

**Problem 1 - (5 points)**

Design an air core solenoid with a length  $l_s = 50$  mm, inner radius  $a = 5$  mm. The solenoid must have a magnetic field on axis of at least 0.05 T, a current density in its wires  $J < 10^7$  A/m<sup>2</sup>, and dissipate < 25 Watts of power. Use a standard wire gauge for the windings. Be sure to calculate the outer radius  $b$  you would expect. Keep  $b$  as small as possible.

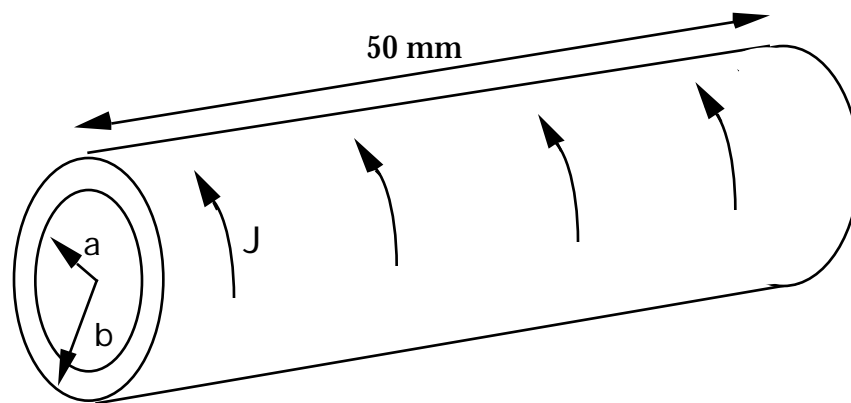
It is OK to ignore any space taken up by insulation in your calculations. You can also assume that the wires pack so that no air spaces are between the wires.

Your design should specify the total number of turns and the wire gauge.

You should calculate numerical values for the following parameters:

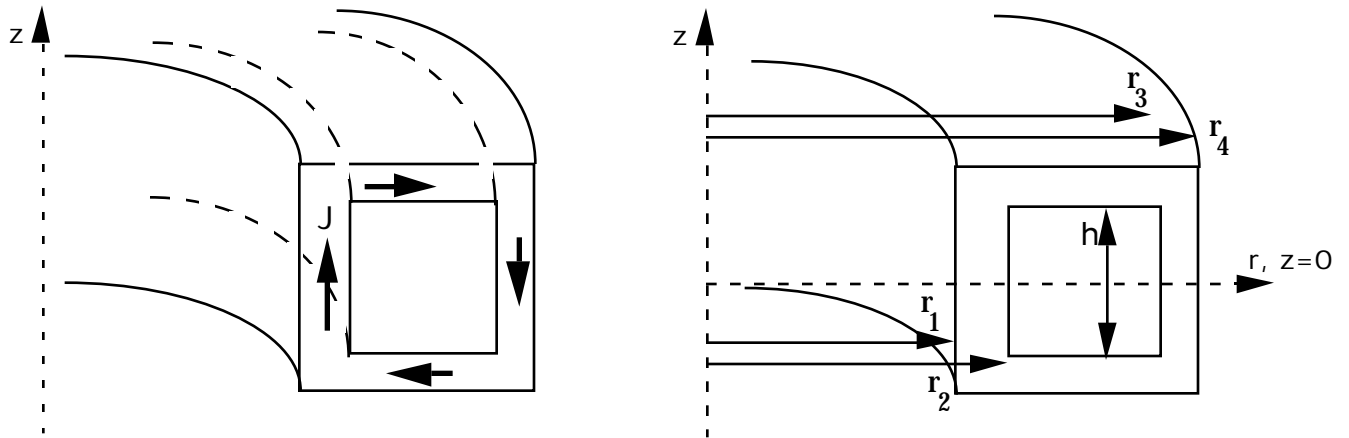
- 1) outer radius  $b$
- 2) current  $I$
- 3) resistance  $R$
- 4) power dissipated
- 5) magnetic field on axis

*Hint: Start with the maximum allowed current density and use the results of Lesson 3.2 to calculate  $b$ .*

**Problem 2 - (8 points)**

A very large number of wires has been wrapped around an air-core toroid. We will model these wires as a current density,  $\mathbf{J}$ . On the inner leg of the toroid, we have a current density of  $\mathbf{J} = J_0 \mathbf{a}_z$  for  $r_1 < r < r_2$ . Do all calculations for  $z=0$  (the horizontal dashed line in the right figure). A section of the toroid is illustrated in the figure on the opposite side. Its shown twice simply because the drawing got too cluttered when only one picture was used.

- a. Calculate  $\mathbf{H}$  for  $r < r_3$ . You will need 3 separate expressions for the different regions.
- b. What is the flux intercepted by a wire that is wrapped on the innermost part of the toroid (i.e. going up at  $r_2$  and down at  $r_3$ ) ?
- c. Since the current is carried by wires with constant cross-section, we can assume that  $\mathbf{J} = -J_0 \mathbf{a}_z$  for  $r_3 < r < r_4$ . What relation must exist between  $r_1$ ,  $r_2$ ,  $r_3$ , and  $r_4$  for this to be true?



**Problem 3 - (7 points)**

This problem looks at the motion of a loop above a magnet. While project 1 deals with rotational motion, this problem uses a linear motion. A circular loop of radius  $a$  moves at a constant velocity of  $v_0$  in the  $z$  direction. Treat the magnetic flux density of the magnet as

$$\mathbf{B} = (B_0/l^2) \{ rz \mathbf{a}_r + (l^2 - z^2) \mathbf{a}_z \}.$$

This isn't accurate (it requires a local current density), but it makes the calculation easier.

- a. Calculate the emf induced around the loop using Eq. B-11a.
- a. Calculate the emf induced around the loop using Eq. B-11b.

