Fields and Waves I

Name _______________ ECSE-2100 Fall 1999 Section ____________

Preparation Assignments for Homework #3

Due at the start of class.

Reading Assignments

Please see the handouts for each lesson for the reading assignments.

20 September Lesson 2.4

No prep assignment to do, but here are two questions anyway.

1. Assume that there are two positive charges of equal value +Q separated by a distance d. Sketch several equipotentials for this combination of charges.

2. What is the dielectric strength and dielectric constant of Teflon?

Due 22 September (2 points) Lesson 2.5

1. What is the approximate capacitance of the earth, if we assume it is an isolated conductor. Make a reasonable assumption for the radius of the earth if you don't know it.

2. Given that the electric field near the surface of the earth is about 100 V/m, estimate the total amount of charge on the earth. Hint: you should be able to get the charge density from the boundary condition for the electric flux density.

Due 24 September (2 Points) Lesson 2.6

1. For a parallel plate capacitor, find the voltage as a function of position.

2. Show that your answer to question 1 satisfies Laplace's equation.

Due 27 September (2 Points) Lessons 2.6 and 3.1

1. A coaxial cable with inner radius a = 1 mm and outer radius b = 3 mm has a voltage of 10 V on the inner conductor and the outer conductor is grounded. Assume that the insulator between the conductors is air. Evaluate the electric field at the outer conductor using the exact expression for the field found in example 3.20. Now evaluate the field with a very crude approximation by taking the difference between the outer and inner potential and dividing by the distance between them. How do the two numbers compare?

2. What is the resistance of a 100 meter long aluminum wire with a diameter of 1mm?

Class time 29 September (Note the date – Wednesday)

Open shop to work on Homework 3. Due at 5 pm on 17 September.

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Homework #3

Problem 1 – (10 points)

There are a small number of simple conductor/dielectric configurations for which we can relatively easily find the capacitance. Students of electromagnetics should be sure that they know how to derive and use all of the expressions for such capacitors. The list of simple capacitors should include the following:

1. Spherical Capacitor – see example 3.21
2. Spherical Capacitor with more than one dielectric – problem 3.9.8
3. Two-wire transmission line – see examples 3.19, 3.25, 3.26 for increasing accuracy
4. Coaxial Cable – see example 3.20 and problem 3.9.2
5. Coaxial Cable with more than one dielectric – problems 3.9.3 and 3.9.7
6. Parallel Plate Capacitor – see equation (111)
7. Parallel Plate Capacitor only partially filled with dielectric or with more than one dielectric – problems 3.9.1, 3.9.4, 3.9.5
8. Parallel plate transmission line – see section 7.1.1

In this problem, we want to look at some variations of these standard configurations. First, we will consider the standard spherical capacitor. Look at the analysis of the spherical capacitor in example 3.21 and answer the following questions.

a. Assuming that there is a grounded conductor at r = b and that the conductor at r = a has a voltage V_o, write the electric field in the region between the plates ($a < r < b$) in terms of V_o instead of in terms of the charge Q. Write your answer for both the case where there is no dielectric between the conductors ($\varepsilon = \varepsilon_o$) and where there is a dielectric between the conductors ($\varepsilon > \varepsilon_o$). Note that it is possible that the two expressions will be the same.

b. Write the corresponding expressions for the voltage V(r) at all radii in the same region.

c. Write the corresponding expressions for the electric flux density **D**(r) in the same region.

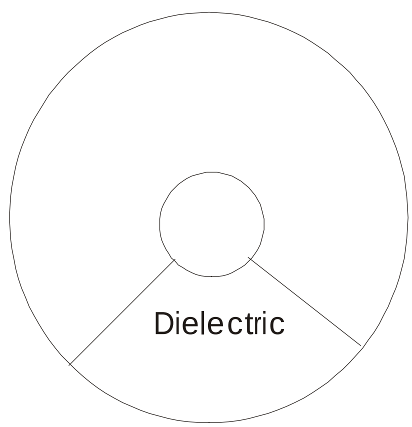
d. Now, using what you know about the flux density and its boundary condition, determine the surface charge density and total charge on each surface (r=a and r=b).

