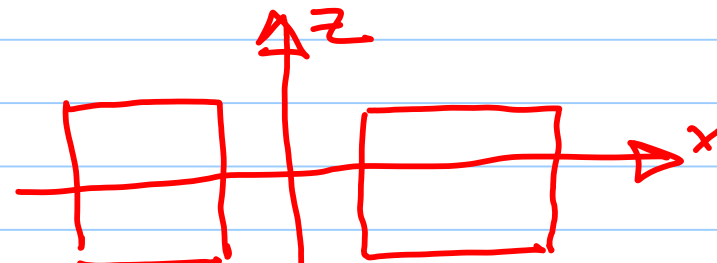
Review

Note Title 5/2/2006

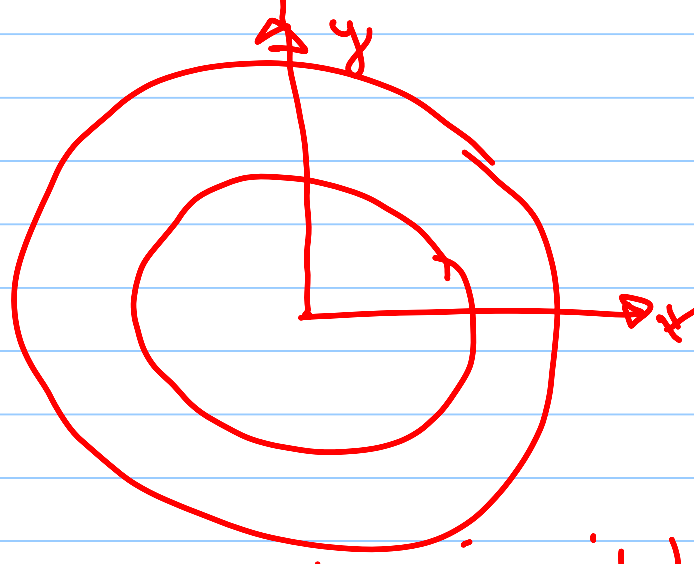
Magnetic Fields

Example – Rectangular cross section torus

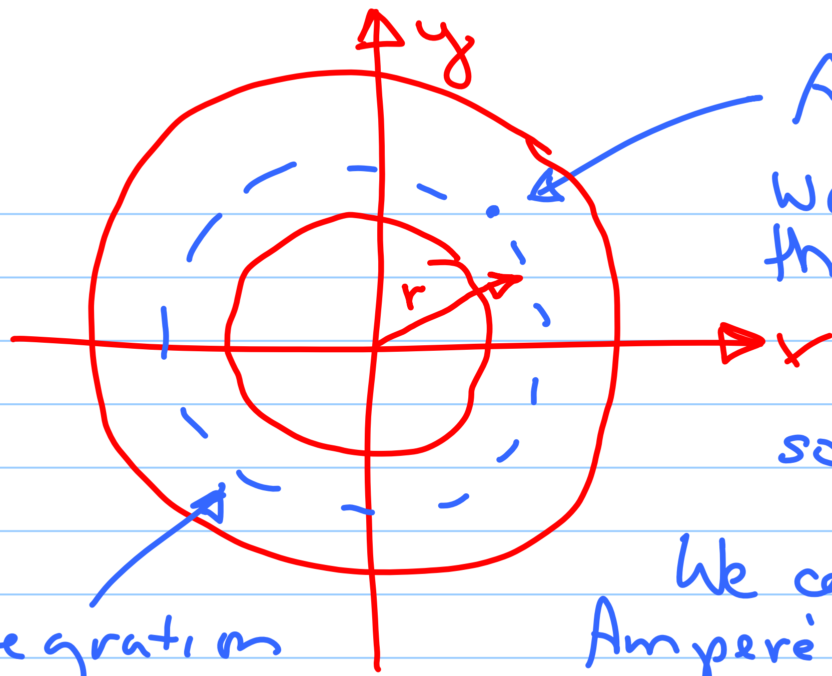
Side View



Top View



With this example we can review inductance, B fields, Ampere's law, boundary conditions (even magnetic circuits)



Assume we want to know the field at some point.

