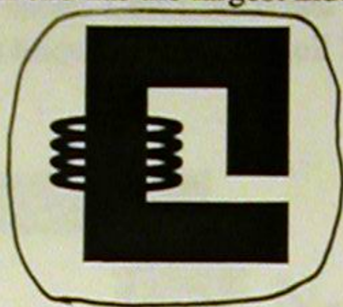
Quiz # 3
Fields and Waves I
Fall 2004

Answer all questions. Where possible, show work for partial credit.

Name Solution

(1) Circle the most appropriate answer (25 pts.)

a) The 2 magnetic circuits have the same number of turns and the same cross sectional area.. Which one has the largest inductance?



b) I was having a conversation with a colleague in humanities who was complaining that the engineering students did not have much culture. I disagree and to prove to him that this is not the case here are 2 questions on classical music.

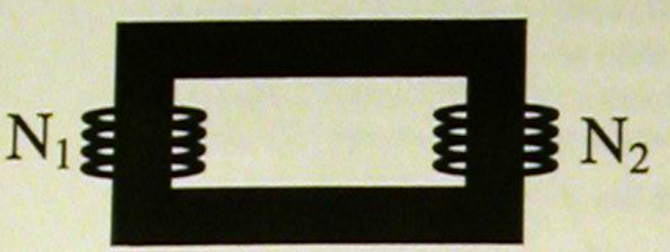
1. **Tchaikovsky**_wrote 5 symphonies. One was the second symphony. Name the other 4.
2. Rearrange the following words to make an opera by Puccini: **Boheme, La**

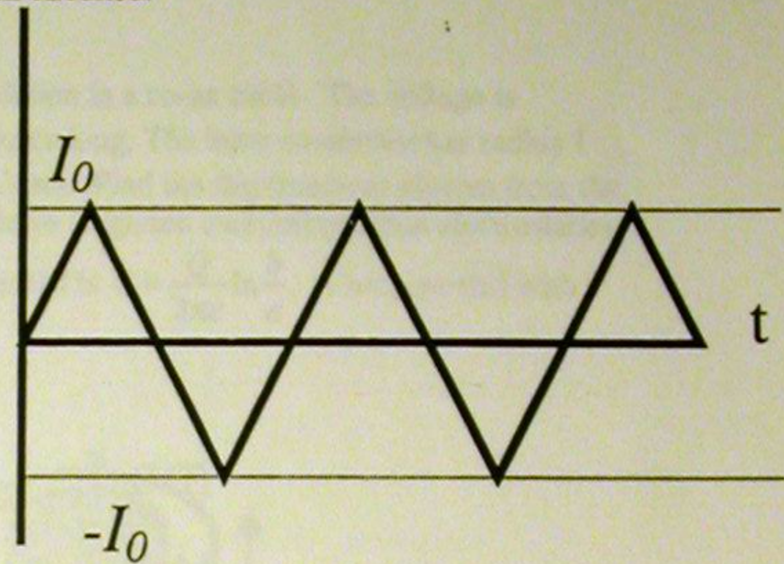
c) In project #2 you had to scrape off some – but not all of the insulation of the coil so that there was conduction over part – but not all of the revolution. Explain why this was necessary. If insulation was completely removed the torque would change direction and the average torque would be zero.

d) One of these conductors is for dc, one is for 60 Hz and one for 5,000 Hz. Indicate which is for which frequency.

60 Hz   5KHz   dc

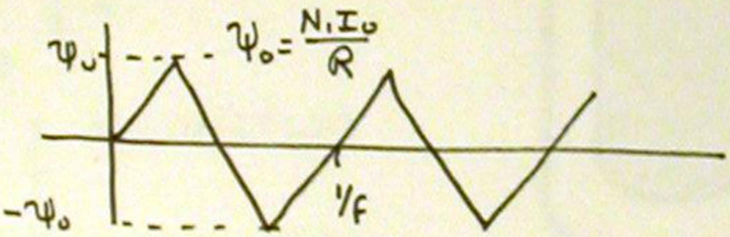
(2) (25 pts) A magnetic circuit is shown below and has length and area of ℓ and A respectively. The permeability is μ. There are two windings on the circuit with N_1 and N_2 turns. Coil 1 is excited with a current (shown below) that has a triangular waveform at frequency f while coil 2 is open circuited.



