Name _______________ ECSE-2100 Fall 2002 Section ____________

Homework 5

Due Thursday 31, October, 2002

1) Resistance

Determine the resistance of a hollow copper sphere. The inner surface is located at $r = a$ and the outer surface is located at $r = b$. Assume current flows from the outer surface to the inner surface and that it is uniform on an arbitrary cross-section perpendicular to current flow.

2) Ampere's Law

The following current distributions can be used to approximate a coaxial cable transmission line, where $a < b$

$$\vec{J}(r,\phi,z) = \begin{cases} J_o r^3 \hat{z} & r < a \quad [A/m^2] \\ -J_s \hat{z} & r = b \quad [A/m] \end{cases}$$

A current density exists in the region $r < a$ and a surface current density exists at the location $r = b$. Furthermore, the total current in the $\hat{z}$ direction is equal and opposite to the total current in the $-\hat{z}$ direction. You may assume the entire domain is free space.

1) What are the regions of the problem?

2) What coordinate system applies to this problem (this better be easy).

3) What direction are the field lines? What direction is the current?

4) Given the answer to part 3, what surface would be appropriate to apply Ampere's Law? What would is $d\vec{S}$? What is $d\vec{l}$ that bounds the surface?

5) Set up, (do not solve) the integral, $\oint_l \vec{H} \bullet d\vec{l} = \iint_S \vec{J} \bullet d\vec{S}$. Indicate the limits and differentials.

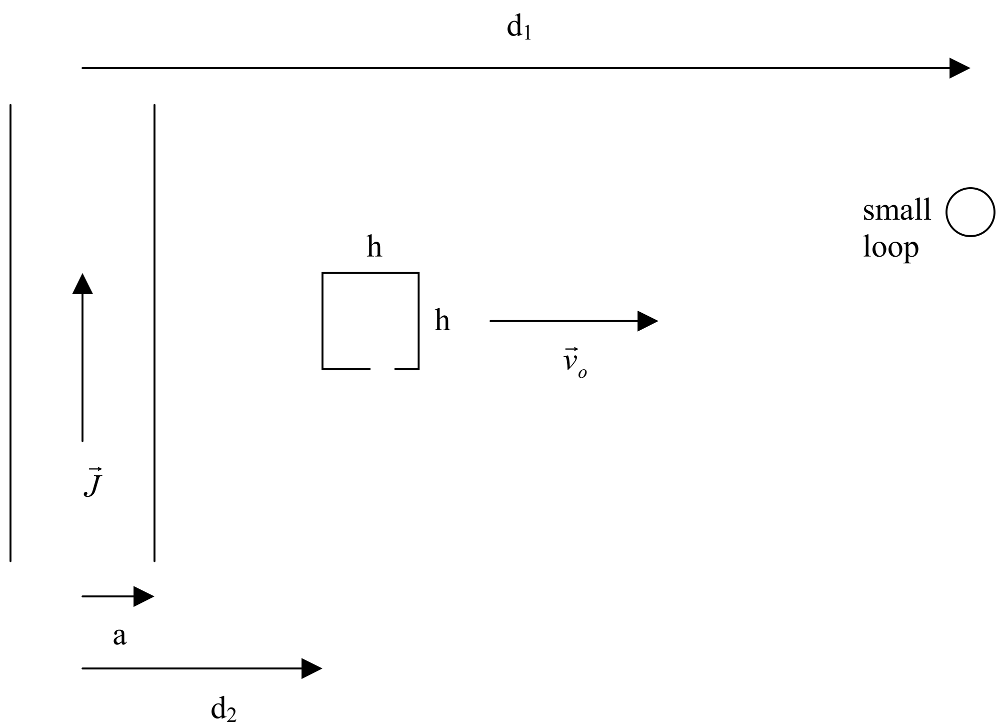
6) For each region, determine the magnetic field. Remember, when you are considering an arbitrary surface inside a given region, the current passing through that surface includes any regions <u>inside</u> the region you are currently considering. Specifically, pay attention to where the current exists. When you have finished determining the field, your solution should include all information: the field in that region, the units, and the specification of the region (similar to the format of the charge distribution given at the top of the page).

7) Verify your solution is correct for each region by using the differential form of Maxwell's Laws as applied to electrostatics, $\nabla \times \vec{H} = \vec{J}$ and $\nabla \bullet \vec{B} = 0$.

3) Ampere's Law

Repeat all the steps in problem 2 for the following current distribution

$\vec{J}(r,\phi,z) = J_o r\hat{\phi} \qquad r < a \quad [A/m^2]$



$\vec{J}(r,\phi,z) = J_o r\,\hat{z} \qquad r < a \quad [A/m^2]$

The above figure represents a lengthwise view of a current density in a wire with radius a.

4) Approximations

If $d_1 >> a$, what is the total field $\vec{B}$ passing through the small circular loop on the right in the above figure? The center of the loop is a distance d_1 from the origin and the loop has radius b. The surface of the loop is in the same plane as the wire.

5) Faraday's Law

A square loop is placed a distance from the wire and oriented such that the wire lays in the same plane as the surface of the loop. The loop moves a constant velocity away from the wire. Determine the EMF induced across the leads of the loop for $t > 0$. At $t = 0$, the left edge of the loop is a distance d_2 from the origin.