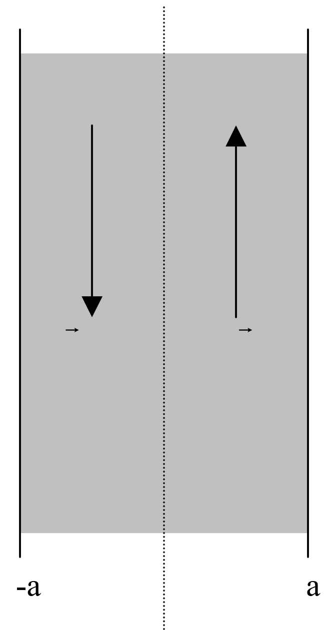
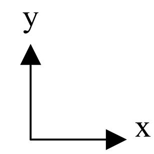
Homework 6

Due Thursday 14, November 2002

1) Ampere's Law





The above figure represents an infinite slab of current. It continues in the *y*-direction and *z*-direction. The current density is $\vec{J} = J_o x\,\hat{y}$ [A/m^2]. When applying Ampere's Law to determine the magnetic field in this problem, consider what symmetry applies.

1) What are the regions of the problem?

2) What coordinate system applies to this problem (this better be easy).

3) What direction are the field lines? What direction is the current?

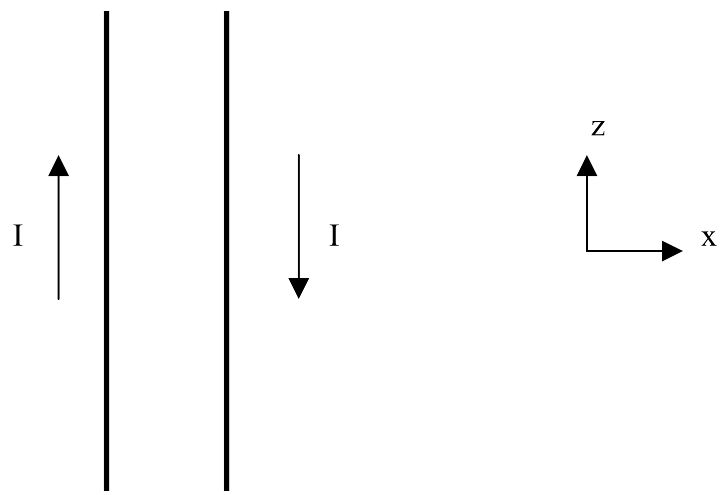
4) Given the answer to part 3, what surface would be appropriate to apply Ampere's Law? What would is $d\vec{S}$? What is $d\vec{l}$ that bounds the surface?

5) Set up, (do not solve) the integral, $\oint_l \vec{H} \bullet d\vec{l} = \iint_S \vec{J} \bullet d\vec{S}$. Indicate the limits and differentials.

6) For each region, determine the magnetic field. Remember, when you are considering an arbitrary surface inside a given region, the current passing through that surface includes any regions inside the region you are currently considering. Specifically, pay attention to where the current exists. When you have finished determining the field, your solution should include all information: the field in that region, the units, and the specification of the region (similar to the format of the charge distribution given at the top of the page).

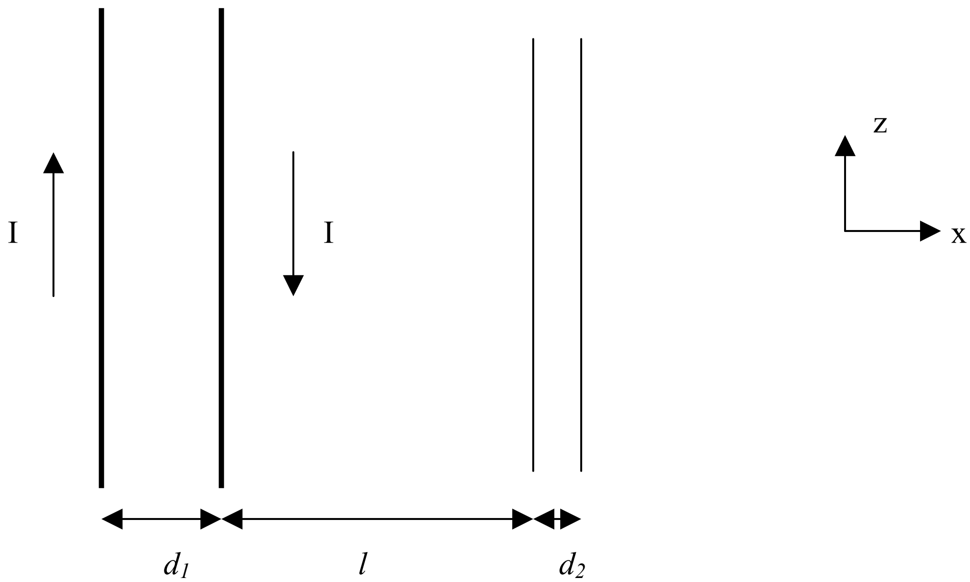
7) Verify your solution is correct for each region by using the differential form of Maxwell's Laws as applied to electrostatics, $\nabla \times \vec{H} = \vec{J}$ and $\nabla \bullet \vec{B} = 0$.

