Fields and Waves I

Name _______________ ECSE-2100 Fall 2002 Section ____________

Homework 4

Due Thursday 17, October, 2002

1) Gauss's Law

The following charge distributions exist in a *spherical* region, where $a < b < c$

$$\rho(r,\theta,\phi) = \begin{cases} \rho_o(1-\frac{r}{a})(1-\frac{r}{b}) & [C/m^3] \quad a < r < b \\ \rho_{sc} & [C/m^2] \quad r = c \\ 0 & \quad elsewhere \end{cases}$$

A volume charge density exists in the region $a < r < b$ and a surface charge density exists at the location $r = c$. Furthermore, the charge densities are given such that the total charge in the domain is zero. You may assume the entire domain is free space.

1) What are the four regions of the problem?

2) What coordinate system applies to this problem (this better be easy).

3) What direction are the field lines?

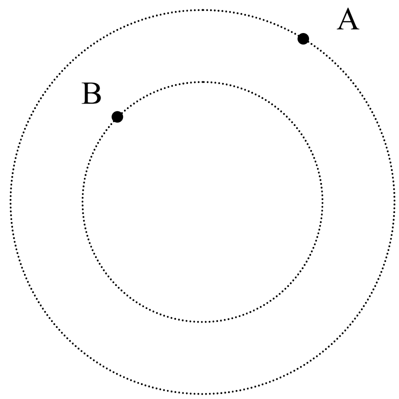
4) Given the answer to part 3, what shape would be appropriate for a Gaussian surface? What would is $d\vec{S}$?

5) Set up, (do not solve) the integral, $\oint\!\!\oint_S \vec{E} \bullet d\vec{S}$, for the closed Gaussian surface that you would use to solve the Gauss's Law problem. Include limits, the $d\vec{S}$, and the result from the dot product.

6) For each region, determine the electric field. Remember, when you are considering an arbitrary surface inside a given region, the charge enclosed includes any regions <u>inside</u> the region you are currently considering. Specifically, pay attention to where the enclosed charge exists. When you have finished determining the field, your solution should include all information: the field in that region, the units, and the specification of the region (similar to the format of the charge distribution given at the top of the page).

7) Verify your solution is correct for each region by using the differential form of Maxwell's Laws as applied to electrostatics, $\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon}$ and $\nabla \times \vec{E} = 0$.

2) Voltage



1) For the above figure, draw an appropriate path to determine the voltage difference between point A and point B. Using the field from problem 1, your path should be one that you can easily apply to the integral relationship between voltage and field. Assume the circles are concentric about the origin. If we change the location of B, but keep it on the inner circle, will the voltage difference between the two points change?

For the previous problem, set the ground at the location $r = b$. (The above figure does not apply to the next two parts.)

2) Determine the voltage as a function of position in all four regions. Again you result should appear similar to that given in the charge distribution. Remember, for each region, your path will include all regions that you pass through between your current location and ground.

3) Verify your solutions by checking the differential relationship between electric field and voltage, $E = -\nabla V$.

4) What can you say about the behavior of the voltage as it crosses any boundary?

3) Capacitance and Energy

When solving electric field problems analytically, there are only a few geometries that have exact solutions. Typically, they are:

1. An infinite plane of charge (special case of 2)
2. An infinite slab of charge where the density is only a function of the coordinate perpendicular to the slab
3. A line of charge (special case of 4)
4. A cylindrical shell of charge (special case of 4)
5. An infinite cylinder of charge where density is only a function or *r*
6. A spherical shell or charge (special case of 6)
7. A sphere of charge where density is only a function of *r*
8. A point charge

We can use superposition to construct more complicated geometries with these shapes. A capacitor is essential an application of superposition with two surface charges. The charges are deposited on the surface of the conductors and the conductors are at different potentials. (Note, many of you assume a conductor that is grounded has zero charge. This is definitely not the case.) The geometries that can be used to create a simple capacitor are:

1. Two parallel planar surfaces
2. Two concentric cylindrical surfaces
3. Two concentric spherical surfaces
4. Two parallel wires

The last case does not have a simple field like the other three, however, we only need to know the field on one field line between the conductors in order to determine capacitance. We can add a dielectric material between the conductors in order to change the capacitance. Again, in order to keep the problem simple enough to be analytic, we must make the surfaces of the dielectric either perpendicular or parallel to the field lines. Problems 4.52-54 in our textbook are capacitor problems. The solution to 4.53 may be found on Fawwaz Ulaby's web page. Dr. Connor and Dr. Salon's notes provide a detailed solution for the coaxial (cylindrical) capacitor with two dielectric materials between the conductors.

1) Considering our geometry in problem 1, we want to replace the volume charge with a surface charge on a conductor located at the position $r = a$. The total charge must be the identical. A conductor is also placed a $r = c$. Given the volume charge from problem 1, what is the surface charge density, ρ_{sa}, in terms of the specified geometry and ρ_o.

2) What is the surface charge density, ρ_{sc}, using the same terms?

3) What is the electric field between the conductors? You may use ρ_{sa} as an expression of the charge density.

4) What is the voltage difference between the conductors, in terms of ρ_{sa} and geometry?

5) What is the capacitance of the conductors?

6) Using the volume integral and the electric field, determine the energy stored between the capacitors.

7) Verify the result of part 6 using voltage and capacitance.

4) Boundary Conditions

We want to increase the capacitance of the spherical capacitor of the previous problem.

1) Adding a material with a relative permittivity of 4, double the capacitance. You will create a capacitor that is partially filled with the dielectric and partially filled with air. There are two possible (easy) solutions to this problem. Draw your result, indicating the location of all surfaces.

2) For the geometry you created, indicated the **all** boundary conditions that apply for both $\vec{E}$ and $\vec{D}$.