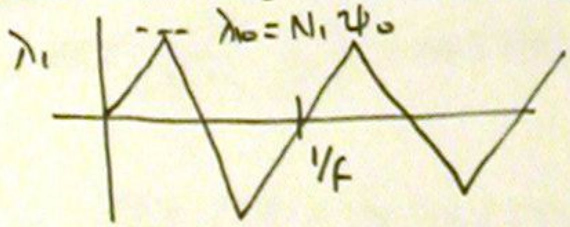


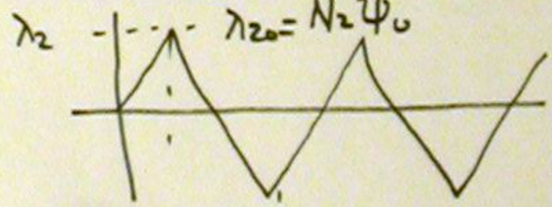
a) Find the flux in the core as a function of time. Sketch it.

$$\mathcal{R} = \frac{\ell}{\mu A} \qquad \psi_m = \frac{N_1 i}{\mathcal{R}}$$



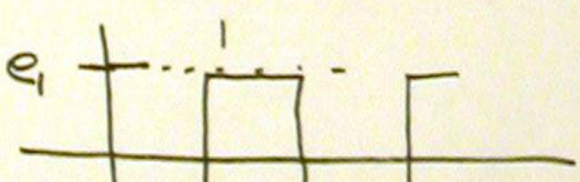
b) Find the flux linkage of each coil as a function of time. Sketch it

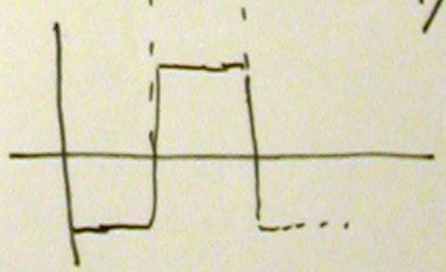




c) Find the voltage on each coil as a function of time. Sketch it.

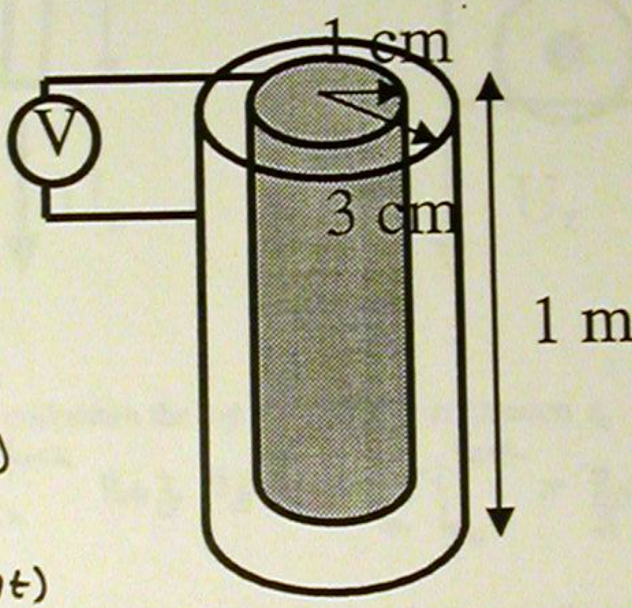
$$\text{Slope} = \frac{\lambda_{10}}{1/4f} = 4f\lambda_{10} = e_1$$





3. (25) The displacement and conduction currents in an ac field are 90 degrees out of phase. True or False? True

A material has $\varepsilon_r = 9$ and is used as the insulation in a co-ax cable. The voltage is $V(t) = 120\sin(377t)$ volts. The cable is 1 meter long. The inner conductor has radius 1 cm. and the radius of the outer conductor is 3 cm. Find the displacement current from the electric flux density. For those of you who have forgotten everything about electrostatics the electric field is $E = \frac{Q}{2\pi\varepsilon r}\hat{a}_r$ and the potential is $V = \frac{Q}{2\pi\varepsilon}\ln\frac{b}{a}$. Compare this with the capacitive current by $i = C\frac{dV}{dt}$



