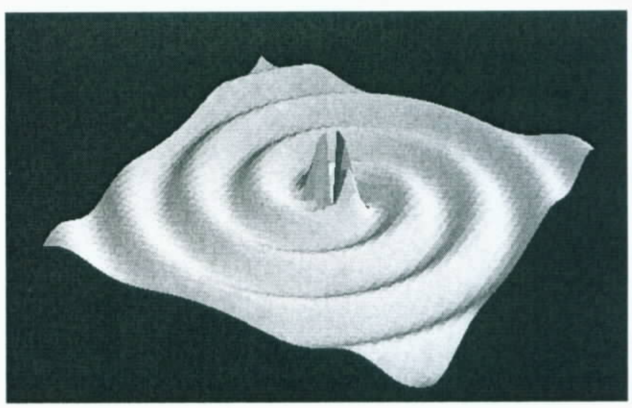
Due Wednesday 30 April 2008 at 6:00 PM (End of day on Wednesday)

Solution

Homework 8

1. **Notation** (1 Pt) _______
2. **Beakman's DC Motor** (2 Pts) _______
3. **Transients on Trans. Lines** (4 Pts) _______
4. **Standing Waves** (3 Pts) _______
5. **Inductance Sensor** (.5 Pt) _______
6. **Reluctance** (.5 Pt) _______
7. **Boundary Conditions** (1 Pt) _______
8. **Capacitance & Inductance** (4 Pts) _______
9. **Plane Waves** (2 Pts) _______
10. **Oblique Incidence** (2 Pts) _______

Total (20 Points) _______



1. **Notation (1 Pt)**

In the following table, identify the symbol with its name, as in the two examples.

	Symbol	Letter for Symbol	Name
a	L	p	Capacitance
b	$\vec{E}$	i	Permeability of Free Space
c	$\vec{J}$	b	Electric Field Intensity
d	$\vec{D}$	c	Volume Current Density
e	ε_o	t	Surface Charge Density
f	Z_o	g	Index of Refraction
g	n	o	Magnetic Flux Linkage
h	$\vec{H}$	l	Conductivity
i	μ_o	d	Electric Flux Density
j	V or ϕ	k	Intrinsic Impedance
k	η	s	Wave Propagation Constant
l	σ	w	Magnetic Vector Potential
m	$\vec{B}$	h	Magnetic Field Intensity
n	Γ	u	Wave Decay Constant
o	Λ	m	Magnetic Flux Density
p	C	j	Electric Scalar Potential
q	ρ_v	a	Inductance
r	R	q	Volume Charge Density
s	β	f	Characteristic Impedance
t	ρ_S	r	Resistance
u	α	v	Surface Current Density
v	$\vec{J}_S$	e	Permittivity of Free Space
w	$\vec{A}$	n	Reflection Coefficient

2. Beakman's DC Motor (2 Pts)

There are actually 2 answers, but only one for the assumed initial position of the coil.

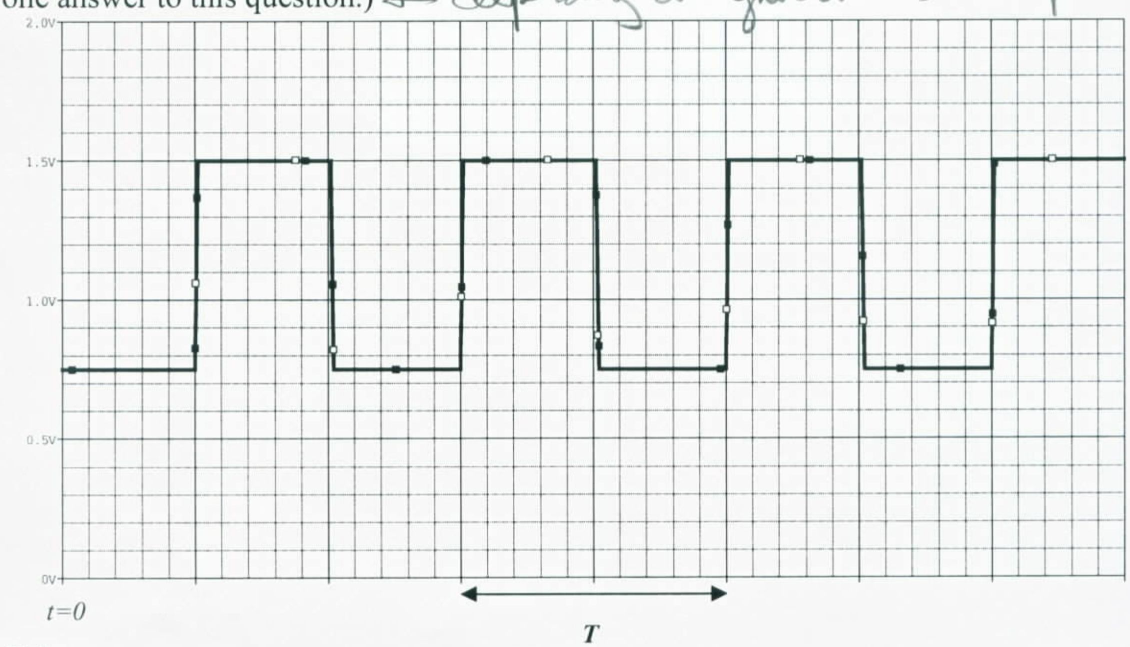
Magnet

Magnet

force at t=0 ⇒ low voltage
No force
No force
force cancels be this end.

