Fields and Waves I

Fall 2008

Quiz 3

Name Solution

1. ____________

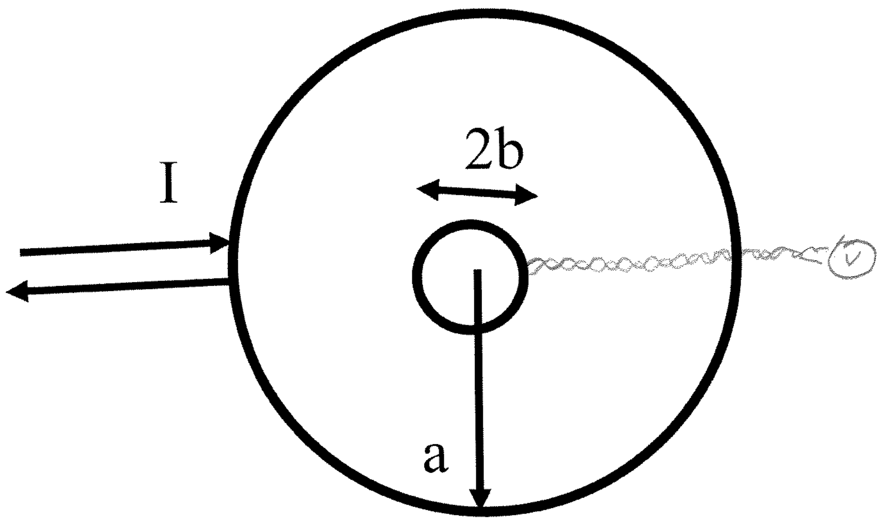
2. _____________

3. _____________

4. _____________

Total ____________

1. (25) Two concentric coils of radii a = 50 cm. and b = 5 cm. with number of turns N1 = 25 and N2 = 100 , both centered at the origin and are in the same plane. We have previously found that the flux density at the center of a circular coil of radius *a* is $B_z = \frac{\mu_0 N I}{2a}\hat{a}_z$.



10 a) Find the mutual inductance between the coils. State any assumptions.

Since b << a assume B is constant over the small coil.

$\lambda = N_2 B \pi b^2 = \frac{N_1 N_2 \pi b^2 \mu_0 I}{2a}$

$L_{12} = \frac{N_2 N_1 \pi b^2 \mu_0}{2a}$ 2.47×10^{-5}

10 b) If the large coil is carrying a current of $I(t) = 10\sin(377t)$, find the induced EMF in the smaller coil.

$B = \frac{4\pi \times 10^{-7} \times 25 \times 10 \sin(377t)}{2 \times 0.5} = 3.14 \times 10^{-4} \sin 377t$

$\lambda_2 = N_2 B \pi b^2$, $\mathcal{E} = -\frac{d\lambda_2}{dt} = -100 \times 2.47 \times 10^{-6} \times 377 \cos(377t)$

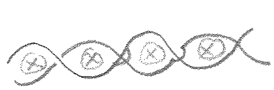
$= -93 \cos(377t)$ mV

$= -0.093 \cos(377t)$ V

5 c) In the figure, the wires connecting the inner coil to the voltmeter are twisted (twisted pair). Why are the wires twisted? Explain using Faraday's law.

$= 931 \times 10^{-4} \cos(377t)$ V $= .093$

$= 0.0931$ V.



If B is as shown the area vector is alternately in and out of the page. So $\int B \cdot dS$ is very small since B S is ≈ 0.

voltages cancel.

$2.47 \times 10^{-5} \times 10 \times 377$

$2.47 \times 377 \times 10^{-6} = 931 \times 10^{-6}$

2. (25) Consider a parallel plate capacitor consisting of two metal plates of 50 cm^2 each separated by a porcelain layer of thickness 1 cm. For porcelain $\varepsilon_r = 5.5, \quad \sigma = 10^{-14}\ S/m$. If a voltage $v(t) = 100\sqrt{2}\cos(120\pi t)$ V is applied across the capacitor plates;

a) Find the electric field and electric flux density in the capacitor

$E = \frac{V}{d} = \frac{100\sqrt{2}\cos(120\pi t)}{0.01} = 1.1\sqrt{2} \times 10^4 \cos(120\pi t)$ V/m

$D = \epsilon E = \epsilon_r \epsilon_0 E = 6.89 \times 10^{-7} \cos(120\pi t)$

b) Find the conduction current density

$J_c = \sigma E = 10^{-14} \times 1.1\sqrt{2} \times 10^4 \cos(120\pi t)$

$= 1.1 \times \sqrt{2} \times 10^{-10} \times \cos(120\pi t)$ A/m² ✓

c) Find the displacement current density

$J_d = \epsilon_r \epsilon_0 \frac{dE}{dt} = 5.5 \times 8.85 \times 10^{-12} * (-1.1\sqrt{2} \times 10^4 \times 120\pi \sin(120\pi t))$

$= -2.66 \times 10^{-4} \sin(120\pi t)$ A/m²

d) Find the total current

$I = \int (J_c + J_d) \cdot ds$, (ignore J_c)

$= 1.33 \times 10^{-6} \sin(120\pi t)$ A

3. (25) A train is traveling at a velocity U along parallel conducting tracks. The normal component of the earth's magnetic field is Bn. The separation of the rails is d.

