Quiz 3
23 April 2007

MULTIPLE CHOICE QUESTIONS
Remember, there can be more than one answer to any of the short questions.

1. Force (8 points)

An actuator is constructed by placing a wire through a solenoid as shown below. This wire carries a fixed current I_o. The solenoid is driven by a capacitor which has been charged up to some voltage V_o. Since the energy available to provide current to the solenoid comes entirely from the capacitor, the current in the solenoid and the resulting magnetic field in the solenoid will depend on the inductance of the solenoid.

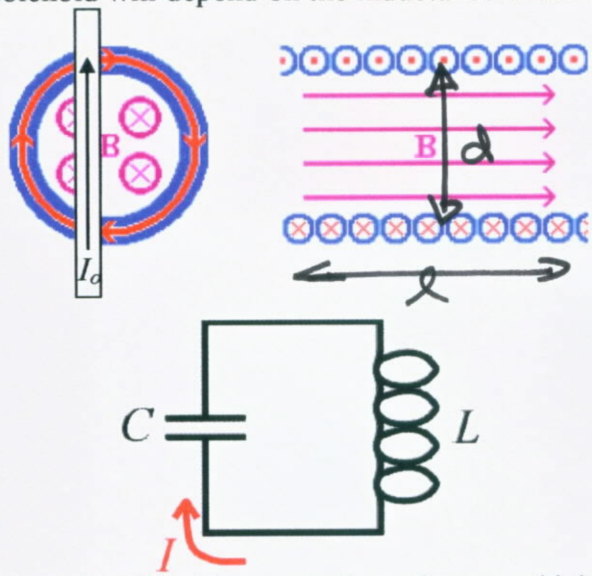
$2a = d$

$B = \frac{\mu_o N I}{l}$

Since $N^2 I^2$ is always constant

B is const

∴ # of turns does not matter



$F = I_o d B$

$\frac{1}{2} C V^2 = \frac{1}{2} L I^2$

For a solenoid

$L = \frac{\mu_o N^2 \pi a^2}{l}$

$N^2 I^2 = $ const no matter what N is

~~If we are free~~ to wind the solenoid with any number of turns, which of the following will produce the maximum force on the wire? Explain your answer.

a. *N = 100* turns
b. *N = 500* turns
~~c. *N = 1000* turns~~
d. *All three choices*
e. *None of the choices*

2. Shielding (8 points)

In order to prevent the electric and magnetic fields from entering or leaving a room, the walls of the room are shielded with 1-mm thick aluminum foil. For which of the following frequencies (if any) will the room be reasonably well shielded?

a. 1 Hz
b. 1 kHz
c. 1 MHz
d. 1 GHz
e. No difference

$\sigma = 3.5 \times 10^7$
$\mu = \mu_o$

Skin depth $= \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{.085}{\sqrt{f}} < .001$

$\sqrt{f} > 85$

$f > 7225$

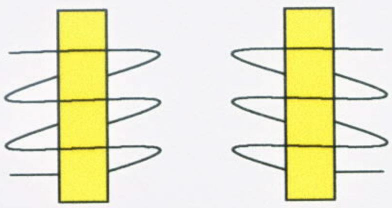
Quiz 3
23 April 2007

3. Mutual Inductance (8 Points)
Assume that we have two identical circular coils, each wound with *N* turns. Which configuration will have the maximum and which will have the minimum mutual inductance? For each case, the coils are wrapped around identical magnetic materials. The distance between the coils is larger for *b* and *c* than for *a* and *d*.

This is like the question from last year except the coils look different.