Integration path

We can use Ampere's Law since

$$\vec{B} = \hat{\phi} B_\phi(r)$$

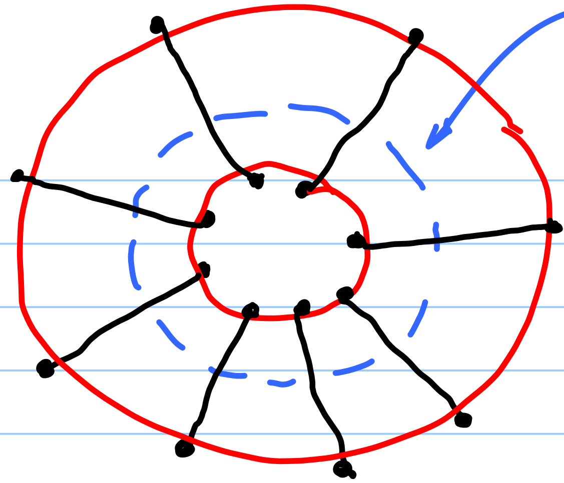
$$\& \ \vec{H} = \hat{\phi} H_\phi(r)$$

The field depends on only one variable. This is critical for Ampere's Law. We can apply

$$\oint \vec{H} \cdot \vec{dl} = I_{encl}$$

on the path shown. (- - - -)

On this path H & B are constants $\Rightarrow \oint \bar{H} \cdot \bar{dl} = H_\phi(r) 2\pi r$



for this path, we enclose all N windings so that

$I_{enc} = NI$

N Turns wrapped around the core.

Thus $\oint \overline{H} \cdot \overline{dl} = H_\phi(r)\, 2\pi r$

$= N\overline{I}$

or $\vec{H} = \hat{\phi} \dfrac{N\overline{I}}{2\pi r}$

& $\vec{B} = \hat{\phi} \dfrac{\mu N\overline{I}}{2\pi r}$

for a core with $\mu = \mu_r \mu_0$

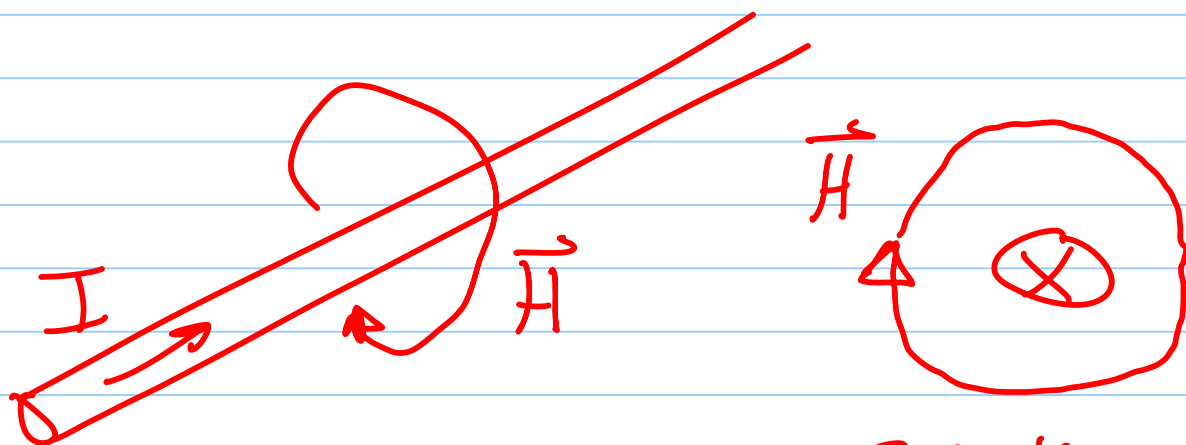
Simplifying $\vec{B}$ & $\vec{H}$: This is one of the most important steps in applying both Ampere's Law & Gauss' Law (for E fields)

Note - In this review we will emphasize the common features found in different of this course. Electromagnetics is not a collection of unrelated topics. What you learn in each part of the course will have some value in other parts of the course.

$\Rightarrow$ To apply Ampere's Law or Gauss' Law, the field must depend on only <u>one variable</u>.

