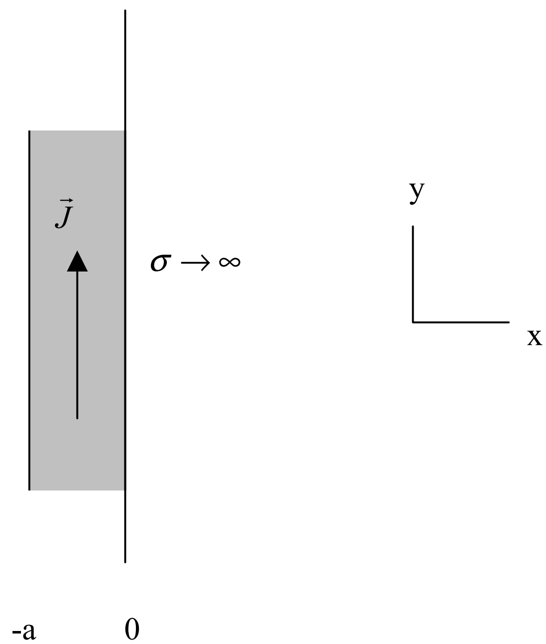
Homework 6

Due Thursday April 4, 2002

1) Ampere's Law



A slab of current, $J = J_o(x+a)\hat{y}$ [A/m^2] for $-a < x < 0$ exists near a conductor with infinite conductivity. A surface current is present on the conductor such that it is equal and opposite to the total current of the slab.

What is the direction of the magnetic field in all regions?

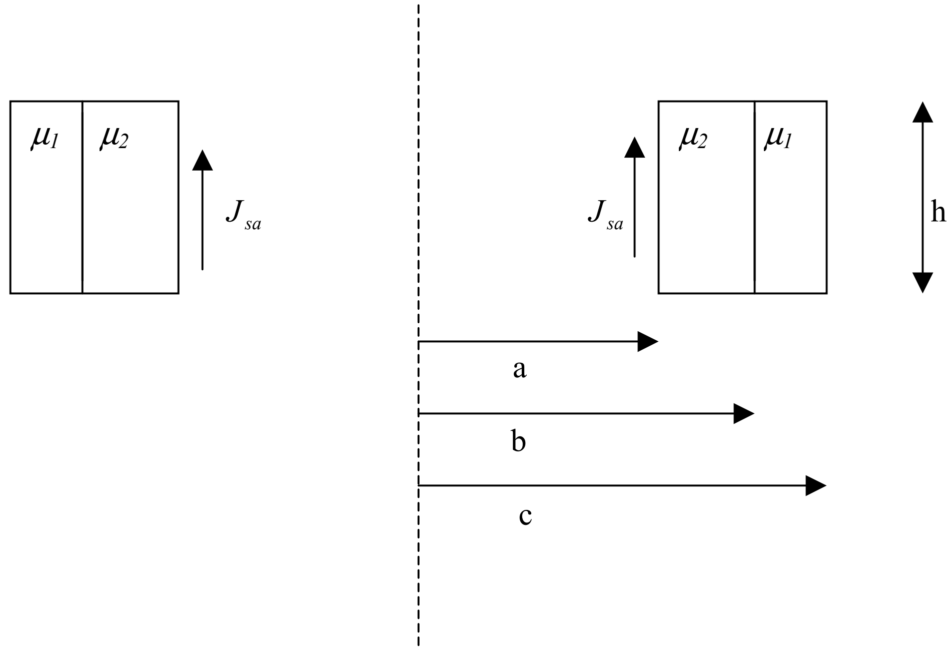
What is the surface current density?

Draw a surface that you would use to apply Ampere's Law.

Determine the magnetic field everywhere.

What boundary condition applies to this geometry?

2) Energy and Inductance



The cross-section of a toroid is shown in the figure. The core consists of two different magnetic materials.

What is the direction of the magnetic field?

What boundary conditions apply in the geometry?