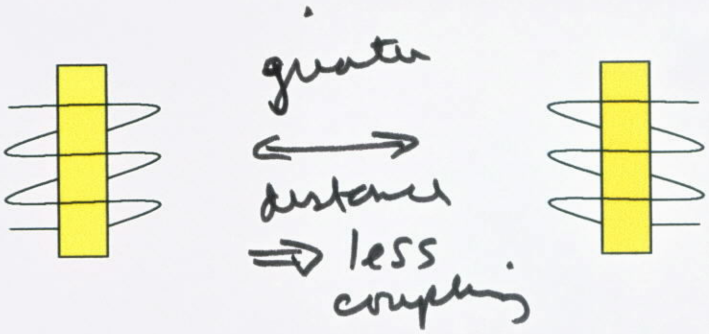
2nd best

a



3rd best

b



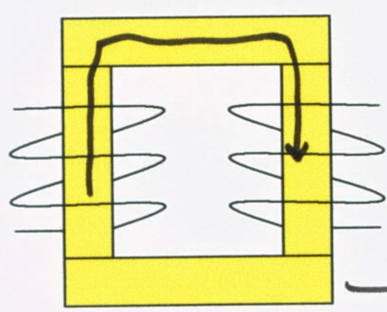
worst

c

Not oriented for the best coupling

Best

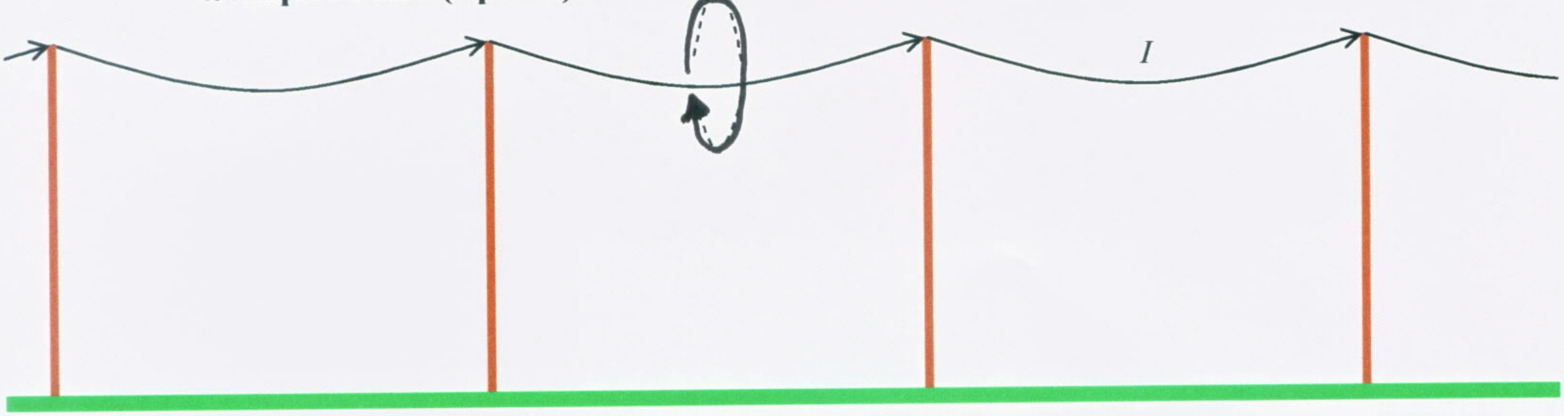
d



— ideally all flux links

a. Figure a will have the maximum and figure d will have the minimum
b. Figure c will have the maximum and figure b will have the minimum
c. Figure d will have the maximum and figure c will have the minimum
d. Figure b will have the maximum and figure a will have the minimum
e. Cannot tell

4. Ampere's Law (8 points)



For an early telegraph line, the current in the line I is carried to the right in the wire and returns through the ground. If one applies Ampere's Law to the dashed loop surrounding the telegraph wire, which of the following describes the integral of the magnetic field intensity $\bar{B}$ around the loop?

a) $\mu_o I$
b) $-\mu_o I$
c) greater than $\mu_o I$
d) less than $-\mu_o I$
e) less than $\mu_o I$ and positive
f) greater than $-\mu_o I$ and negative
g) zero

$\oint \bar{B} \cdot d\bar{l} = \mu_0 I_{encl}$

$I_{encl} = I$ for loop around wire

Quiz 3
23 April 2007

5. Applications of Fields and Waves I (8 points)

Lower frequency RFID coil circuits must be designed to be resonant to be effective. Which of the following is a correct description of the how this resonant circuit is constructed? *You might want to sketch the circuit.*

a) a resistor is placed in series with the coil. The value of the resistor is chosen to be identical to the inductive impedance of the coil at the resonant frequency. The resonant frequency of this circuit is thus determined by its total resistance and inductance.

b) an inductor is placed in parallel with the coil. The value of the inductor is chosen to be the inverse of the inductance of the coil. The resonant frequency of the circuit is determined by value of the inductance.

c) a capacitor is placed in parallel with the coil. The value of the capacitor is chosen so that the combination of the capacitance and inductance determines the resonant frequency.