To show that $\vec{H} = \vec{H}(r)$ ← only

go back to the long-straight wire



For current into the page, the right hand rule gives us the direction of $\vec{H}$ shown. Since the long wire is specified only by r, the field must also only dep. on r.
(The current exists in $0 \leq r \leq a$)

$$\Rightarrow \vec{H} = \hat{\phi}\, H_\phi(r)$$

The magnetic field of a long straight wire (see the text) is

$$\vec{H} = \hat{\phi}\, \frac{I}{2\pi r}$$

The evaluation of Ampere's Law is the same

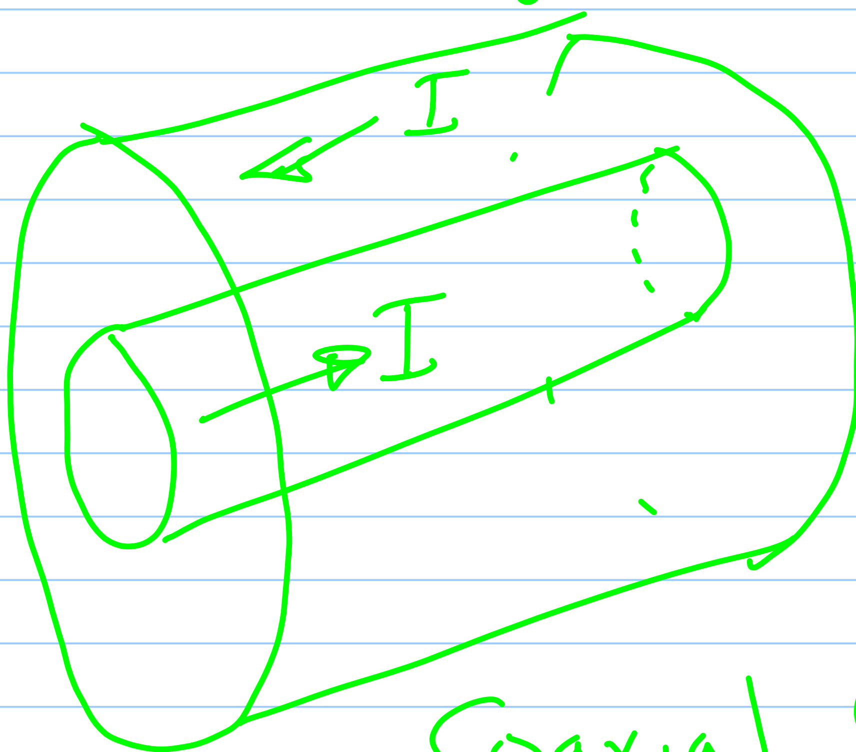
$$\oint \vec{H} \cdot \vec{dl} = H_\phi(r)\, 2\pi r$$

$$= I_{encl} = I$$

outside the wire $\Rightarrow \vec{H} = \hat{\phi}\, \frac{I}{2\pi r}$

Note that, except for the N, this is the same as for the torus. Why is that?

This happens because:
The square or rectangular cross-section torus & a coaxial cable are essentially the same configuration

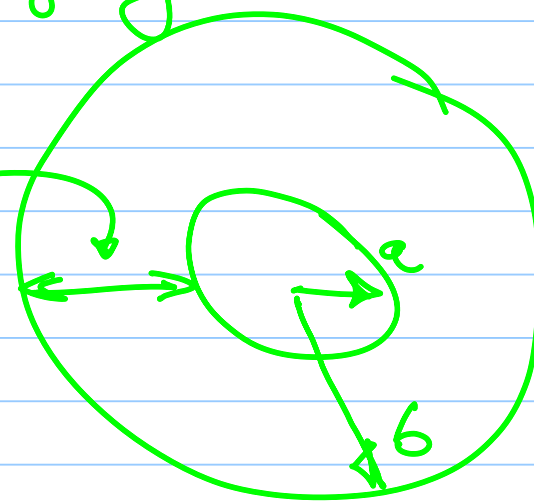


Coaxial Cable

For a coax, the H field in the region between the inner & outer conductors is given by $\vec{H} = \hat{\phi} \dfrac{I}{2\pi r}$

The outer conductor does not change this. Rather it just provides a boundary to define the edge of the field region

H exists here but not outside.



Let's assume that the coax is built 2 very thin foil conductors at $r=a$ & $r=b$

That is, the conductors are thin surfaces at $r=a$ & $r=b$.