$$C = \frac{2\pi\epsilon}{\ln b/a}$$

$$E = \frac{2\pi\epsilon V}{\ln b/a\,(2\pi\epsilon r)}\,\hat{a}_r$$

$$= \frac{V}{\ln b/a}\,\frac{1}{r}\,\hat{a}_r$$

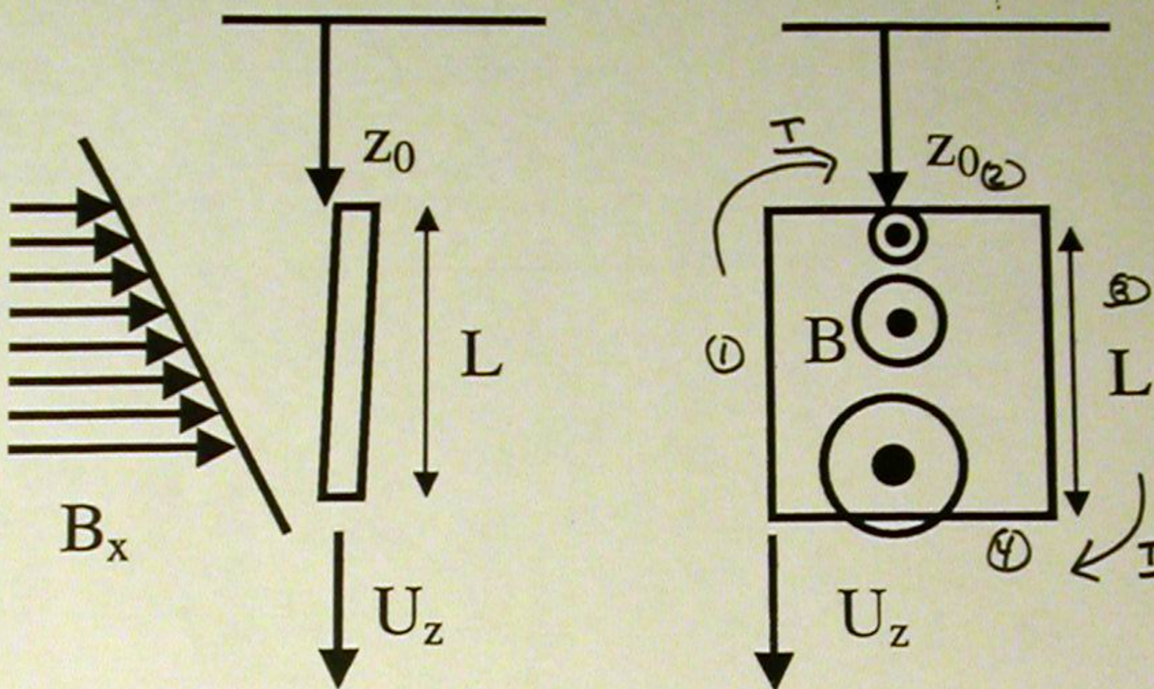
$$D = \frac{\epsilon}{\ln b/a}\,\frac{1}{r}\,120\sin(377t)$$

$$\frac{dD}{dt} = \frac{377\,\epsilon}{\ln b/a}\,\frac{1}{r}\,120\cos(377t)$$

$$I_d = \frac{\partial D}{\partial t}A = 2\pi\cancel{r}\left[\frac{377\epsilon}{\ln b/a}\,\frac{1}{\cancel{r}}\right]120\cos(377t)$$

$$I_c = C\frac{dV}{dt} = \frac{2\pi\epsilon}{\ln b/a}\left[377 * 120\cos(377t)\right.$$ same.

4. (25) A rectangular loop at velocity U_z is falling through a magnetic field that is increasing linearly with z so that $B = B_0 z\hat{a}_x$. The coil is one meter deep (into the page) and L in the z direction. The area vector is in the x direction so all of the flux is passing through the coil.



a) Find the flux linking the coil when the top conductor is at location z_0.

$$\Psi = \int B\cdot ds = \int_{z_0}^{z_0+L} B_0 z\, dz = B_0 \frac{z^2}{2}\Big|_{z_0}^{z_0+L} = \frac{B_0}{2}\left[\cancel{z_0^2} + 2z_0L + L^2 - \cancel{z_0^2}\right]$$

b) Find the UxB voltage on the four sides of the loop. Add these to find the total voltage.

no UxB on sides (1) + (3).

$$U\times B_{(2)} = U_z(B_0 z_0), \quad U\times B_{(4)} = U_z(B_0(z_0+L))$$

$$emf = U_z(B_0(z_0+L) - B_0 z_0) = U_z B_0 L$$

c) Find the voltage by using $EMF = -\frac{d\psi_m}{dt}$ and compare the results to part b

$$\Psi = \frac{B_0}{2}\left[2zL + L^2\right], \quad \frac{d\Psi}{dt} = \frac{B_0}{2}\left[2LU_z\right] = B_0 L U_z$$

d) If current were allowed to flow, show the direction of the force on the loop. Would the electromagnetic force be in the direction of the velocity or opposed to it? Explain.

force on (4) is up, force on (2) is down but force on (4) is greater. Force opposes motion