Preparation Assignments

Due Monday, February 11

A vector field is $\vec{A} = 3x\hat{x} + z\hat{y} + 2\hat{z}$.
i)Evaluate $\oint \vec{A} \bullet d\vec{l}$ for a square loop with sides of length *a* that lies in the *z = 0* plane.
ii)Check that Stoke's theorem $\int_S (\nabla \times \vec{A}) \bullet d\vec{S}$ is consistent with this result.
iii) Problem from the last class: For the following field
$\vec{E}(\vec{r}) = \frac{qd}{4\pi\varepsilon_o R^3}(\hat{R}2\cos\theta - \hat{\theta}\sin\theta)$, evaluate the integral $\oint \vec{E} \cdot d\vec{s} = ?$ over a spherical surface with radius *R=a.*

Due Wednesday, February 13

i)In cylindrical coordinates the following field exists:

$$\vec{E} = E_o \ \hat{R} \qquad R < a$$

$$\vec{E} = \frac{E_o a}{R} \ \hat{R} \qquad a < R$$

Evaluate $\nabla \times \vec{E}$ and $\nabla \bullet \vec{E}$ for both regions.

ii)In cylindrical coordinates the following field exists:

$$\vec{H} = H_o \ \hat{\phi} \qquad R < a$$

$$\vec{H} = \frac{H_o a}{R} \ \hat{\phi} \qquad a < R$$

Evaluate $\nabla \times \vec{H}$ and $\nabla \bullet \vec{H}$ for both regions.

Homework #3

Problem 1 – (5 points) Stokes Theorem

a. A vital first step in solving static field problems is recognizing the coordinate system that would be best applied to the geometry. The scale or size of the current source is not always relevant. The location where you are trying to determine the field is much more important. With that in mind, what coordinate system would be best applied to these geometries:
i) The field inside a TV cathode ray tube where the current source is an electron beam
ii) The field in CMOS chip due to the current flowing between the gates
iii) The far (distant) field of a magnetic dipole antenna. This type of antenna is a simple wire loop, like an old TV antenna.

b. We will tend to analyze classical problems in this course. The geometries may be very long or infinite to simplify the problems. An example would be a current carrying wire that is infinitely long and has a radius, *a*. The current has a certain distribution in that wire and a magnetic field exists due to that current. The field may be expressed as:

$$H_\phi = \frac{\pi r^2}{6} \text{ for } r < a$$

$$H_\phi = \frac{\pi a^3}{6r} \text{ for } a < r$$

In both regions, determine $\nabla \times \vec{H} = \vec{J}$.

c. Define a surface that is perpendicular to $\vec{J}$. The normal to the surface should 'point' in the same direction as $\vec{J}$. What is the $d\vec{S}$ we would use to integrate on this surface.

d. For an arbitrary surface $r < a$, find the current passing through the surface you just defined, $I = \int \vec{J} \cdot d\vec{S}$. Note, only one integral sign is given, however, a surface integral is always a double integral. This notation is very common.

e. Repeat part d for an arbitrary surface $a < r$. Be careful when you perform the surface integral. There are two expressions for $\vec{J}$, each specific to a region. You need to apply the appropriate integration limits and expression for each region.

f. Now, evaluate the closed line integral $\oint \vec{H} \cdot d\vec{l}$ that bounded the surface you defined in part d. How does your solution compare with the result in part d? Verify to yourself that the steps you just performed were solutions to the left and right side of Stoke's theorem.

g. Repeat part f for the closed loop that bounds the surface for part e. Again, compare your result with the solution from part e.