We can model these as surface currents

$$\vec{J}_{sa} = \frac{I}{2\pi a}\hat{z}$$

$$\vec{J}_{sb} = -\frac{I}{2\pi b}\hat{z}$$

For the region $r > b$, we have no magnetic field $\vec{H} = 0$

Since $\vec{H}$ is in the ϕ direction, $\vec{H}$ is tangent to $r = b$. Thus the B.C. here is

$$H_{tan_1} - \underset{\substack{\| \\ 0 \text{ outside}}}{H_{tan_2}} = J_s$$

or $\quad H_{tan} = H_\phi = J_s$

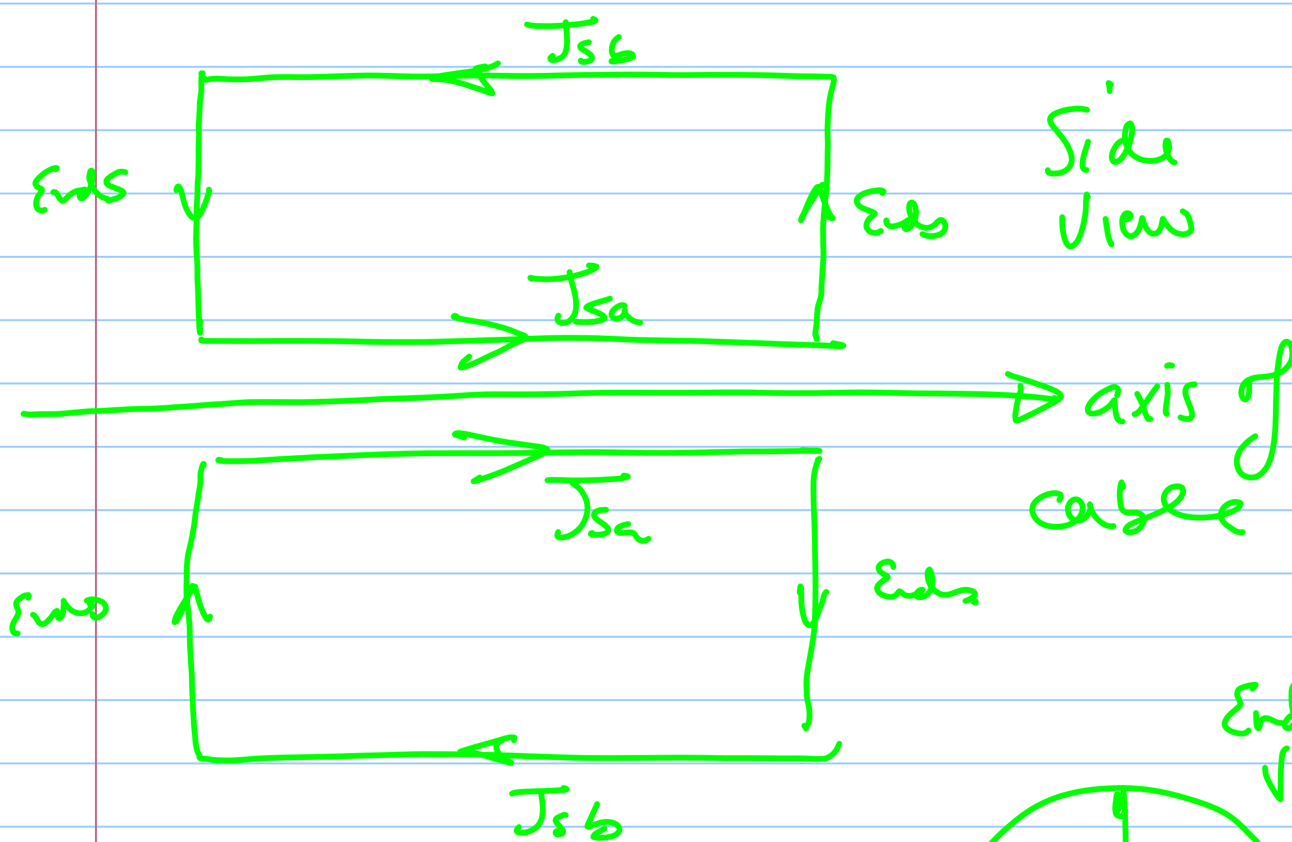
Checking, we see that

$$H_\phi = \frac{I}{2\pi b} = J_{sb}$$

as it must. Thus the outer conductor not only helps create the field, it provides the B.C. so that there is no field for $r > b$. Since we have assumed thin conductors at $r = a$ & b the same thing occurs at $r = a$ & $\bar{H} = 0$ for $0 < r < a$.