No!

d) the coil is mounted on a plate that vibrates at the desired resonant acoustic frequency. The frequency is determined entirely by the mechanical properties of the plate.

This is just a ckt conductor.

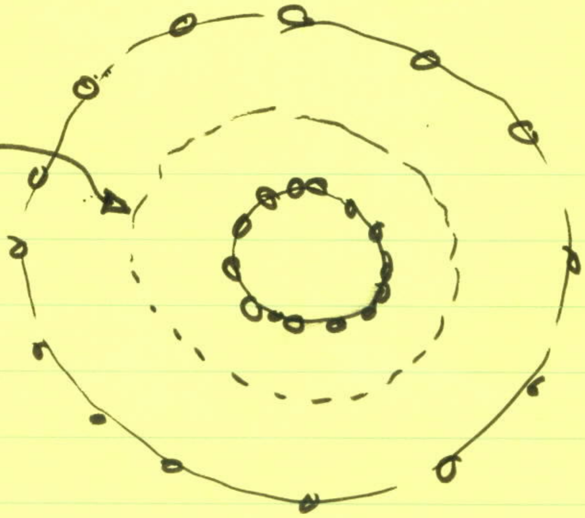
e) the rate at which the transmitting and receiving circuits are programmed to send and receive signal pulses determines the so called resonant frequency of the system

f) all of the above

g) none of the above

A simple resonant circuit requires both C & L

6.a. $\oint \vec{H} \cdot d\bar{l} = I_{encl}$



$H_\phi 2\pi r = NI$

$H_\phi = \frac{NI}{2\pi r}$

In numbers $= \frac{(40{,}000)(1000)}{2\pi r} = \frac{4\times 10^7}{2\pi r}$

$B_\phi = \mu_0 H_\phi = \frac{(4\pi \times 10^{-4})(4\times 10^{+7})}{2\pi r} = \frac{8}{r}$

b. $\phi_m = \int \bar{B}\cdot d\bar{S}$

$= \int_0^{12} dz \int_4^{12} B\, dr = 12 \int_4^{12} \frac{8}{r} dr$

$= 12 \cdot 8 \ln \frac{12}{4} = 96 \ln 3$

$= 105.5$

Total flux linked $= N\phi_m$

$= (40000)(96 \ln 3)$

$= 4.2 \times 10^6$ Wb.

c. $W_m = \frac{1}{2} L I^2$

$$= \frac{1}{2} \int B \cdot H \, dv$$

$$= \frac{1}{2} \int_0^{2\pi} d\phi \int_0^{12} dz \int_4^{12} \cancel{r} \, dr \; \frac{8}{r} \frac{2\times10^7}{\pi \cancel{r}}$$

$$= \frac{\cancel{2\pi}}{\cancel{2}} 12 \frac{16\times10^7}{\cancel{\pi}} \ln 3$$

$$= 12 \cdot 16\times10^7 \ln 3 = 2.1\times10^9$$

d. $\Lambda = 4.2\times10^6 = L I$

$$\Rightarrow L = \frac{4.2\times10^6}{10^3} = 4.2\times10^3 \text{ H}$$

Huge!

$$W_m = \frac{1}{2} L I^2 \Rightarrow L = \frac{2 W_m}{I^2}$$

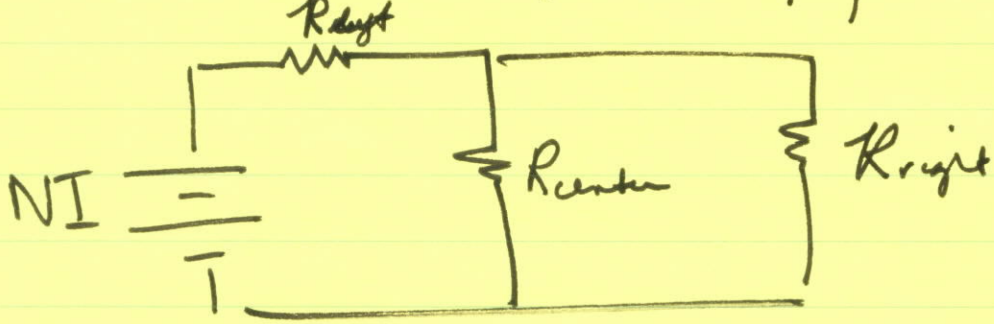
$$= \frac{4.2\times10^9}{10^6} = 4.2\times10^3 \text{ H}$$

Huge!