The surface current on the inner radius $r = a$ is given. What is the surface current on the other three sides? Include direction in your answer. *Hint: your answer is not necessarily a constant on all surfaces.*

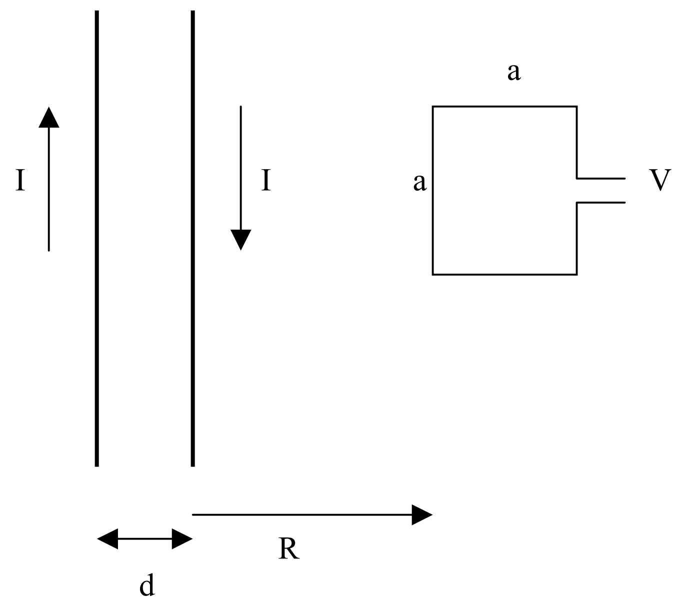
Draw a surface you would use to apply Ampere's Law. *Hint: you may want to draw some other views of the torus.*

What is the magnetic field inside the toroid? That is, what is it inside the magnetic material?

What is the stored energy of this inductor?

What is the self-inductance?

3) Faraday's Law



A two wire transmission line is shown in the figure. We want to use the rectangular loop as a pulse detector. Assume the current in the line is *I(t)*.

What is the total flux through the rectangular loop? Assume that the loop has N turns.

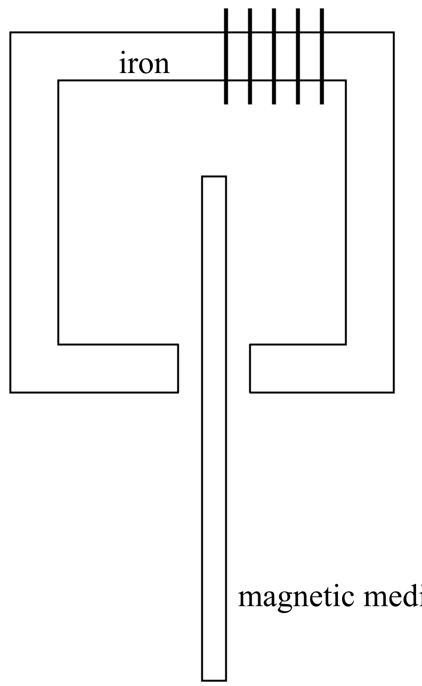
What is the mutual inductance, L_{21}?

What *EMF* will be measured in the loop?

A surge occurs in the line and it appears to be a short square pulse. Plot the flux through the loop during the pulse and the *EMF*.

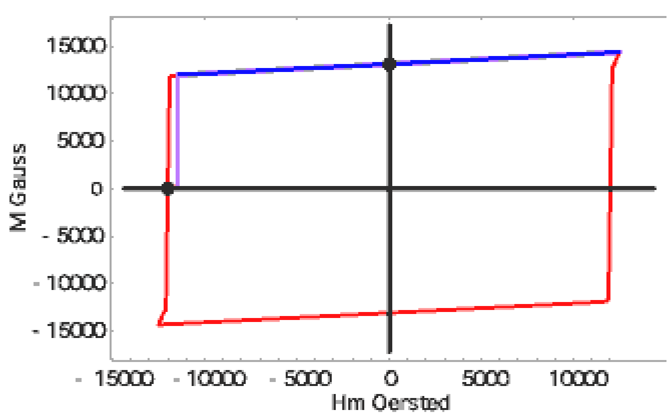
Extra Credit: Using equipment in the studio, plot the *EMF* for a square pulse. You must completely describe your experiment so that we can properly assess your answer.