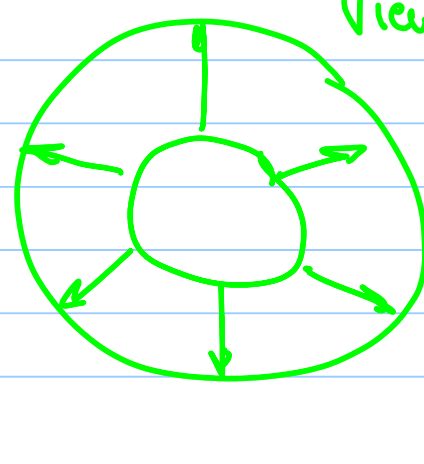
Going one step further, assume that we have thin washer like conductors connecting the inner & outer conductors at each end.

Side View



End View

Note the current flowing radially at the ends.



The current flows radially in the ends, spreading out as r increases. In fact $\vec{J}_s = \hat{r}\,\dfrac{I}{2\pi r}$ at one end &

$$\vec{J}_s = -\hat{r}\,\frac{I}{2\pi r}$$

at the other end. Again this is exactly the B.C. necessary to support a field in the cable & no field beyond the ends. (Try the B.C. to see this.)

Note that a coax with end caps looks just like a turns. The only difference is that there is only 1 turn. We could have made such a torus a toroidal form around which we wrap a layer of conducting foil. This explains why the fields for a coax & a torus look the same except for the # of turns N.

$$\vec{H} = \hat{\phi}\,\frac{NI}{2\pi r} \qquad \vec{H} = \hat{\phi}\,\frac{I}{2\pi r}$$

$$\vec{B} = \hat{\phi}\,\frac{\mu NI}{2\pi r} \qquad \vec{B} = \phi\,\frac{\mu_0 I}{2\pi r}$$

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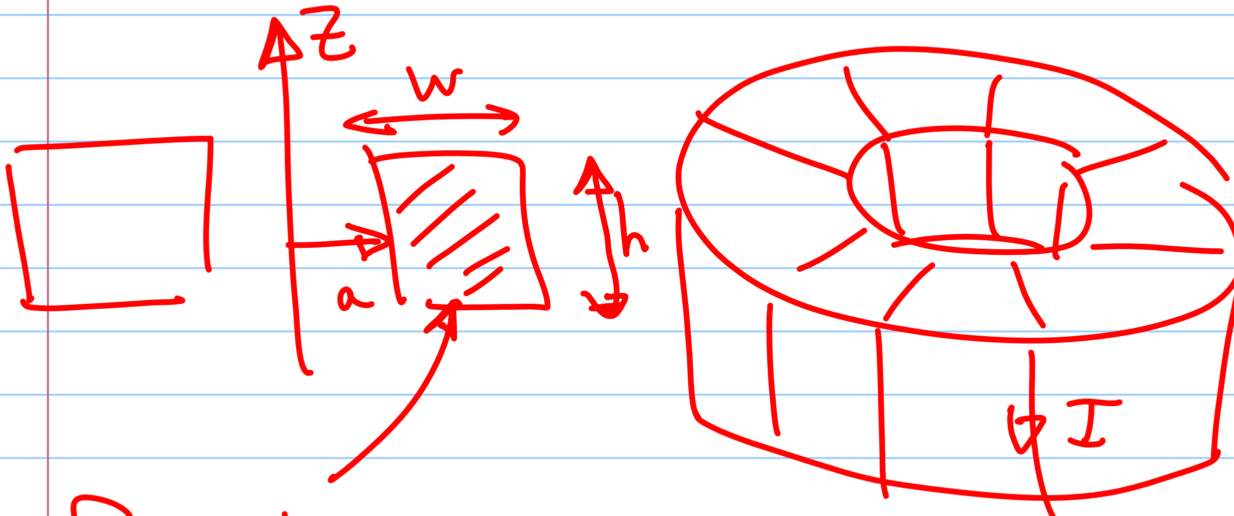
What can we do with $\vec{B}$ once we know it?



Flux through one turn.

