Fields and Waves I

Name _______________ ECSE-2100 Spring 2000 Section ____

Preparation Assignments for Homework #5

Due at the start of class.

These assignments will only be accepted from students attending class.

Reading Assignments

Please see the handouts for each lesson for the reading assignments.

23 February Lesson 3.5 Inductance

1. Write the formulas for calculating the magnetic flux in terms of the magnetic flux density $\vec{B}$ and the magnetic vector potential $\vec{A}$.

2. The small inductor we usually use in the studio ($L = 1mH$) is made by winding N wires around a plastic cylinder. If the radius of the cylinder is *a* and the length of the cylinder is *l*, write the expressions for the magnetic flux density $\vec{B}$ and the magnetic vector potential $\vec{A}$ that are correct for the plastic region ($r < a$).

3. Determine the magnetic flux in the plastic region ($r < a$).

4. What is the total flux in the plastic region linked by this solenoid? This is the flux that will be used to determine the external inductance of the solenoid. The external inductance is due to the flux outside of the wires. The internal inductance is due to the flux inside the wires (inside the copper).

Class time 24,25 February

Open shop to work on Homework 5. If there is room, you are free to attend any of the open shop sessions. Due at 5 pm on 25 February.

28 February Lesson 3.6

1. Read over problem 1 of lesson 3.6 in which a toroidal core is added around a long straight cylindrical wire. From the information provided, determine the additional inductance caused by adding the toroid. Don't try to figure out the inductance per unit length of the wire since it will be infinite. Rather, figure out the inductance of the core region, with and without the core material and take the difference. To answer this question, you can use the results of example 4.14 and assume that there is only one turn of wire ($N = 1$).

2. Read over example 4.13. Write out the general expressions for the boundary conditions used in this problem.

3. Search the web and find a diagram of the earth's magnetic field that shows the direction of the magnetic field lines. Print out the diagram and give the url where you found it.

4. Here are two pictures of cow magnets. What are they used for? Where might you purchase them?


Homework #5

Problem 1. Standard Self Inductances (10 Points)

There are a small number of simple conductor/magnetic material configurations for which we can relatively easily find the inductance by analytic determination of magnetic fields. There is also a large class of configurations that can be analyzed by a technique known as *Magnetic Circuits*, which will address after the second quiz. Most of the standard configurations can also be analyzed using magnetic circuits, but we will focus here only on those that do not need this latter technique. Students of electromagnetics should be sure that they know how to derive and use all of the expressions for such inductors. The list of simple inductors that can be analyzed without magnetic circuits should include the following:

1. Solenoid – example 4.8 and problem 4.8.3. Chap VII 22-24 of notes.
2. Solenoid with more than one magnetic material
3. Torus – example 4.14 (square cross-section), problem 4.4.4 and 4.8.4 (circular cross-section). *A torus can have any cross-sectional shape.*
4. Torus with more than one magnetic material – example 4.21 (Magnetic Circuits)
5. Two-wire transmission line – see section 7.5.1 (one wire above a ground plane is also similar).
6. Coaxial Cable – see examples 4.16 and 4.20. Chap VIII 11-12 of notes.
7. Coaxial Cable with more than one magnetic material (can be done with Magnetic Circuits)
8. Parallel Plate Inductor – see parallel plate transmission line and Chap IX 20-24 of notes (this discussion does not relate directly to inductance, but rather develops the criteria for the quasi-static approximation). The parallel plate inductor configuration is also thoroughly discussed in Appendix II of the notes.
9. Parallel Plate Capacitor only partially filled with magnetic material or with more than one magnetic material
10. Parallel plate transmission line – see sections 7.1.1 and 7.5.1

There are also some inductances that you should know, even if you cannot totally derive them from first principles. Most of these can be found at http://emcsun.ece.umr.edu/new-induct/ from the Electromagnetic Compatibility Laboratory at the University of Missouri-Rolla. Variations of other standard inductance configurations relevant to printed circuit board applications are also included.

1. Single turn circular loop of wire or several turns of wire forming a single loop.
2. Single turn square and triangular loops of wire
3. Two parallel traces on a circuit board

You should notice that there is no inductance formula for a single long-straight wire. That is because the inductance of such a wire is infinite. Also, there is no such thing in practice as a single long-straight wire, since there always has to be a return path for the current.

a. The magnetic field of a long-straight cylindrical wire of radius $r = a$ is analyzed in example 4.6. Show that the external inductance per unit length of this wire is infinite by first determining the flux per unit length and then find the inductance.