4) Magnetic Circuits

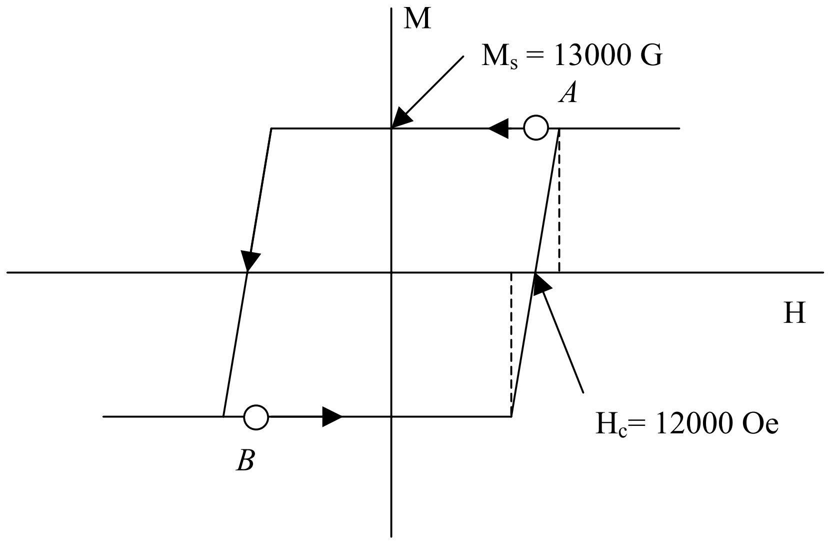


The figure illustrates a crude magnetic storage method. The current through the loop induces a magnetic field that passes around the core and through the magnetic media. If the field is strong enough, the media will be magnetized in the same direction as the applied field. The direction will determine the '1' or '0' state of the bit.

The magnetic media's response to the field is dependent on the current direction of the magnetization. This type of response is called hysteresis. A typical hysteresis curve is shown below. The x-axis is ***H***, measured in Oersteds (1Oe = 79.5 A/m). The y-axis is ***M***, measured in Gauss (1G = 1E-4 Tesla).



For approximations we can use an idealized hysteresis curve where the saturation lines are horizontal. In other words, a limit is placed on the magnetization.



Movement on the loop is in the counter-clockwise direction. So, reducing *H* below zero when the material has +*M* magnetization won't change the direction of the grains until a certain threshold is reached. The slopes on the sides are exaggerated but are useful for approximating the behavior of the magnetization as the applied magnetic field is increased or decreased. Note, you cannot move backwards on the curve. For example if you were moving up the 'right side' and turned off the field before you saturated the material, the magnetization would follow a hysteresis curve that was 'inside' that shown in the figure. You can use this property to demagnetize a material by gradually reducing the field, spiraling in toward $M = 0$.

The saturation magnetization is shown in the figure. Another common characteristic is the coercitivity, the level of applied field when the magnetization is zero in the direction of the field. The linear equations for the sides may be approximated as

Left: $M = 6000H + 72E6$ [G]
Right: $M = 6000H - 72E6$ [G]

In the region where the magnetization is changing, we are interested in the differential permeability, which can be obtained by $\mu_d = \frac{\partial M}{\partial H}\mu_o$. Locally, we can use the approximation $B = \mu_d H = 6000\mu_o H$

Simple question: using magnetic circuits, determine the inductance of this configuration. Assume the core has sides of length, *l*, and cross-sectional area, *A*, an air gap exists on both sides of the magnetic material, *g*, and the material has thickness, *t*.

Extra credit: what *MMF* is necessary to change the direction of the magnetization of the material? Assume that the magnetic media is magnetized at position *A* and that you want to change it to position *B*. Both of these points are marked with circles on the hysteresis curve.