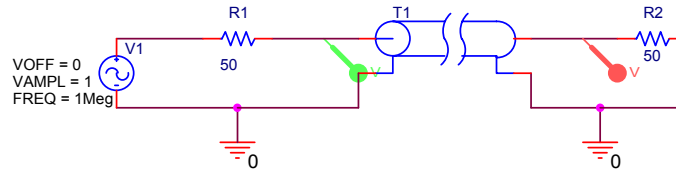3. Input and output voltages measured on an ideal, lossless transmission line with a propagation velocity of $v = 2x10^8 \, m/_s$.

a. A simple transmission line is driven by a 1volt, 1MHz source with a 50 ohm internal impedance as shown. It has a 50 ohm resistor as a load. Given that the input and output voltages appear as shown below under matched conditions, we are to determine the voltage and current on the transmission line as a function of space and time and in phasor form. Also, we need to determine the shortest length possible for this line. The same line and the same source voltage are used for all parts of this problem. Note that the time scale does not begin at $t=0$ because we wish to show the signal in steady state.
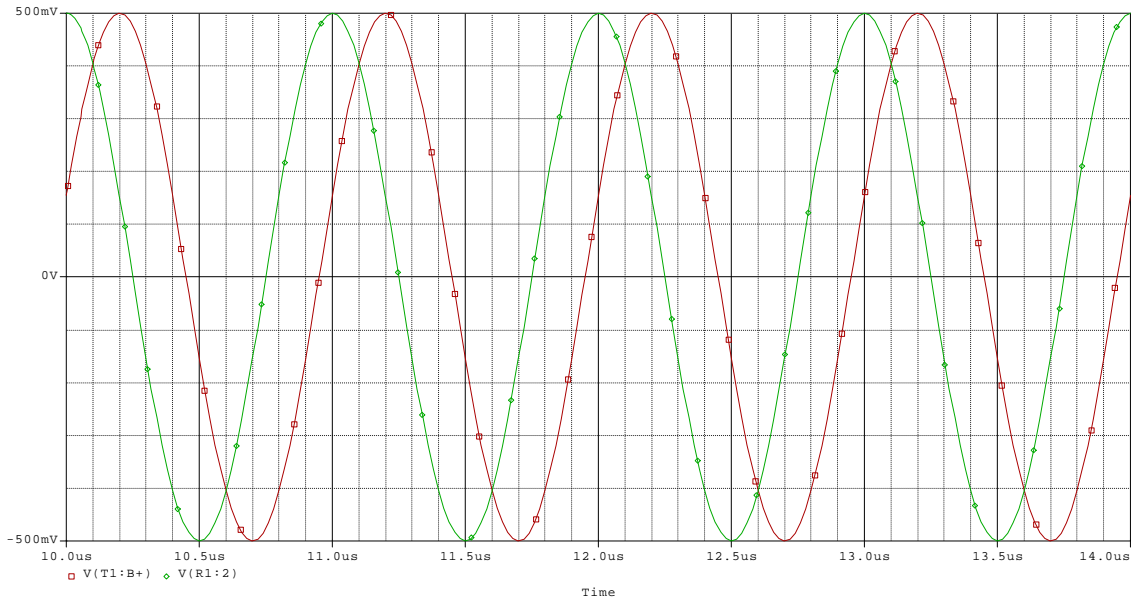
i) Find the β for this line. *ans:*

$$\beta = \frac{\omega}{v} = \frac{(2\pi)(10^6)}{2x10^8} = \frac{\pi}{100}$$

ii) Write the voltage and current on the line in general phasor form for the case of a matched load. *ans:* Since the load is matched, there will be no reflected wave. Thus,

$$\hat{V}(z) = V_m^+ e^{-j\beta z} \text{ and } \hat{I}(z) = I_m^+ e^{-j\beta z}$$

iii) Write these expressions in space and time dependent form. *ans:*

$$V(z,t) = V_m^+ \cos(\omega t - \beta z) \text{ and } I(z,t) = I_m^+ \cos(\omega t - \beta z)$$

Note that both of the preceding expressions for the current could be rewritten in terms of the characteristic impedance with $I_m^+ = \dfrac{V_m^+}{Z_o}$, with Z_o=50 ohms.

iv) We still need to evaluate the voltage magnitude. To do so, we need the real time dependent voltage at the source end (input) of the transmission line. We will also include the current for completeness. *ans:*

$$V(0,t) = V_m^+ \cos \omega t \text{ and } I(0,t) = I_m^+ \cos \omega t$$

Looking at the plot for the voltage at the source end, we see that it has a magnitude of 0.5 volts, so that is the value of V_m^+. The magnitude of the current is just this number divided by 50ohms.