N turns.

$$\Psi_m = \int \vec{B}\cdot d\vec{S} = \frac{\mu N I}{2\pi}\int_0^h dz \int_a^{a+w} \frac{1}{r}\, dr$$

$$= \frac{\mu N I}{2\pi}\, h \ln \frac{a+w}{a}$$

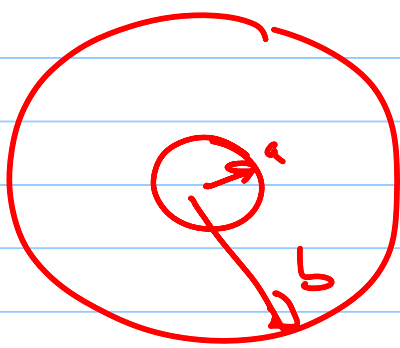
Note: we could also have used $b = a+w$

The total flux linked by the toroidal configuration is

$$\lambda = N\Psi_m = \frac{\mu N^2 I}{2\pi} h \ln\frac{a+w}{a}$$

$$= L I$$

$$\Rightarrow L = \frac{\mu N^2}{2\pi} h \ln\frac{a+w}{a}$$



For a coaxial cable (at the left) the L per unit length is

$$\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{b}{a}$$

(You can look this up)

Again note that this is the same as the torus without the N^2 term.

$$L_{coax} = \frac{\mu_0}{2\pi} \ell \ln\frac{b}{a} \Rightarrow L_{torus} = \frac{N^2\mu}{2\pi} h \ln\frac{b}{a}$$

ℓ becomes h for torus

We can also find the L from

$$W_m = \frac{1}{2} L I^2 = \frac{1}{2}\int \bar{B}\cdot\bar{H}\,dV$$

Integrate over the volume of the field.

$$\frac{1}{2}\int \bar{B}\cdot\bar{H}\,dV = \frac{1}{2}\int_0^{2\pi} d\phi \int_0^h dz \int_a^b \frac{\mu N^2 I^2}{(2\pi)^2}\frac{r}{r^2}\,dr$$

$$= \frac{1}{2}\,2\pi h\,\frac{\mu N^2 I^2}{(2\pi)^2}\ln\frac{b}{a}$$

or $L = \frac{\mu}{2\pi} h N^2 \ln \frac{b}{a}$

as before.

For Inductance & Capacitance there are two general approaches. The most reliable is

$$W_e = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \int \bar{D} \cdot \bar{E} \, dV$$

& $$W_m = \frac{1}{2} L I^2 = \frac{1}{2} \int \bar{B} \cdot \bar{H} \, dV$$

called the energy method. We can also use $Q = CV$ &

$\lambda = LI$

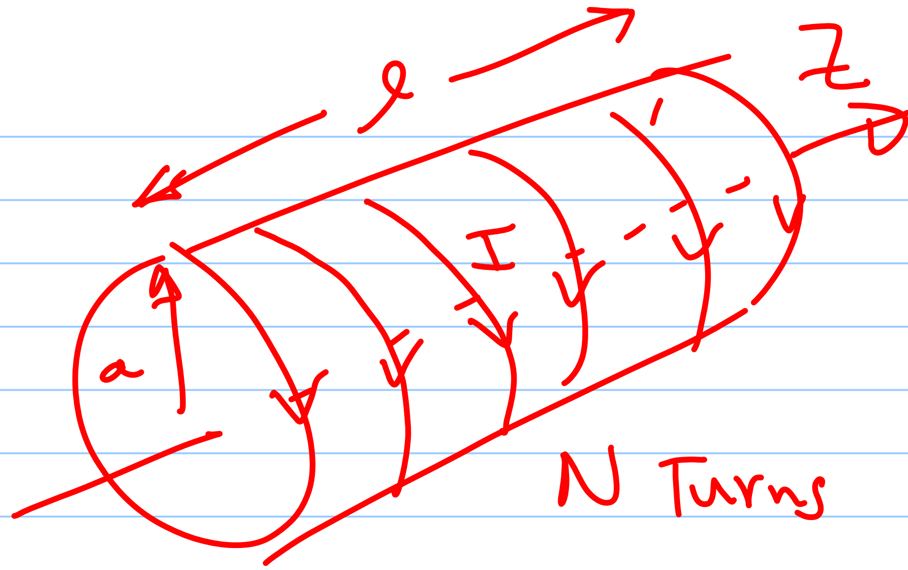
but flux linkage is sometimes hard

to apply inside currents.
We don't have this problem for conductors & $\vec{E}$ fields so either method works well for capacitance.

There are 4 basic inductors that we can analyze with Ampere's Law. Torus, Coax, Solenoid, Parallel Plate (Stripline)

Just as the torus & coax are similar the solenoid & p.p. inductor are essentially the same.

