Transmission Lines
Steady State

Phasor Notation

$$V(z,t) = \mathrm{Re}(\hat{V}(z)e^{j\omega t})$$
$$I(z,t) = \mathrm{Re}(\hat{I}(z)e^{j\omega t})$$

Voltage Wave

$$\hat{V}(z) = \hat{V}_m{}^{+}e^{-\gamma z} + \hat{V}_m{}^{-}e^{+\gamma z} = \hat{V}_m{}^{+}e^{-\gamma z}(1+\hat{\Gamma}(z)) \text{ for lossy lines}$$
$$\hat{V}(z) = \hat{V}_m{}^{+}e^{-j\beta z} + \hat{V}_m{}^{-}e^{+j\beta z} = \hat{V}_m{}^{+}e^{-j\beta z}(1+\hat{\Gamma}(z)) \text{ for lossless lines}$$

Current Wave

$$\hat{I}(z) = \frac{\hat{V}_m{}^{+}}{Z_o}e^{-\gamma z} - \frac{\hat{V}_m{}^{-}}{Z_o}e^{+\gamma z} = \frac{\hat{V}_m{}^{+}}{Z_o}e^{-\gamma z}(1-\hat{\Gamma}(z)) \text{ for lossy lines}$$
$$\hat{I}(z) = \frac{\hat{V}_m{}^{+}}{Z_o}e^{-j\beta z} - \frac{\hat{V}_m{}^{-}}{Z_o}e^{+j\beta z} = \frac{\hat{V}_m{}^{+}}{Z_o}e^{-j\beta z}(1-\hat{\Gamma}(z)) \text{ for lossless lines}$$

Wavelength

$$\lambda f = u \text{ where } u \text{ is the propagation velocity; } \beta = \frac{2\pi}{\lambda}$$

Propagation Constant

$$\hat{\gamma} = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC} \text{ for lossless lines}$$

Characteristic Impedance

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} \text{ for lossless or low loss lines}$$

Phase Velocity

$$v_{ph} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} \text{ for lossless or low loss lines}$$

Total Wave Impedance

$$\hat{Z}(z) = \frac{\hat{V}(z)}{\hat{I}(z)} = Z_o\frac{1+\hat{\Gamma}(z)}{1-\hat{\Gamma}(z)}$$

Reflection Coefficient

$$\hat{\Gamma}(z) = \frac{\hat{V}^{-}{}_m}{\hat{V}^{+}{}_m}e^{2\gamma z} = \hat{\Gamma}(z')e^{2\gamma(z-z')} \text{ for lossy lines}$$

Note: The symbols R, L, G and C refer to resistance, inductance, conductance and capacitance ***per unit length.*** *In the textbook, these are written as* R', L', G' *and* C' *while in the class lessons these are written as* ***r, l, g*** *and* ***c****.*

$$\hat{\Gamma}(z) = \frac{\hat{V}^{-}{}_{m}}{\hat{V}^{+}{}_{m}} e^{j2\beta z} = \hat{\Gamma}(z')e^{j2\beta(z-z')} \text{ for lossless lines}$$

$$\text{Reflection from load } \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\text{Reflection from generator } \Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o}$$

Input Impedance of a Transmission Line of Length d with load Z_L

$$Z_{in}(z=0) = Z_o \frac{Z_L + Z_o \tanh\gamma d}{Z_o + Z_L \tanh\gamma d} \text{ for a lossy line}$$

$$Z_{in}(z=0) = Z_o \frac{Z_L + jZ_o \tan\beta d}{Z_o + jZ_L \tan\beta d} \text{ for a lossless line}$$

Z_o is real when the line is lossless

Average Power

$$P_{ave}(z) = \frac{1}{2}\mathrm{Re}(\hat{V}(z)\hat{I}^{*}(z))$$

Standing Wave Ratio

$$SWR = \frac{|V_{\max}|}{|V_{\min}|} = \frac{|I_{\max}|}{|I_{\min}|} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Low Loss Lines (for the usual case where G can be neglected and $R << \omega L$)

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \approx \sqrt{\frac{R+j\omega L}{j\omega C}} \approx \sqrt{\frac{j\omega L}{j\omega C}}\sqrt{1+\frac{R}{j\omega L}} \approx \sqrt{\frac{L}{C}}\left(1 - j\frac{R}{2\omega L}\right)$$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} \approx \sqrt{(R+j\omega L)(j\omega C)} \approx \sqrt{(j\omega L)(j\omega C)}\sqrt{1+\frac{R}{j\omega L}}$$

$$j\beta \approx j\omega\sqrt{LC} \text{ and } \alpha \approx \omega\sqrt{LC}\left(\frac{R}{2\omega L}\right) = \frac{R}{2Z_o}$$

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Transients

Lossless Wave Equation

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2}$$

$$\frac{\partial^2 I}{\partial z^2} = LC\frac{\partial^2 I}{\partial t^2}$$

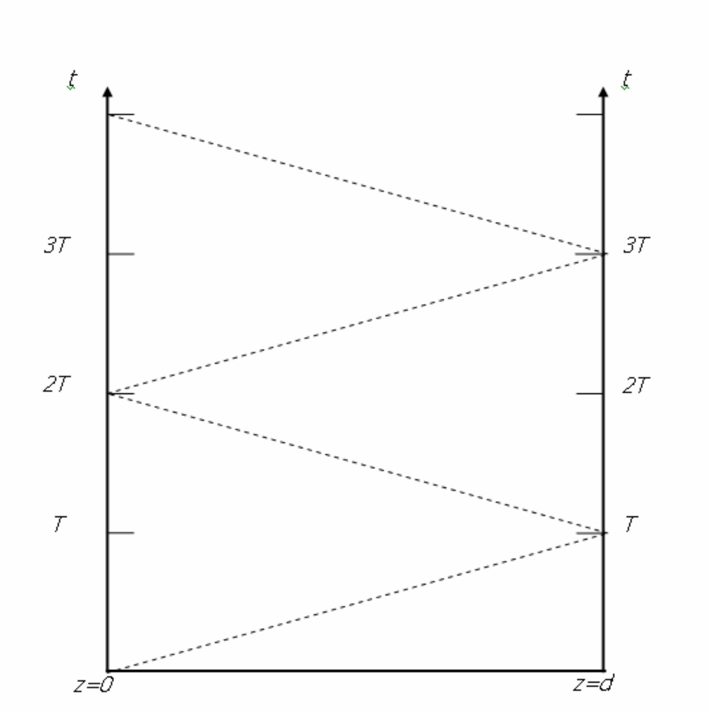
Voltage and Current Waves

$$V(z,t) = V^+\left(t - \frac{z}{v_{ph}}\right) + V^-\left(t + \frac{z}{v_{ph}}\right) \equiv V^+ + V^-$$

$$I(z,t) = \frac{V^+}{Z_o} - \frac{V^-}{Z_o}$$

Z_o is also called the surge impedance

Bounce or Lattice Diagram



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