Fields and Waves I

Name _______________ ECSE-2100 Spring 2002 Section ____________

Homework 4
Due 7 March 2002

Problem 1 (10 points) Capacitance and Energy

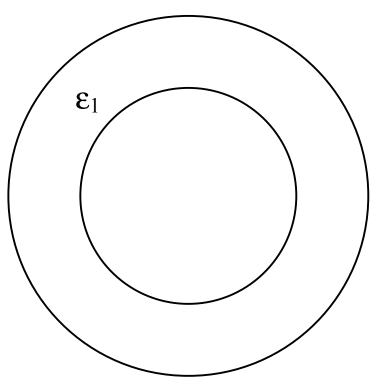
When solving electric field problems analytically, there are only a few geometries that have exact solutions. Typically, they are:

1. An infinite plane of charge
2. An infinite slab of charge where the density is only a function of the coordinate perpendicular to the slab
3. A cylindrical shell of charge
4. An infinite cylinder of charge where density is only a function or *r*
5. A spherical shell or charge
6. A sphere of charge where density is only a function of *r*
7. A point charge

We can use superposition to construct more complicated geometries with these shapes. A capacitor is essentially an application of superposition with two surface charges. The charges are deposited on the surface of the conductors and the conductors are at different potentials. (Note, many of you assume a conductor that is grounded has zero charge. This is definitely not the case.) The geometries that can be used to create a simple capacitor are:

1. Two parallel planar surfaces
2. Two concentric cylindrical surfaces
3. Two concentric spherical surfaces
4. Two parallel wires

The last case does not have a symmetric field like the other three, however, we only need to know the field on one field line between the conductors in order to determine capacitance. We can add a dielectric material between the conductors in order to change the capacitance. Again, in order to keep the problem simple enough to be analytic, we must make the surfaces of the dielectric either perpendicular or parallel to the field lines. Problems 4.52-54 in our textbook are capacitor problems. The solution to 4.53 may be found on Fawwaz Ulaby's web page. Dr. Connor and Dr. Salon's notes provide a detailed solution for the coaxial (cylindrical) capacitor with two dielectric materials between the conductors. (This problem is identical to the prep assignment for week 6). Your homework will consider a spherical capacitor, with inner conductor of radius *a* and outer conductor of radius *b*. The outer conductor is grounded.



a. What type of charge distribution will exist on the conductors, (point, line, surface, volume)?

b. How can we characterize that distribution? That is, what do we know about it?

c. Apply Gauss's Law to determine the field in the region between the conductors. What is the field outside this region?

d. What is the voltage as a function of position everywhere? Assume the outer conductor is grounded. Plot your result.

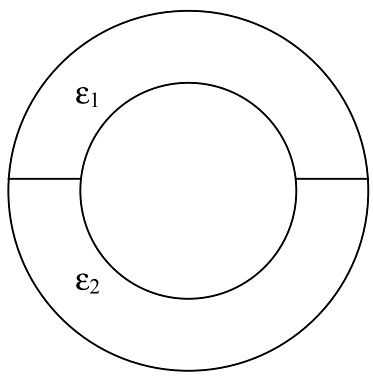
e. As we have discussed, we don't completely know the charge distribution, but we do know the voltage difference between the conductors. Assume a voltage difference of V_o. Determine the charge density in terms of voltage and geometry.

f. What is the capacitance?

g. What is the stored energy of the capacitor?

h. Confirm this result by checking the stored energy using another method.

i. Using the same geometry, we fill the capacitor with two dielectrics such that the volume of each dielectric is the same. That is, each fills half of the space between the conductors.



j. Maintaining the same voltage difference as in part e, is E larger, smaller or the same in each region, compared to the E for the single dielectric case? What boundary condition must be enforced at the dielectric/dielectric boundary?

k. Determine the capacitance of this geometry.