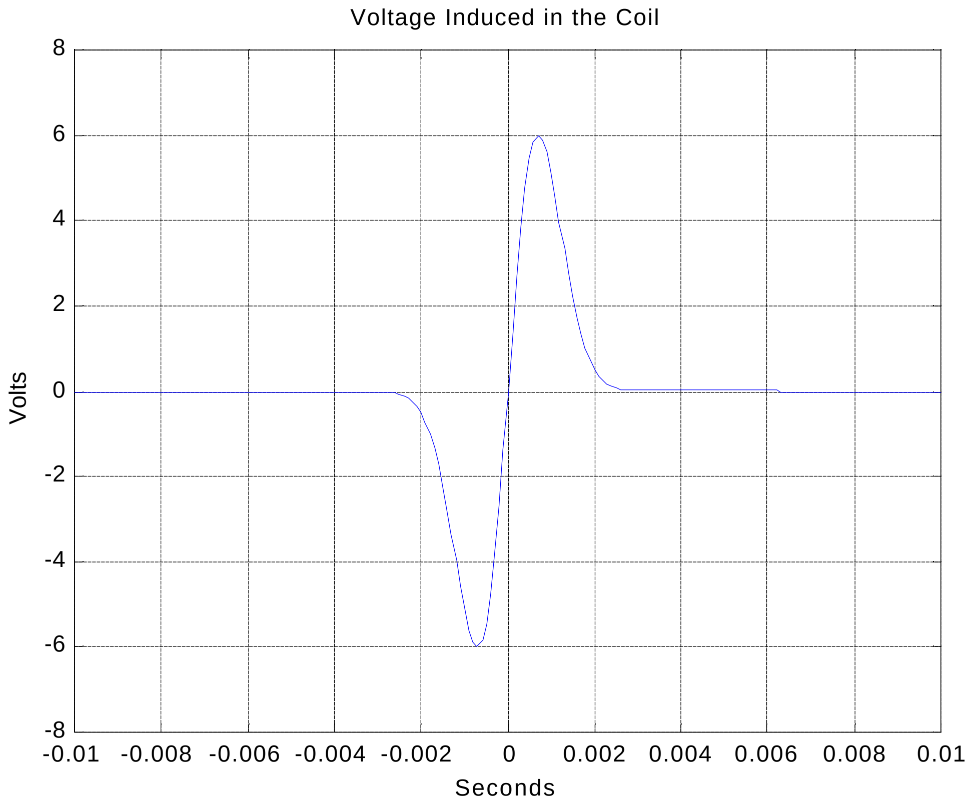
b. The inductance of a circular cross-section torus is determined in problems 4.4.4 and 4.8.4. An approximate expression for such a torus can also be found (when $a >> b$) by using the value of the magnetic flux density in the center of the field region ($x = a, y = 0$) and multiplying it by the cross-sectional area to find the flux. Use this approximate method to find the self inductance of the torus in problem 4.4.4. Show that your answer is the same as is given in the back of the text when $b << a$.

c. Using the expression for a single circular loop of wire from the University of Missouri-Rolla, find the inductance of a seven turn coil with a mean diameter of 4.3 cm. Assume that you are using 22 gauge wire. Also, determine the resistance of the coil if it is made from copper.

d. In addition to being able to find the self inductance of these basic geometries, you should also be able to find the mutual inductance between two similar configurations (e.g. a second wire wrapped around a solenoid or a torus). In example 4.15, two methods are used to determine the mutual inductance between two coaxial, filamentary conducting loops. Repeat the calculation by first finding the flux produced by coil 1that links coil 2 using the standard expression $y_m = \int \vec{B} \cdot d\vec{S}$. Then find the mutual inductance.

Problem 2. Faraday's Law (10 Points)

A small permanent magnet is dropped through a coil like the one analyzed in part c of the previous problem (with 30 turns of wire and a radius of 1 cm). The voltage measured across the terminals of the coil looks like the following figure as a function of time. A good approximation to this voltage is given by the expression $V(t) = 10^4 te^{-(1000t)^2}$ volts.



a. Using the simple relationship between the induced voltage and the flux linked by the coil, $V(t) = -\frac{d\Lambda}{dt}$, determine the magnetic flux linked by the coil $\Lambda(t)$ as a function of time during the entire period of the measurement. Plot your result.

b. Determine the average flux density $B_{axial}(t)$ of the magnet as a function of time. (The coil sensing the magnetic field is coaxial with the magnet axis, so you will be determining the average flux density in the axial direction.) Plot your result. *Hint: Remember the difference between the flux in a cross-sectional area* ψ_m *and the total flux linked by the circuit* Λ .

c. Assuming that the peak radial magnetic flux density of the magnet seen by the coil is about 43% of your answer to part b, determine the velocity of the magnet as it passes through the coil. *Note: There are several ways to analyze the voltage signal from the coil. In parts a and b, you have been using the relationship between magnetic flux and magnetic flux density. Because the coil is oriented with its axis in the same direction that the magnet moves, this analysis uses the axial component of the magnetic field. Since all the components of the magnetic field are related through Maxwell's equations, which in a region with no currents are the very simple expressions* $\nabla \cdot \vec{B} = 0$ *and* $\nabla \times \vec{H} = 0$, *we can also analyze the induced voltage by looking at the radial component of the field. Using the expression for the induced voltage (aka emf)* $V(t) = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$ *and the information you have been given about the peak radial magnetic field, you should be* *able to figure out the velocity of the magnet. Be sure that you use the entire length of the coil.*

d. Using your velocity from part c, determine the average axial directed magnetic field as a function of axial position. That is, if the magnet is assumed to be falling in the z-direction, find $B_z(z)$. Plot your result.

e. The magnetic field outside of the magnet (which of course is the only region of use to us and where the coil is) can generally be modeled quite accurately as a magnetic dipole field. (See equation 69 on page 220 of the text.) The earth is also a simple permanent magnet, so it too can be modeled with as a dipole, with reasonable accuracy. Using the field lines shown in the figure found at the Encyclopedia Britannica web page

http://www.britannica.com/bcom/eb/article/single_image/0,5716,3138+asmbly%5Fid,00.html

explain the general shape of the induced voltage plot. *Hint: Print out a copy of this plot and then show the path of the wire through the field.*

f. The voltage sensed by the coil is produced by an electric field. What is the direction of this electric field relative to the coil axis?

Note: An experiment like this will be demonstrated in class. If you have the time, you should try it yourself.