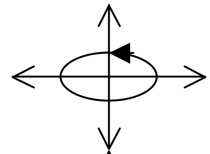
1. (25) Determine the polarization of the following uniform plane wave fields. The electric field is of the form $E(z,t) = A\cos(\omega t - \beta z)\hat{a}_x + B\sin(\omega t - \beta z + \phi)\hat{a}_y$.

   a. (7) $A = 4 \text{ V/m}$, $B = 3 \text{ V/m}$, $\phi = 0°$.

   $E = 4\cos(\omega t - \beta z)\hat{a}_x + 3\sin(\omega t - \beta z)\hat{a}_y$

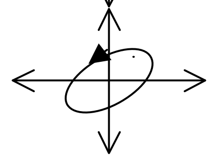
   Right elliptical.



   b. (7) $A = 3$, $B = 3 \text{ V/m}$, $\phi = 45°$.

   $E = 3\cos(\omega t - \beta z)\hat{a}_x + 3\sin(\omega t - \beta z + 45°)\hat{a}_y$

   Right elliptical.



   c. (7) Write the electric field in air, $E = -j30\hat{a}_x + 10\hat{a}_y$ at 100MHz, in time domain form.

   $E = 30\cos\left(2\pi \times 10^8 t - \pi/2\right)\hat{a}_x + 10\cos\left(2\pi \times 10^8 t\right)\hat{a}_y$

   $E = 30\sin\left(2\pi \times 10^8 t\right)\hat{a}_x + 10\cos\left(2\pi \times 10^8 t\right)\hat{a}_y$

   d. (4) Write the electric field $E = 100\cos(\omega t - \beta x)\hat{a}_z$ in phasor form.

   $E = 100 e^{-j\beta x}\hat{a}_z$

2. (25) A wave in free space has a magnitude of 100 V/m and is normally incident of a perfect dielectric of $\varepsilon_r = 25$.
   Find the following:
   a. (10) The reflection and transmission coefficients.

   $$\eta_0 = 377, \quad \eta_1 = \frac{377}{5}$$

   $$\Gamma = \frac{\eta_1 - \eta_0}{\eta_1 + \eta_2} = \frac{\frac{1}{5} - 1}{\frac{1}{5} + 1} = -\frac{2}{3}$$

   $$\tau = \frac{2\eta_1}{\eta_2 + \eta_1} = \frac{2/5}{6/5} = \frac{1}{3} = \Gamma + 1$$

   b. (5) The standing wave ratio in the free space region.

   $$\text{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 2/3}{1 - 2/3} = 5$$

   c. (10) The power densities (Poynting vector) of the incident, reflected and transmitted wave.

   $$S_{avg} = \frac{1}{2}\frac{E^2}{\eta}$$

   incident: $\frac{1}{2}\frac{(100\,\text{V}/\text{m})^2}{377} = 13.263$

   reflected: $\frac{1}{2}\frac{\left(2/3 \times 100\,\text{V}/\text{m}\right)^2}{377} = 5.895$

   transmitted: $\frac{1}{2}\frac{\left(1/3 \times 100\,\text{V}/\text{m}\right)^2}{377} \times 5 = 7.368$

3. (25) A plane wave in free space with magnitude $200\,{}^{\mathrm{V}}\!/_{\mathrm{m}}$ at 100MHz is traveling in the z direction and is normally incident on a material which has $\mu_r = 1,\ \varepsilon_r = 10,\ \sigma = 20\times10^{-9}\,{}^{\mathrm{S}}\!/_{\mathrm{m}}$.

a. (5) Is the material a good conductor, good insulator or neither? Explain.

$$\frac{\sigma}{\omega\varepsilon} = \frac{20\times10^{-9}}{2\pi\times10^{8}\times\frac{10}{36\pi}\times10^{-9}} = 3.6\times10^{-7} \ll 1$$

Low loss dielectric. Good insulator.

b. (5) Find the propagation constants in each material $(\alpha_1 + j\beta_1$ and $\alpha_2 + j\beta_2)$.

In free space:

$$\alpha_1 = 0,\ \beta_1 = \frac{\omega}{c} = \frac{2\pi\times10^{8}}{3\times10^{8}} = 2.0944$$

In the dielectric:

$$\alpha_2 = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{20\times10^{-9}}{2}\frac{377}{\sqrt{10}} \approx 1.1922\times10^{-6}$$

$$\beta_2 = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r} = 6.6231$$

c. (5) Find the wave impedance in each region η_1, η_2.

In free space:

$$\eta_1 = 120\pi$$

In the dielectric:

$$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}}\left[1 + j\cancel{\frac{\sigma}{2\omega\varepsilon}}\right] \approx \frac{377}{\sqrt{10}} = 119.2151$$

d. (5) Find the transmission coefficient, τ.

$$\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2/\sqrt{10}}{1 + 1/\sqrt{10}} = 0.4805$$

e. (5) Write an expression for the electric field in the lossy dielectric.

$$E = \tau 200 e^{-\alpha_2 z}\cos\left(2\pi\times10^{8}t - \beta_2 z\right)\hat{a}_x$$

$$E = 96.1012 e^{-1.2\times10^{-6}z}\cos\left(2\pi\times10^{8}t - 6.6z\right)\hat{a}_x$$

4. (25)
   a. (20) A long wire is carrying 10A (peak) at 1000Hz. A rectangular coil is near the long wire as indicated in the figure. The wire has $N = 100$ turns. The width of the wire coil is 0.2m and the height is 0.4m. The distance to the wire is 0.1m. Find the induced voltage in the coil.

$$I = \cos(\omega t) = \cos(2\pi \times 1000t)$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$d\Psi = \frac{\mu_0 I}{2\pi r}(0.4)dr$$

$$\Psi = \int_{0.1}^{0.3} d\Psi$$

$$\Psi = \frac{0.4\mu_0 I}{2\pi}\ln(3)$$

$$\lambda = N\Psi = (100)\frac{0.4\mu_0 I}{2\pi}\ln(3)$$

$$E = -\frac{d\lambda}{dt} = -(100)\frac{0.4\mu_0}{2\pi}\ln(3)\omega\sin\omega t$$

$$E = 4\mu_0 \times 10^5 \ln(3)\sin\omega t \approx 0.5522\sin\omega t$$

   b. (5) The displacement current in a material is related to the conduction current in which of the following ways (circle one)
      - The two currents are in phase.
      - Displacement current leads conduction current by 90 degrees.
      - Displacement current lags conduction current by 90 degrees.
      - The currents are 180 degrees out of phase.