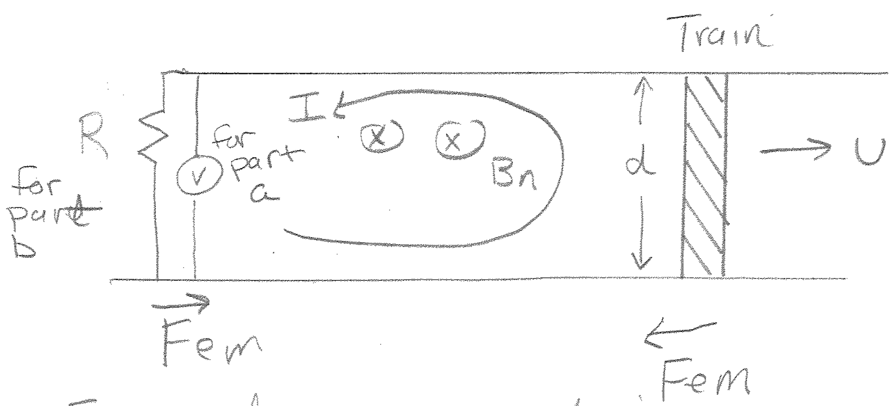
a) Assume that you are measuring a voltage across the tracks as the train speeds away. Find an expression for the voltage. State assumptions.

$\lambda = B_n d x$, $\frac{d\lambda}{dt} = B_n d U$

b) Now connect a resistor of resistance R Ohms across the track and measure the current in the resistor. Find the force on the conductor (train) and the resistor (magnitude and direction). Explain if the force depends on whether the train is approaching or traveling further from your location.

$F = I d B_n$, $I = \frac{\mathcal{E}}{R} = \frac{B_n d U}{R}$ so

$F = \frac{B_n^2 d^2 U}{R}$

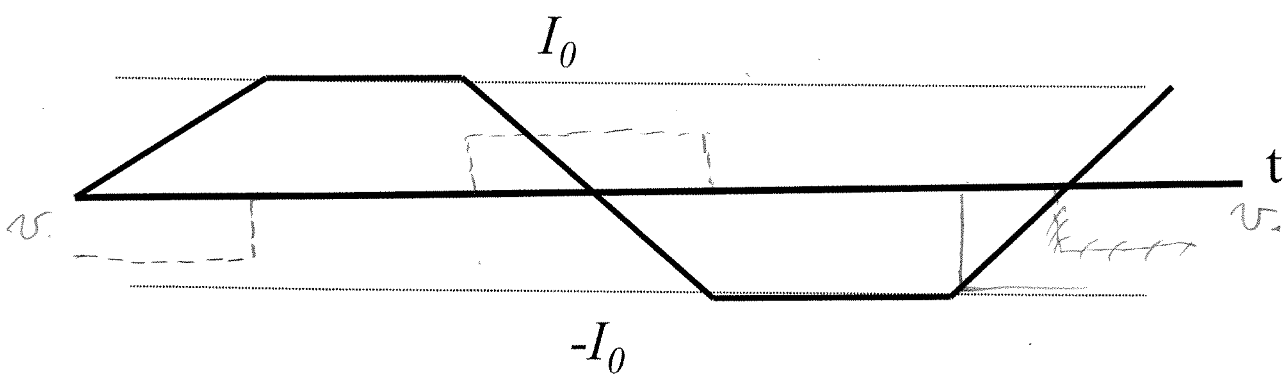


yes, Force always opposes motion

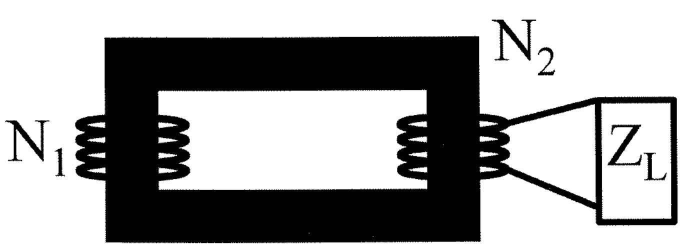
4. (25) Consider the ideal transformer below. In part a there is current in the primary winding but none in the secondary. The primary winding has N1 turns and the secondary has N2 turns. The transformer is ideal so there are no losses and no leakage flux.

a) If the current in the primary (secondary current is zero) has the waveform shown below, sketch (on the graph below) the voltage on the primary winding which was necessary to produce this current.

$\Psi_m \propto I$, so $\mathcal{E} \propto -\frac{d\Psi_m}{dt}$



Now assume that the secondary winding has a load connected of Z_L Ohms. The primary is connected to an ideal voltage source $v(t) = V_0 \cos(\omega_0 t)$



b) Find the voltage on the load impedance.

$v_2 = v_1 \frac{N_2}{N_1} = \frac{N_2}{N_1} V_0 \cos \omega_0 t$

c) Find the current in the secondary winding.

$$I_2 = \frac{V_2}{Z_L} = \frac{N_2}{N_1 Z_L} V_0 \cos \omega t$$

d) Find the current in the primary winding

$$N_1 I_1 = N_2 I_2$$

$$I_1 = \frac{N_2}{N_1} \cdot \frac{N_2}{N_1 Z_L} V_0 \cos \omega t$$

e) Explain why, for the same voltage magnitude, if the frequency is higher the transformer can be smaller.

$$\mathcal{E} = N \frac{d\Psi_m}{dt} = N A \frac{dB}{dt} = jNA\omega B$$

if $\omega \uparrow$ then $A \downarrow$