Checking

$$\frac{1}{2} L I^2 = \frac{1}{2} \, 4.2\times10^3 \; 10^6 = 2.1\times10^9 \checkmark$$

7.a. The simplest approach is to look for the dominant reluctance, usually from the gap.



$$R_{left} = \frac{\frac{d}{2} + \frac{d}{2} + d}{\mu \; W^2} = \frac{2d}{\mu W^2}$$

$$R_{right} = \frac{\frac{d}{2} + \frac{d}{2} + d}{\mu \; W^2} = \frac{2d}{\mu W^2}$$

$$R_{center} = \frac{d-g}{\mu W^2} + \frac{g}{\mu_0 W^2} \simeq \frac{g}{\mu_0 W^2}$$

↑ much smaller due to $\mu >> \mu_0$

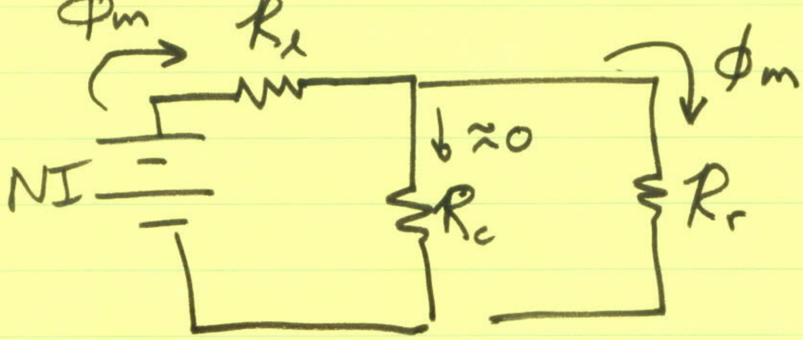
Parallel combo of R_{center} & R_{right} $\approx R_{right}$

since the smaller reluctance dominates

$$\Rightarrow R_{TOTAL} \cong R_{left} + R_{right}$$

$$\simeq \frac{4d}{\mu W^2}$$

$$\Phi_m = \frac{NI}{\mathcal{R}_{TOTAL}} = \frac{NI}{4d/\mu w^2} = \frac{\mu N I w^2}{4d}$$



Flux in left & right legs are both about $\Phi_m = \frac{\mu N I w^2}{4d}$ & zero in center leg.

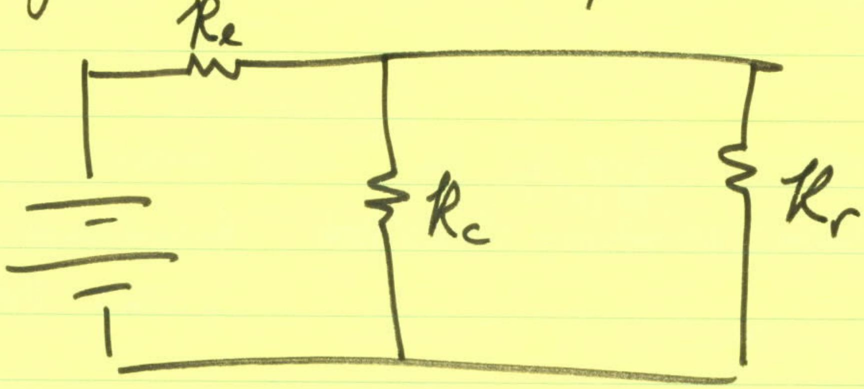
zero is fine as an answer.

