ds = Q_{enc.} = D \cdot \cancel{4\pi r^2} = \frac{\cancel{4\pi} K r^{\cancel{5}3}}{5}$$

$$D = \frac{K r^3}{5} \hat{a}_r$$

c) Find the flux density outside the sphere. Let $Q = \frac{4\pi K a^5}{5}$

$$D = \frac{Q}{4\pi r^2} \hat{a}_r$$