Solenoid



A solenoid is formed by wrapping N turns of wire around some cylinder. For this configuration

$$L = \frac{\mu N^2 \pi a^2}{\ell}$$

where we have assumed that the core permeability is μ. The general form is

$$L = \mu N^2 \frac{\text{Area}}{\text{Length}}$$

Compare with a stripline. From Ulaby the inductance per unit length is

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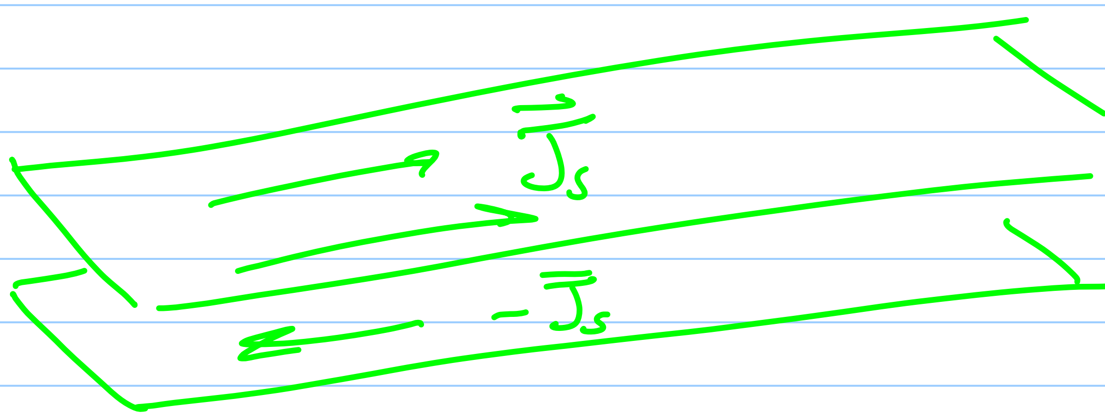
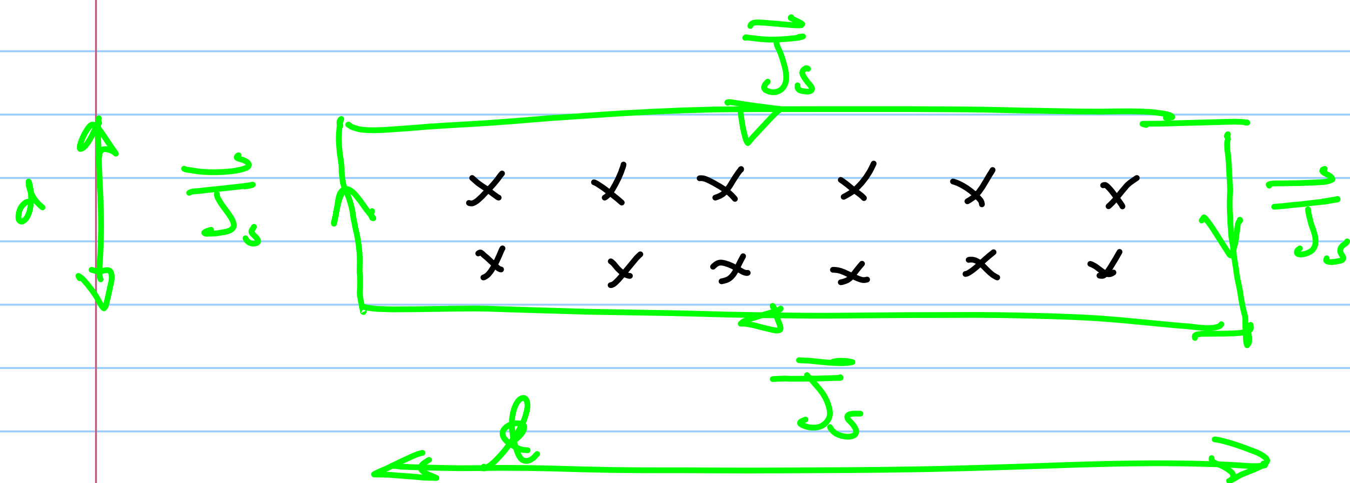
$$L/\ell = \frac{\mu_0 d}{w}$$

So that the inductance of a line of length ℓ is $L = \frac{\mu_0 d \ell}{w}$



A side view



Where we have added end caps again to create a closed structure. The x's show the direction of the $\vec{H}$ field. The area of the field is $d\ell$ & the depth is w. Thus for the entire line

$$L = \mu_0 \frac{d\ell}{w} = \mu_0 \frac{\text{Area}}{\text{Length}}$$

or the same as the solenoid.

One can also look at the B.C. in the same manner as we did with the Torus & Coax.

