Fields and Waves I

Name _______________ ECSE-2100 Spring 2003 Section ____________

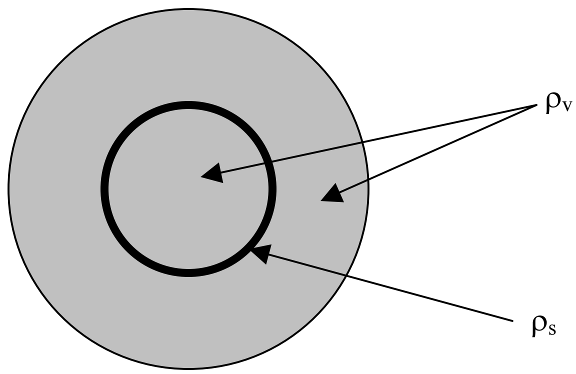
Homework 3

Due Wednesday 19 February, 2003

1) Gauss's Law

The following charge distributions exist in a *cylindrical* region, where $a < b$

$$\rho(r,\theta,\phi) = \begin{cases} \rho_v = \rho_o r(1-\frac{r}{b}) & [C/m^3] & 0 < r < b \\ \rho_s = \rho_{sa} & [C/m^2] & r = a, where\ a < b \\ 0 & & elsewhere \end{cases}$$



A volume charge density exists in the region $0 < r < b$ and a surface charge density exists at the location $r = a$, inside the volume charge density. This geometry is mathematically possible, but would be difficult to implement. Furthermore, the charge densities are given such that the total charge in the domain is zero. We often using the phrase, "the total charge is equal and opposite", meaning $Q_{vol} = -Q_{surf}$, where capital Q indicates the total charge (per unit length) for that distribution. You may assume the entire domain is free space, $\varepsilon = \varepsilon_o$.

1) In terms of geometry and the known expression for the volume charge density, ρ_v, determine an expression for the surface charge density, ρ_s. (Determine what ρ_{sa} must equal.)

2) What are the regions of the problem? (Indicate the regions where you will find the field)

3) What coordinate system applies to this problem (this better be easy).

4) What direction are the field lines? (just provide the coordinate direction, not the sign).

5) Given the answer to part 3, what shape would be appropriate for a Gaussian surface? Make your shape closed, not infinite in length. What is $d\vec{S}$ on all sides of your shape?

6) Set up, (do not solve) the integral, $\oint\!\!\oint_S \vec{E} \bullet d\vec{S}$, for the closed Gaussian surface that you would use to solve the Gauss's Law problem. Include limits, the $d\vec{S}$, and the result from the dot product. (Your equation should have 3 integrals, two of which are trivial.)

Set up the integral(s) for enclosed charge as well, $\frac{1}{\varepsilon}\int_V \rho \; dV$. Depending on your region, you need to be careful. A surface charge is a "special" type of enclosed charge, requiring a different integral than the volume integral.

8) For each region, determine the electric field. Remember, when you are considering an arbitrary surface inside a given region, the charge enclosed includes any regions <u>inside</u> the region you are currently considering. Specifically, pay attention to where the enclosed charge exists. When you have finished determining the field, your solution should include all information: the field in that region, the units, and the specification of the region (similar to the format of the charge distribution given at the top of the previous page).

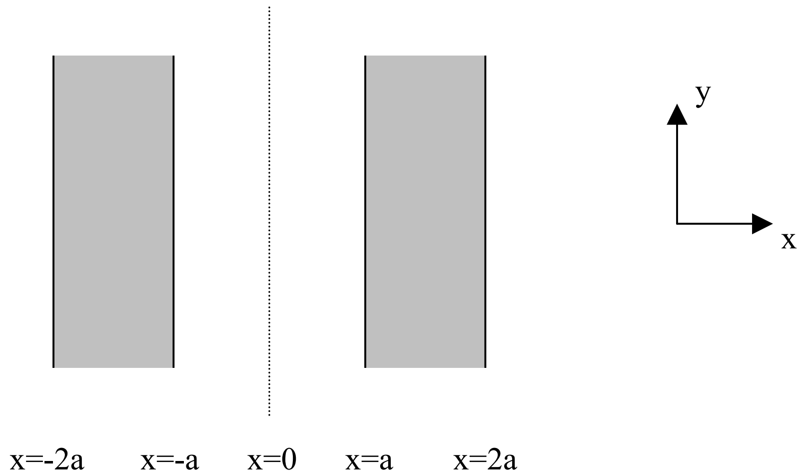
9) What can you say about the electric field when you cross the cylindrical shell at r = a?

10) Verify your solution is correct for each region by using the differential form of Maxwell's Laws as applied to electrostatics, $\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon}$ and $\nabla \times \vec{E} = 0$.

(Alternatively, $\nabla \bullet \vec{D} = \frac{\rho}{\varepsilon}$ and $\nabla \times \vec{D} = 0$)

11) Determine the voltage everywhere as a function of position. You may assume ground is placed at $r = b$.

12) Determine the voltage difference between the location $r = 0$ and $r = b$. Is it possible that this value can be zero?



In the above figure, two parallel slabs of uniform charge are shown. They are infinite in the *y* and *z* directions. (They are shown as finite since I would run out of paper. Use your imagination.). The charge distribution is specified as:

$$\rho(x,y,z) = \begin{cases} \rho_v = \rho_o \quad [C/m^3] & -2a < x < -a \\ \rho_v = \rho_o \quad [C/m^2] & a < x < 2a \\ 0 & elsewhere \end{cases}$$

Repeat questions 1-8) and 10) of the previous problem. Several Gaussian surfaces are possible. Choose one that you feel simplifies the problem.

11) Determine the voltage difference between *r = -2a* and *r = 2a*. When determining the voltage difference between two positions, is the location of ground important? What can you say about the voltage in the region *-a < r < a*?