1. (25) Determine the polarization of the following uniform plane wave fields. The electric field is of the form

\[ E(z,t) = A\cos(\omega t - \beta z)\hat{a}_x + B\sin(\omega t - \beta z + \phi)\hat{a}_y. \]

a. (7) \( A = 4 \, \text{V/m}, \quad B = 3 \, \text{V/m}, \quad \phi = 0^\circ. \)

\[ E = 4\cos(\omega t - \beta z)\hat{a}_x + 3\sin(\omega t - \beta z)\hat{a}_y \]

Right elliptical.

b. (7) \( A = 3, \quad B = 3 \, \text{V/m}, \quad \phi = 45^\circ. \)

\[ E = 3\cos(\omega t - \beta z)\hat{a}_x + 3\sin(\omega t - \beta z + 45^\circ)\hat{a}_y \]

Right elliptical.

c. (7) Write the electric field in air, \( E = -j30\hat{a}_x + 10\hat{a}_y \) at 100MHz, in time domain form.

\[ E = 30\cos(2\pi \times 10^8 t - \frac{\pi}{2})\hat{a}_x + 10\cos(2\pi \times 10^8 t)\hat{a}_y \]

\[ E = 30\sin(2\pi \times 10^8 t)\hat{a}_x + 10\cos(2\pi \times 10^8 t)\hat{a}_y \]

d. (4) Write the electric field \( E = 100\cos(\omega t - \beta x)\hat{a}_z \) in phasor form.

\[ E = 100e^{-j\beta x}\hat{a}_z \]
2. (25) A wave in free space has a magnitude of 100\( \gamma_m \) and is normally incident of a perfect dielectric of \( \varepsilon_r = 25 \).

Find the following:

a. (10) The reflection and transmission coefficients.

\[ \eta_0 = 377, \quad \eta_i = \frac{377}{5} \]

\[ \Gamma = \frac{\eta_i - \eta_0}{\eta_i + \eta_0} = \frac{\frac{1}{5} - 1}{\frac{1}{5} + 1} = -\frac{2}{3} \]

\[ \tau = \frac{2\eta_i}{\eta_2 + \eta_1} = \frac{\frac{2}{5}}{\frac{6}{5}} = \frac{1}{3} = \Gamma + 1 \]

b. (5) The standing wave ratio in the free space region.

\[ \text{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \frac{2}{3}}{1 - \frac{2}{3}} = 5 \]

c. (10) The power densities (Poynting vector) of the incident, reflected and transmitted wave.

\[ S_{\text{avg}} = \frac{1}{2} \frac{E^2}{\eta} \]

incident: \( \frac{1}{2} \left( \frac{100\gamma_m}{377} \right)^2 = 13.263 \)

reflected: \( \frac{1}{2} \left( \frac{\frac{2}{3} \times 100\gamma_m}{377} \right)^2 = 5.895 \)

transmitted: \( \frac{1}{2} \left( \frac{\frac{1}{3} \times 100\gamma_m}{377} \right)^2 \times 5 = 7.368 \)
3. (25) A plane wave in free space with magnitude $200 \frac{V}{m}$ at 100MHz is traveling in the $z$ direction and is normally incident on a material which has

$\mu_r = 1, \ \epsilon_r = 10, \ \sigma = 20 \times 10^{-9} \frac{A}{m}.$

a. (5) Is the material a good conductor, good insulator or neither? Explain.

$$\frac{\sigma}{\omega \epsilon} = \frac{20 \times 10^{-9}}{2\pi \times 10^8 \times \frac{10}{36\pi} \times 10^{-9}} = 3.6 \times 10^{-7} \ll 1$$

Low loss dielectric. Good insulator.

b. (5) Find the propagation constants in each material ($\alpha_i + j\beta_i$ and $\alpha_2 + j\beta_2$).

In free space:

$$\alpha_1 = 0, \ \beta_1 = \frac{\omega}{c} = \frac{2\pi \times 10^8}{3 \times 10^8} = 2.0944$$

In the dielectric:

$$\alpha_2 = \frac{\sigma}{2\sqrt{\epsilon}} = \frac{20 \times 10^{-9}}{2} \frac{377}{\sqrt{10}} \approx 1.1922 \times 10^{-6}$$

$$\beta_2 = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r} = 6.6231$$

c. (5) Find the wave impedance in each region $\eta_1, \eta_2$.

In free space:

$$\eta_1 = 120\pi$$

In the dielectric:

$$\eta_2 = \sqrt{\frac{\mu}{\epsilon}} \left[ 1 + j\frac{\sigma}{\omega \epsilon} \right] \approx \frac{377}{\sqrt{10}} = 119.2151$$

d. (5) Find the transmission coefficient, $\tau$.

$$\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2}{\frac{1}{\sqrt{10}} + 1} = 0.4805$$

e. (5) Write an expression for the electric field in the lossy dielectric.

$$E = \tau 200 e^{-\alpha_2 z} \cos(2\pi \times 10^8 t - \beta_2 z) \hat{a}_x$$

$$E = 96.1012 e^{-1.2 \times 10^{-6} z} \cos(2\pi \times 10^8 t - 6.6z) \hat{a}_x$$
4. (25)
a. (20) A long wire is carrying 10A (peak) at 1000Hz. A rectangular coil is near the long wire as indicated in the figure. The wire has \( N = 100 \) turns. The width of the wire coil is 0.2m and the height is 0.4m. The distance to the wire is 0.1m. Find the induced voltage in the coil.

\[
I = \cos(\omega t) = \cos(2\pi \times 1000t)
\]

\[
B = \frac{\mu_0 I}{2\pi r}
\]

\[
d\Psi = \frac{\mu_0 I}{2\pi r} (0.4)dr
\]

\[
\Psi = \int_{0.1}^{0.3} d\Psi
\]

\[
\Psi = \frac{0.4 \mu_0 I}{2\pi} \ln(3)
\]

\[
\lambda = N\Psi = (100) \frac{0.4 \mu_0 I}{2\pi} \ln(3)
\]

\[
E = -\frac{d\lambda}{dt} = -(100) \frac{0.4 \mu_0}{2\pi} \ln(3) \omega \sin \omega t
\]

\[
E = 4 \mu_0 \times 10^3 \ln(3) \sin \omega t \approx 0.5522 \sin \omega t
\]

b. (5) The displacement current in a material is related to the conduction current in which of the following ways (circle one)

- The two currents are in phase.
- **Displacement current leads conduction current by 90 degrees.**
- Displacement current lags conduction current by 90 degrees.
- The currents are 180 degrees out of phase.