We can also determine V_m^+ from the input impedance of the line. Since the line is matched, $Z_{in} = Z_o = 50 ohms$ and the equivalent circuit seen by the source is:
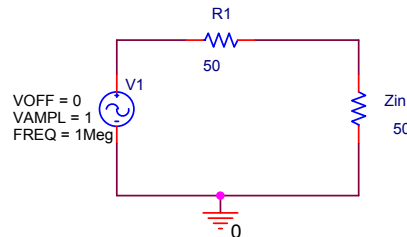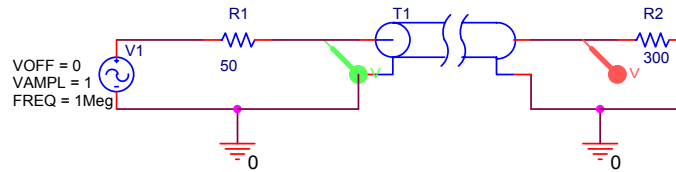

Thus, for a 1 volt source, the voltage across the input is 0.5volt, as we found above.

v) The length of the line can be determined from the phase shift at the output end. The voltage at the end of the line is given by
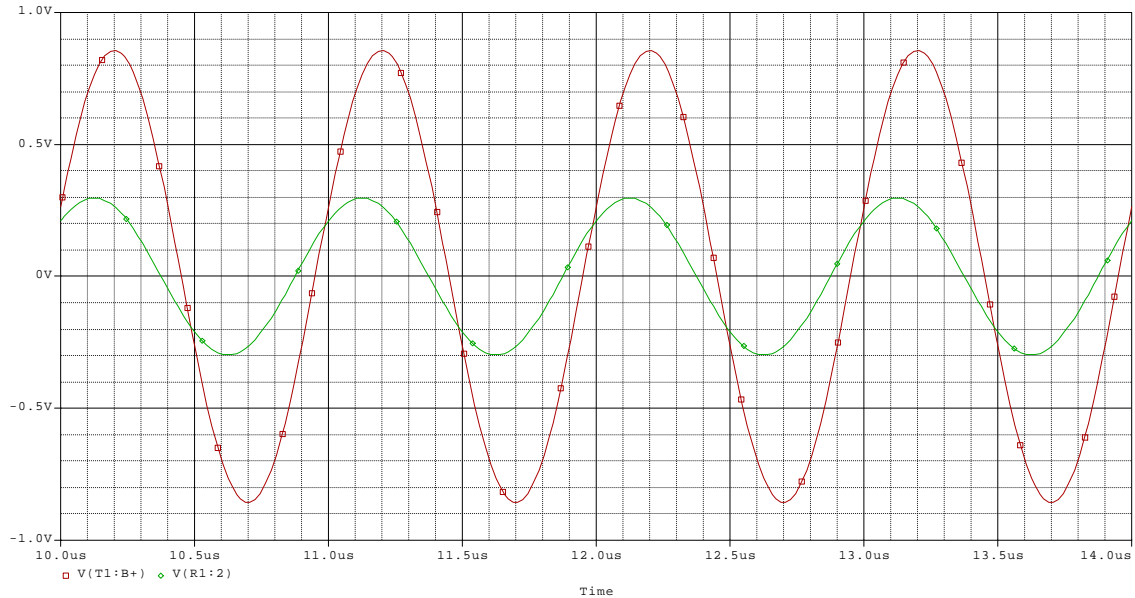$$V(l,t) = V_m^+ \cos(\omega t - \beta l)$$
The phase shift between the input and output voltages is 0.4π. Thus, the length of the line is 40 meters. ($\beta l = \dfrac{\pi}{100} l = 0.4\pi$)

b. Now the load resistor is changed to 300 ohms.

The input and output voltages become the following. The first peak for the input wave occurs at 10.12microseconds, while the first peak for the output voltage occurs at 10.2microseconds.

At the end of this question, you will be asked to show that your solution is consistent with the input and output voltages given. You will probably find that at least one aspect of this plot is not correct. For now, however, disregard the voltage plot. Please note that, while these questions are similar to those asked for the matched line, here you can expect that the input impedance and voltage magnitudes will be complex. Since you will be doing some complex arithmetic, you can make your life a little easier if you use Maple, Matlab or a calculator for this purpose. You need to know how to do this by hand, but you might as well take advantage of the tools you have at hand.

i) First, write the voltage and current on the line in general phasor form. Since you do not know the magnitudes of the positive and negative traveling waves, leave those as unspecified in your general form. Since this is the same transmission line as in part *a*, you should use the same value for β.

ii) Determine the reflection coefficient at the load.

iii) Determine the input impedance of the transmission line Z_{in}.

iv) The only unknown you have at this point is the amplitude of the positive traveling wave. To find this, you must first find the voltage at the source end from the equivalent circuit of the transmission line as seen by the source. This circuit will be just like the one drawn for the first case, but the input impedance of the line will now be different. Draw this circuit and solve for the voltage across the input of the line in both phasor and time varying form. From the phasor form, you should be able to identify the value of the amplitude of the positive traveling wave.

v) Now that you have values (some complex) for all of the constants, write the voltage on the line again and then evaluate it at the load end $z=l$ in both phasor and time varying form.

vi) Finally, compare the results you obtained for the voltages at the source and the load ends of the line with the plots given above. These were generated with Pspice. It appears that Pspice does not always give the correct answer. Which aspects of the plots look correct and which do not? Remember that you need to be concerned with both the magnitude and the phase of sinusoidal signals.