Problem 2 – (5 points)

Assume that we have an electric field given by the following generic expression $\vec{E}(\vec{r}) = E_o \dfrac{R}{a}\hat{R} + E_1\hat{R}$ for $0<R<a$ and $\vec{E}(\vec{r}) = E_2 \dfrac{a^2}{R^2}\hat{R}$ for $a<R$**.** The coordinate system is spherical.

a) According to Maxwell and Gauss, the divergence of the electric field tells us what charge distribution produced the field. Thus find the charge distribution by evaluating $\rho = \nabla \cdot \varepsilon_o \vec{E}$ Note that you must apply this expression in both regions and that ε_o is a fundamental constant called the permittivity of free space.

b) The integral $Q_{Enclosed} = \int_V \rho dv$ is the charge contained in a volume V defined by a closed surface S. Thus, it is the charge enclosed by the surface S. Let V be a spherical volume of radius R, where $R < a$. Determine the charge contained in this volume $Q(R)$. Begin by writing out this integral fully with its limits and then evaluate it. Note that for this question, R can take on any value from 0 to a.

c) Now evaluate the integral for a sphere of radius $R>a$.

d) When we consider Gauss' Law for electric fields, we will see that $D_R(R)4\pi R^2 = Q(R)$ will give us the electric flux density $D_R(R)$. Evaluate $D_R(R)$ using this formula.

e) Your answer for $D_R(R)$ can be used to determine the electric field $E_R(R)$, since they are usually related by a constant. In empty space (also called free space), $\vec{D} = \varepsilon_o \vec{E}$. Show that your answer for $D_R(R)$ is consistent with the given electric field for a particular relationship between E_o, E_1, and E_2? What is the relationship?

Problem 3 – (5 points) Total Charge

a. During this course, we will need to know both the total charge present in a distribution and the charge density of that distribution. The first is measured in Coulombs [C]. However, for a problem that is infinite in one dimension, we will want total charge in a unit length [C/m]. The following geometries have a uniform (constant) charge density with a total charge Q C or total charge per unit length Q C/m. Determine the charge density. *For both parts of this problem it might be a good idea to sketch the geometry before answering the question.*
i) a point charge (be careful with this one)
ii) a spherical shell with finite thickness $a < R < b$
iii) an infinitely long cylinder of radius a
iv) an infinitely long cylindrical surface located at $r = a$

b. Determine the total charge, total charge per unit area or total charge per unit length of the following geometries, which ever is appropriate. *For both parts of this problem it might be a good idea to sketch the geometry before answering the question.*
i) an infinitely long thin wire with 0.25 C/m
ii) a spherical charge distribution $\rho(R) = 2R^2$ C/m^3 with radius a
iii) a planar slab of uniform charge density ρ_o with finite thickness located in the region $-d < x < d$
iv) a cylindrical charge distribution with uniform density ρ_o located in the region $a < r < b$

Problem 4 – (5points)

The following charge distributions exist in a *cylindrical* region:

$\rho = \rho_o\left(1 - \frac{r}{a}\right)$ for *0<r<b* and $\rho_s = -\rho_{so}$ at *r=a* where *0<a<b*.

The first expression is a volume charge while the second is a surface charge.

a. To determine the electric field produced by these charge distributions, we must first decide what Gaussian surface or surfaces to use. Draw the appropriate Gaussian surface for this configuration and indicate what surface element you will use from the following expressions for cylindrical coordinates $d\vec{S}_z = \hat{z} r d\phi dr, d\vec{S}_r = \hat{r} r d\phi dz, d\vec{S}_\phi = \hat{\phi} dr dz$ for each surface (if there is more than one).

b. For Gauss' Law $\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enclosed}}{\varepsilon_o}$, evaluated the right hand side for all values of *r*. That is, find three expressions: one for *0<r<a*, one for *a<r<b* and one for *b<r*.

c. Assuming that the total charge is zero, what is the relationship between ρ_s and ρ?

d. Now evaluate the left hand side of Gauss' Law

e. Solve for the electric field $\vec{E}$ in all three regions.

f. If you have done this problem correctly, your answers for $\vec{E}$ should give back the original charge distribution when you apply $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_o}$. Note that this will only give you ρ.