Your answers to b, c and d should be in terms of V_o, a, b and ε or ε_o.

Now, assume that we have an air-filled spherical capacitor in which we fill one quarter of the space between the conductors with a dielectric. The sides of the dielectric region align with the radial direction. This dielectric could be used to support the center conductor in an otherwise empty space.

e. Find both the electric field **E**(r) and the electric flux density **D**(r) in both the dielectric and empty regions. Assume that the dielectric has a permittivity of ε, while the empty region (air) has a permittivity ε_o.

f. Using your expressions from part e, show that the appropriate boundary conditions are satisfied at the interface between the dielectric and empty regions.



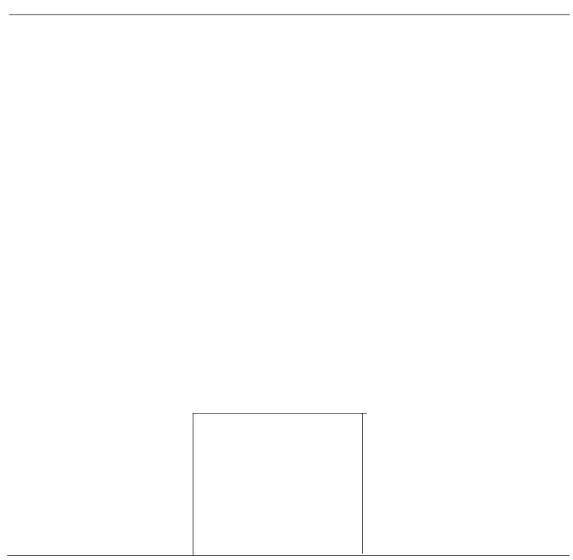
Problem 2 – (10 points)

Nearly all practical electromagnetics problems are analyzed using numerical methods rather than the analytical methods we have been addressing. One of the most common techniques is called the Finite Difference Method, which we have considered in class. In this problem, we will try to find the capacitance of a simple two-dimensional problem with a somewhat odd geometry. To address this geometry, we will use a spreadsheet to solve Laplace's equation. Review the materials on numerical methods in the class notes and on reserve in the library.

The method is reasonably simple. First, identify the cells you will use to represent the conductors. Set the value in these cells equal to the potential on the conductor. Second, all exterior cells must have either a specific potential or be set equal to their nearest interior neighbor. This is the equivalent to setting the normal derivative of the potential equal to zero. This boundary condition is quite accurate if the boundary in question is a line of symmetry for the problem. As we will see, this will be the case in this problem. Third, for all interior points, set the voltage in each cell equal to the average of its four nearest neighbors. This is the finite difference equivalent of Laplace's equation. After you enable iteration, the spreadsheet values will eventually converge to something near the correct voltages at each location. Use this information to determine the capacitance per unit length of this configuration, following the method we discussed in class. For your solution, use at least a 21 x 21 array. Whatever information you use from your spreadsheet has to be included with your analysis.

The geometry you are to consider is shown below. Assume that it continues for a very long distance into the page, just like a coaxial cable. Thus, you will be finding the capacitance per unit length, just as is the case with the cable. Also, assume that the structure is periodic in the horizontal direction. That is, it repeats over and over. That means that the left and right boundaries are lines of symmetry, so that the normal derivative boundary condition is the appropriate choice. It is easier to address this problem if we assume that this region is 20mm by 20mm and the conducting block on the bottom is 5mm by 5mm and in the exact center of the bottom. Assume that the top electrode is at 100 volts and the bottom is grounded.

To summarize – solve for the potential at all points represented by cells and use this information to determine the capacitance per unit length of this structure.



Extra Credit – Fill the regions to the right and left of the conducting pedestal with Teflon. Determine the capacitance for this more complex configuration.