2) Self-Inductance



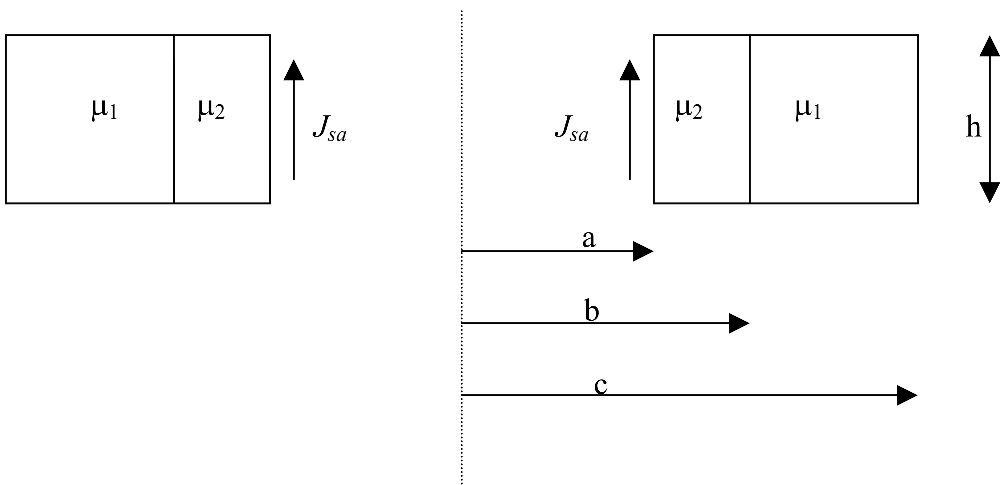
A two-wire transmission line is shown above, with the wires continuing in the z-direction. The wires have radius *a* and are separated by distance *d*. Determine the self-inductance per unit length for this transmission line. You may ignore any field that is inside the wires.

3) Mutual-Inductance



A second two-wire transmission line is added to the geometry in problem 2. The line is oriented such that all the wires lie in the same plane. Determine the mutual inductance L_{21}.

4) Energy and Inductance

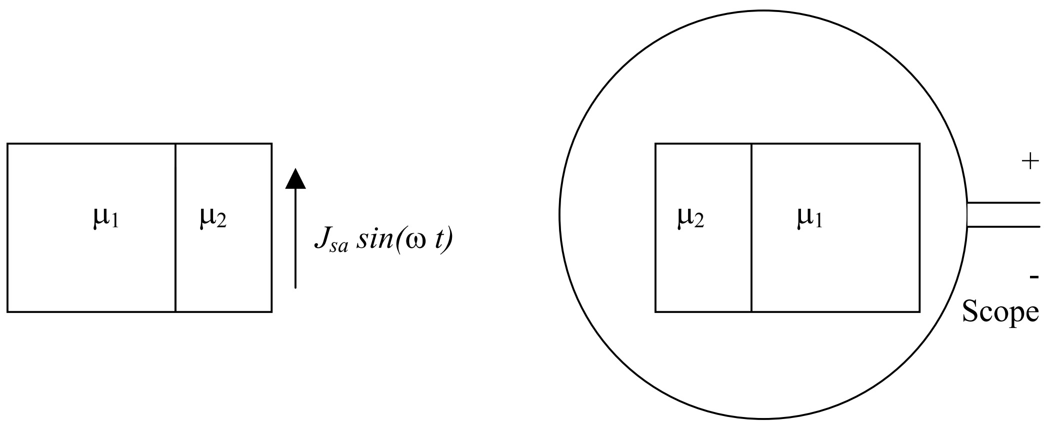


A cross-section of a toroid is shown in the above figure. Instead of wires wrapping around the surface, a surface current exists. The surface current on the inner surface is shown. You should recognize that the surface current density changes on the other sides of the rectangle. The toroid itself consists of two high permeability materials.

What is the self-inductance of the toroid?

Determine the total magnetic energy using the relationship between energy and magnetic field.

Verify that this answer is consistent with your self-inductance calculation.



A loop is placed around the toroid as shown in the above figure. The positive and negative connections indicate the polarity of the oscilloscope reading. In other words, if the current is flowing in the clockwise direction, the oscilloscope will indicate positive voltage. Counter-clockwise current would indicate negative voltage.

The current is sinusoidal. Plot the total current, the total flux and the EMF measured on the oscilloscope.

Extra Credit:

Problem 6.9 in the book. The solution is given in the back of the book. To receive any credit for this problem, you must show your work.