Vector Calculus-Extra Credit (5pts)

due Friday, October 11, 5:00pm

1) We will tend to analyze classical problems in this course. The geometries may be very long or infinte to simplify the problems. An example would be a current carrying wire that is infinitely long and has a radius, *a*. The current has a certain distribution in that wire and a magnetic field exists due to that current. The field may be expressed as:

$$H_\phi = \pi r^2/6 \ \hat{\phi} \quad \text{for} \quad r < a$$

$$H_\phi = \pi a^3/6r \ \hat{\phi} \quad \text{for} \quad a < r$$

   In both regions, determine $\nabla \times \mathbf{H} = \mathbf{J}$.
2) Define a surface that is perpendicular to **J.** The normal to the surface should 'point' in the same direction as **J**. What is the d**S** we would use to integrate on this surface.
3) For an arbitrary surface r < a, find the current passing through the surface you just defined, $I = \int_S \mathbf{J} \bullet d\mathbf{S}$. Note, only one integral sign is given, however, a surface integral is always a double integral. This notation is very common.
4) Repeat part 4 for an arbitrary surface a < r. The result should be a function of the radius, *r*. There are two expressions for **J**, each specific to a region. You need to apply the appropriate integration limits and expression for each region.
5) Now, evaluate the closed line integral $\int \mathbf{H} \bullet d\mathbf{l}$ that bounded the surface you defined in part 4. How does your solution compare with the result in part 4? Verify to yourself that the steps you just performed were solutions to the left and right side of Stoke's theorem
6) Repeat part 6 for the closed loop that bounds the surface for part 5. Again, compare your result with the solution from part 5.