Back to Ampere's Law

If we calculate $\vec{B}, \vec{H}$ inside a current, then we can check the answer by apply

$$\nabla \times \vec{H} = \vec{J} \quad \text{(Static)}$$

to see if we get back the original $\vec{J}$ (Source)

We can do the same thing for $\vec{D}, E$ by taking

$$\nabla \cdot \vec{D} = \rho$$

Basic Capacitors

Spherical Cap.

Coax

Parallel Plate

From charge dist find $\vec{D}$ using Gauss' Law

$$\oint \bar{D} \cdot d\bar{S} = Q_{encl}$$

From $\vec{D} \longrightarrow \vec{E} = \dfrac{\vec{D}}{\varepsilon}$

From $\vec{E}$

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

usually choose to be $V(a) = 0$

$$\begin{tikzcd} Q \arrow[r] & D \arrow[r] & E \arrow[r] & V \arrow[dl] \\ & & C \arrow[ull] & \end{tikzcd}$$

Obtain Capacitance

Can also use $W_e = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C}$

$$= \frac{1}{2} \int \vec{E} \cdot \vec{D} \, dv$$

We use $\vec{D}$ instead of $\vec{E}$ in application of Gauss' law because, when all aspects of a configuration are given by a single coordinate (e.g. r for cylindrical or spherical or x for parallel plates) then $\vec{D}$ does not see the dielectric boundaries

To go beyond the very simple geometries we can address with Gauss' law, we need a numerical method.

The spread sheet method gives us V everywhere from which we can estimate $\vec{E}$ & $\vec{D}$ which we can use to find ρ_s through the B.C. at conductors & then find the charge on each conductor.

$$\underbrace{V \to \vec{E} \to \vec{D} \to \rho_s \to Q}_{C}$$

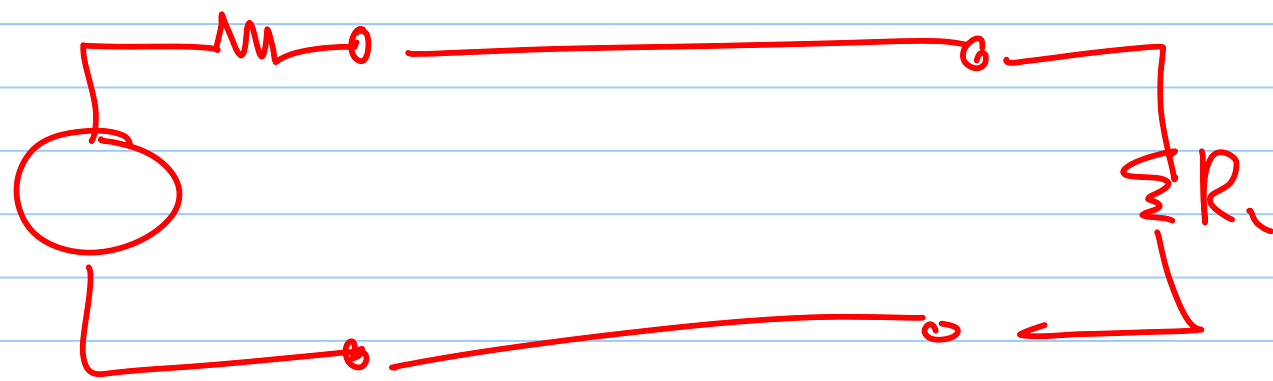
find Capacitance this way

Note - You should know the basics of the general shunt method.

Transmission Lines

We have already discussed the similarities between trans. lines & uniform plane waves normally incident on material boundaries. Be sure to review this material.

Basic Trans Line Config



Source Line Load

For the line, L & C can be given, then $Z_0 = \sqrt{\frac{L}{C}}$ $u = \frac{1}{\sqrt{LC}}$

or Z_0, u can be given

The source can be sinusoidal or pulsed. For both cases the key is Z_{in}.

For pulses, $Z_{in} = Z_0$

for sinusoidal

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta d}{Z_0 + jZ_L \tan\beta d}$$

Then the input voltages are given by voltage divider action at the source. For a lossless line, power in will be delivered to the load.

Be sure you can take a simple trans line and analyze it for both sinusoidal & pulsed input.

The period one inputs will either be shorter than the transit time T or very much longer (as in switching on a D.C. source).

Note — these comments were motivated by looking at HW8 for Spring 2005 & the 8 extra credit questions from Spring 2005