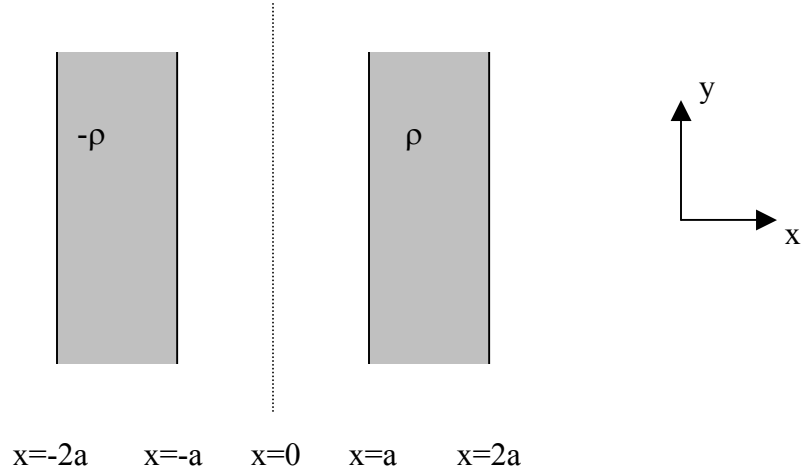


Homework 4
 Due March 6, 2003

1) Gauss's Law



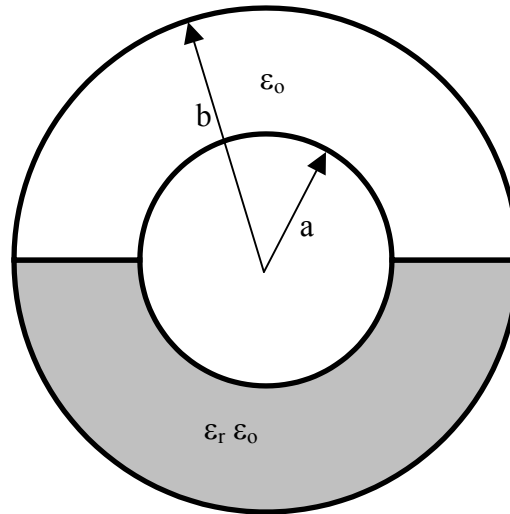
In the above figure, two parallel slabs of uniform charge are shown. They are infinite in the y and z directions. (They are shown as finite since I would run out of paper. Use your imagination.) The charge distribution is specified as:

$$\rho(x, y, z) = \begin{cases} \rho_v = -\rho_o & [C/m^3] & -2a < x < -a \\ \rho_v = \rho_o & [C/m^3] & a < x < 2a \\ 0 & & \text{elsewhere} \end{cases}$$

This geometry is similar to that seen in Homework 3, however, the charge distribution for $x < 0$, is now negative. Therefore, you need to make new observations about the location of a Gaussian surface.

- 1) Determine the electric field, \vec{E} , and displacement field, \vec{D} , everywhere. Show all your work. I advise that you follow the steps used in Homework 3.
- 2) Verify that your solution is correct using the differential form of both of Maxwell's that apply to electrostatics.
- 3) Indicate a reasonable location for ground. Why did you pick that location? Determine the voltage as a function of position for the range $-2a < x < 2a$. Verify that this solution is correct by determining the electric field, \vec{E} , using the differential relationship between field and voltage.

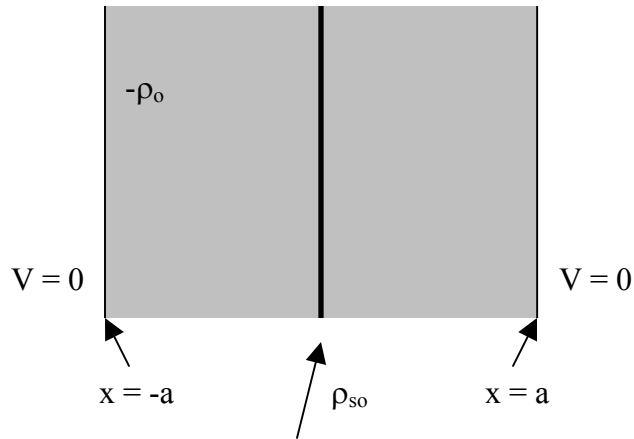
2) Laplace's Equation, Boundary Conditions, Capacitance



The above figure represents a spherical capacitor, half filled with a dielectric material. The relative permittivity is $\epsilon_r = 3$. The two conductors are located at $r = a$ and $r = b$. The inner conductor has voltage, $V = V_0$, and the outer conductor has voltage, $V = 0$.