There is actually a small amount of flux in the center as determined from a current divider relationship

$$\Rightarrow \Phi_{\text{in center}} \approx \Phi_m \frac{\mu_0}{\mu} \frac{d}{g}$$

which is still very small.

b. Again choose the simplest solution



$$2R_c = R_r = \frac{2d}{\mu w^2} = R_l$$

$$R_{TOTAL} = R_l + R_c \| R_r$$

$$\left(R_c \| R_r = \frac{\frac{d}{\mu w^2} \frac{2d}{\mu w^2}}{\frac{3d}{\mu w^2}} = \frac{2}{3} \frac{d}{\mu w^2} \right)$$

$$= \frac{2d}{\mu w^2} + \frac{2d}{3\mu w^2} = \frac{8d}{3\mu w^2}$$

$$\phi_m = \frac{NI}{8d} 3\mu w^2 = \phi_{m\ left}$$

$$\phi_{m_{center}} = \frac{2}{3}\phi_m = \frac{NI\, 2\mu w^2}{8d}$$

$$\phi_{m\ right} = \frac{NI\, \mu w^2}{8d} = \frac{1}{3}\phi_m$$

c. $B = \frac{\Phi_m}{W^2}$ since area $= W^2$

This question was not specific about where B was measured so we include all 3 parts of the core.

air $\quad \Phi_{left} = \frac{\mu N I W^2}{4d} = \Phi_{right} \qquad \Phi_{center} = 0$

magnetic $\quad \Phi_{left} = \frac{3\mu N I W^2}{8d} \qquad \Phi_{right} = \frac{2\mu N I W^2}{8d}$

$$\Phi_{center} = \frac{N I \mu w^2}{8d}$$

	left	center	right
B for air	$\frac{\mu N I}{4d}$	0	$\frac{\mu N I}{4d}$
B for mag	$\frac{3\mu N I}{8d}$	$\frac{\mu N I}{8d}$	$\frac{2\mu N I}{8d}$

d. $L = \frac{N\Phi_m}{I} = \frac{\mu N w^2}{4d}$ for air

$= \frac{3\mu N w^2}{4d}$ for μ.

Prop in air

8. a. $\omega = 2\pi f = 2\pi\,(9\times10^8) = 18\,\pi\times10^8 =$

$$\beta_o = \frac{\omega}{c} = \frac{18\pi\times10^8}{3\times10^8} = 6\pi$$

$$\lambda = \frac{2\pi}{\beta_o} = \frac{2\pi}{6\pi} = \frac{1}{3}$$

b. $1000\ \frac{W}{m^2} = \frac{1}{2}\frac{E_o^2}{\eta_o}$ $\qquad \eta_o = 120\pi$

Magnitude of E: $E_o = \sqrt{2\eta_o 1000}$

$= 868$

$$\vec{E}_i = \hat{x}\,E_o\,e^{-j\beta_o z}$$

$$\vec{H}_i = \hat{y}\,\frac{E_o}{\eta_o}\,e^{-j\beta_o z}$$

} Incident Only

c. From d. P_{av} Reflect $= 150$

$$= \frac{\Gamma^2 E_o^2}{2\eta_o}$$

$\Gamma^2 =$ [redacted] 0.15

$\Gamma = \pm .3875$ Not sure of sign yet

air | diel

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$\eta_2 < \eta_1$ since η_2 is diel

$\Rightarrow \Gamma$ is negative.

$$\Rightarrow \Gamma = -.3875 = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}}$$

↑ diel

$$(1 + \sqrt{\varepsilon_r})(-.3875) = 1 - \sqrt{\varepsilon_r}$$

$\varepsilon_r = 5.13$ (answers near this are OK)

d. Reflected E & H

$$\vec{E}_r = \hat{x}\,\Gamma E_o\, e^{+j\beta_o z}$$

$$\vec{H}_r = -\hat{y}\,\frac{\Gamma E_o}{\eta_o}\, e^{+j\beta_o z}$$

Transmitted

$$\vec{E}_t = \hat{x}\,\underline{\tau E_o}\, e^{-j\beta z}$$

$$\vec{H}_t = \hat{y}\,\frac{\tau E_o}{\eta}\, e^{-j\beta z}$$

$$\beta = \beta_o\sqrt{\varepsilon_r}$$
$$= \beta_o\, 2.26$$
$$= 13.6\pi$$

$$\eta = \frac{\eta_o}{\sqrt{\varepsilon_r}} = 53\pi$$

Note: For part c, assuming that $\Gamma > 0$ will give a reflection coefficient with the right magnitude but $\Rightarrow \varepsilon_r = .545$ which is not realistic since it is less than 1.