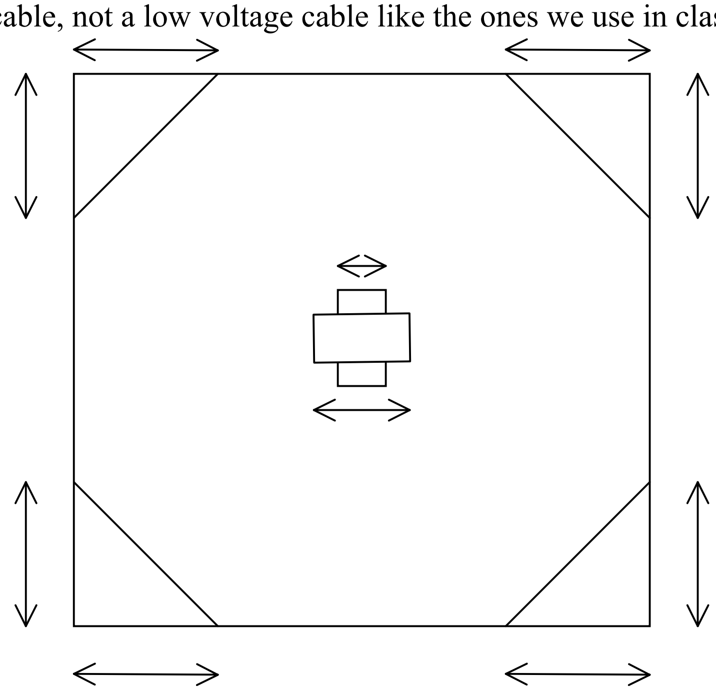
l. If we let the radius a grow very large and keep the distance *(b-a)* constant, approximate the capacitance. What type of geometry does this approach?

Problem 2 – (10 points) Numerical Methods

Nearly all practical electromagnetics problems are analyzed using numerical methods rather than the analytical methods we have been addressing. One of the most common techniques is called the Finite Difference Method, which we have considered in class. In this problem, we will try to find the capacitance of a simple two-dimensional problem with a somewhat odd geometry. To address this geometry, we will use a spreadsheet to solve Laplace's equation. Review the materials on numerical methods in the class notes and on reserve in the library.

The method is reasonably simple. First, identify the cells you will use to represent the conductors. Set the value in these cells equal to the potential on the conductor. Second, all exterior cells must have either a specific potential or be set equal to their nearest interior neighbor. This is the equivalent to setting the normal derivative of the potential equal to zero. This boundary condition is quite accurate if the boundary in question is a line of symmetry for the problem. Third, for all interior points, set the voltage in each cell equal to the average of its four nearest neighbors. This is the finite difference equivalent of Laplace's equation. After you enable iteration, the spreadsheet values will eventually converge to something near the correct voltages at each location. Use this information to determine the capacitance per unit length of this configuration, following the method we discussed in class. For your solution, use a 33 x 33 array. Whatever information you use from your spreadsheet has to be included with your analysis.

The geometry you are to consider is shown below. This is a more accurate representation of the coaxial cable than we used in class. The dimensions chosen are meant to represent a high voltage coaxial cable, not a low voltage cable like the ones we use in class.



The outside box is 32mm x32mm. The arrows giving the dimensions of the corner conductors are 10mm in length. The center conductor has dimensions of 2mm and 4mm. The space between the conductors is empty (free space). Use the spreadsheet method to find the potential distribution of this problem (you can assume any voltage on the inner conductor (say 100 volts) and that the outer conductor is grounded. The table can be laid out as shown. Note that the table is 33 cells wide, which represents 32mm, since the distance from cell 1 to cell 33 is 32.

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To summarize – solve for the potential at all points represented by cells and use this information to determine the capacitance per unit length of this structure following the procedure we used in class.

Next, we would like to show that it is not necessary to solve for the entire structure. Find the lines of symmetry for this structure, then set up a new spreadsheet that incorporates only ¼ of the geometry (e.g. a 17 cell x 17 cell spreadsheet), where the boundaries are now either conductors or lines of symmetry. Show that you get the same potential structure and that the charge per unit length is ¼ of the charge per unit length of the entire structure and, thus, the total charge per unit length and capacitance per unit length is the same as you obtained above. Make sure that you use the correct boundary conditions on the symmetry lines.

Extra Credit
Finally, add a small dielectric region to each corner, as shown. Assume that the dielectric constant of this region is 20 (chosen to produce a large effect). Set up the spreadsheet analysis for this case and then find the capacitance per unit length of this odd cable. You can use either the whole structure of the ¼ structure for this part.