- 1) Using Laplace's equation, determine the voltage as a function of position in the region between the conductors. What is the voltage for the regions $r < a$ and $r > b$?
- 2) Determine the electric field, \vec{E} , and displacement field, \vec{D} , everywhere.
- 3) In terms of the known voltage and the geometry, determine the charge density, ρ_s , that exists on each conductor.
- 4) At each boundary (the surface between materials), determine the continuity boundary condition that applies. In other words, what must be true about the relationship between the field on one side of the boundary relative to the field on the other side? You should have answer part of this problem previously.
- 5) Using your results from part 3, determine the total charge, Q , on the inner conductor. What is the capacitance, C , of this geometry?
- 6) Using your field results from part 2, determine the total stored energy, W_E , of this geometry.
- 7) Using your result from part 6, verify that your solution in part 5 is correct.

3) Poisson's Equation



In the above figure, a volume charge density, ρ_o , exists in the region $-a < x < a$. Additionally, a surface charge, ρ_{so} , exists on the plane $x = 0$. The geometry is infinite in the y- and z-directions.

1) Determine the voltage as a function of position.

4) Approximate Capacitance



The above figure represents the cross-section of a type of coaxial capacitor. Not all capacitors are circular. There is an inner conductor and outer conductor, and the geometry is infinite perpendicular to the paper. The "straight" part of the capacitor has parallel conductors. The ends are concentric, and the radius of curvature is indicated in the figure.

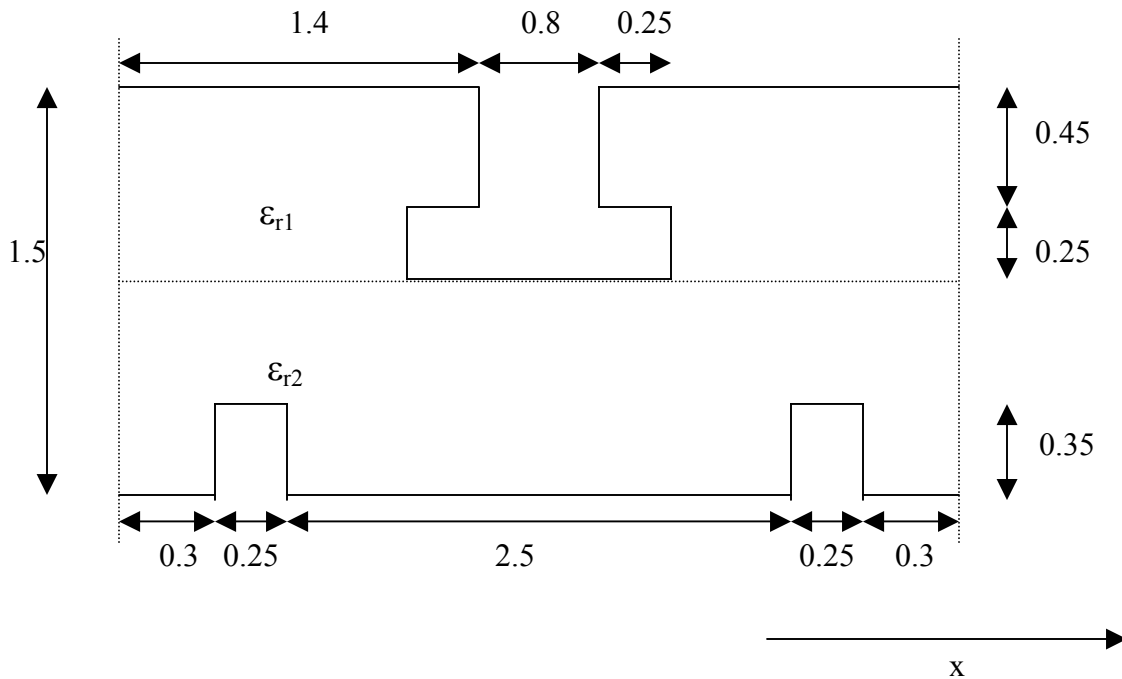
1) Using any method you consider reasonable, analytically estimate the capacitance per unit length of this geometry.

(Hint: Assume the charge density on a conductor is uniform.)

(Bigger Hint: When I look at the geometry, I see 4 capacitors in parallel.)

Extra Credit: Using the Finite Difference Method, determine the capacitance of the geometry and compare it with your analytic result. You may arbitrarily designate one of the conductors as ground and the other as 100 [V]. Since you are determining capacitance, the result should not be dependent on the voltage difference between the conductors. If you make the grid resolution sufficiently dense, the finite difference method will be significantly more accurate. Why?

5) Finite Difference Methods



The above figure can be interpreted as a (admittedly strange) microchip structure. You need to determine the capacitance. The “top plate” is $V = 5$ [V] and the “bottom plate” is grounded. The dimensions given are in microns. There is a 5 micron thickness, the z -dimension into the paper. The horizontal dashed line represents a boundary between dielectric materials, with $\epsilon_{r1} = 2.3$ and $\epsilon_{r2} = 4.4$ respectively. The vertical dashed lines are a division in the periodic structure. This geometry repeats in the x -direction. Therefore, the potential on the dashed lines is identical for the left and right line.

1) Use Excel to create a finite difference solution for the Laplace’s equation. Determine the capacitance per period of this structure.