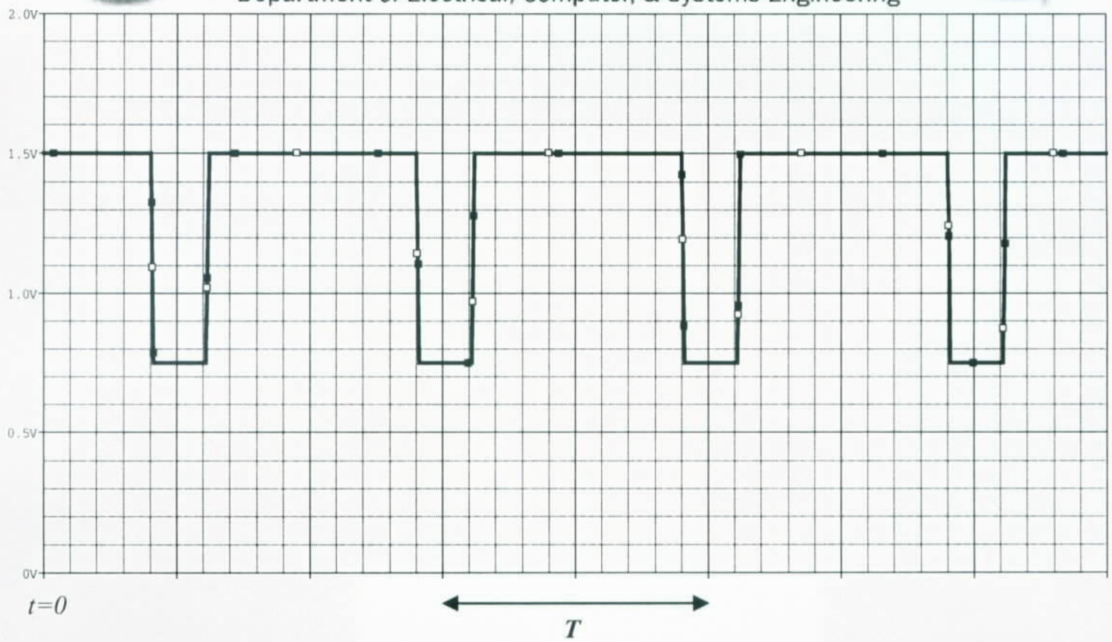
Position shown is for $t = 0$

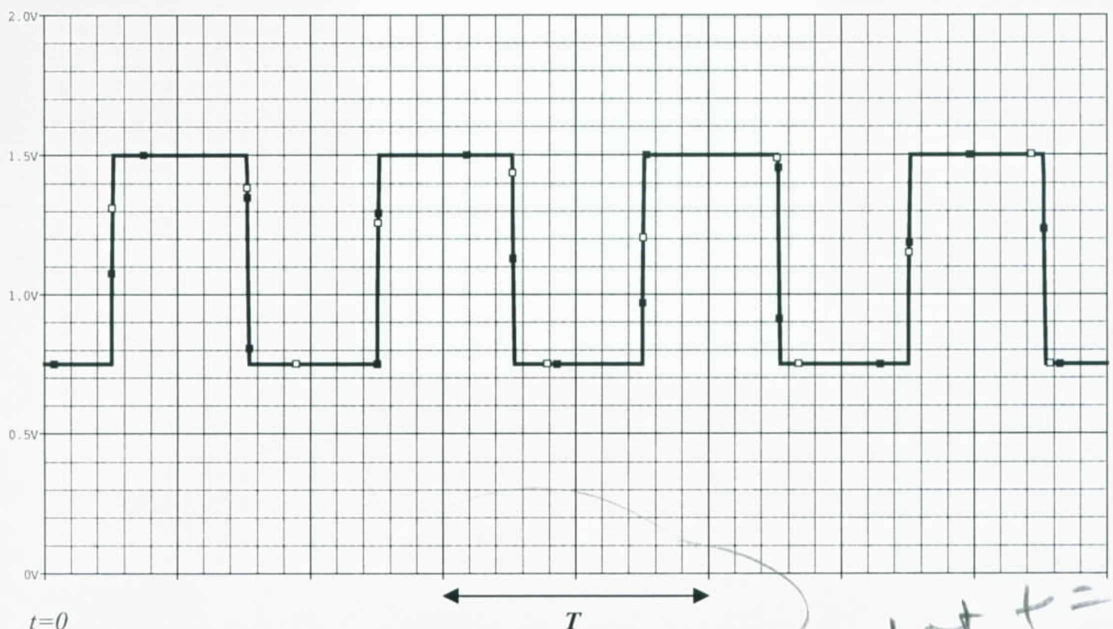
A simplified version of a Beakman's Motor coil (in this case rectangular) is shown at its position at time $t=0$. One of the key design issues in making this coil is how much and where the insulating enamel is removed from one of the pieces of wire that make up the axle of the coil. Assume that all of the enamel has been effectively removed from the other part of the axle. Also assume that the coil rotates counter-clockwise with a frequency ω as shown in the figure. Four experiments are done with the coil for which everything is exactly the same (balance, number and size of windings, resistance, inductance, etc) <u>except</u> for how the enamel is removed from the one side. Sometimes the sanding is not complete so that some residue of the insulation remains or some oxidation occurs so that the contact is not perfect; sometimes the enamel is only removed from a third or less of the wire rather than half; sometimes the enamel is removed from different sides of the wire (e.g. top, left, right, bottom). The following voltages are observed across the coil (with the usual setup for measuring the motor speed). Note that the time scale is not shown; only the period of rotation is indicated. For which of the following will the motor most likely be turning the fastest? Be sure to explain your answer. (There is only one answer to this question.) ← depending on your initial assumption



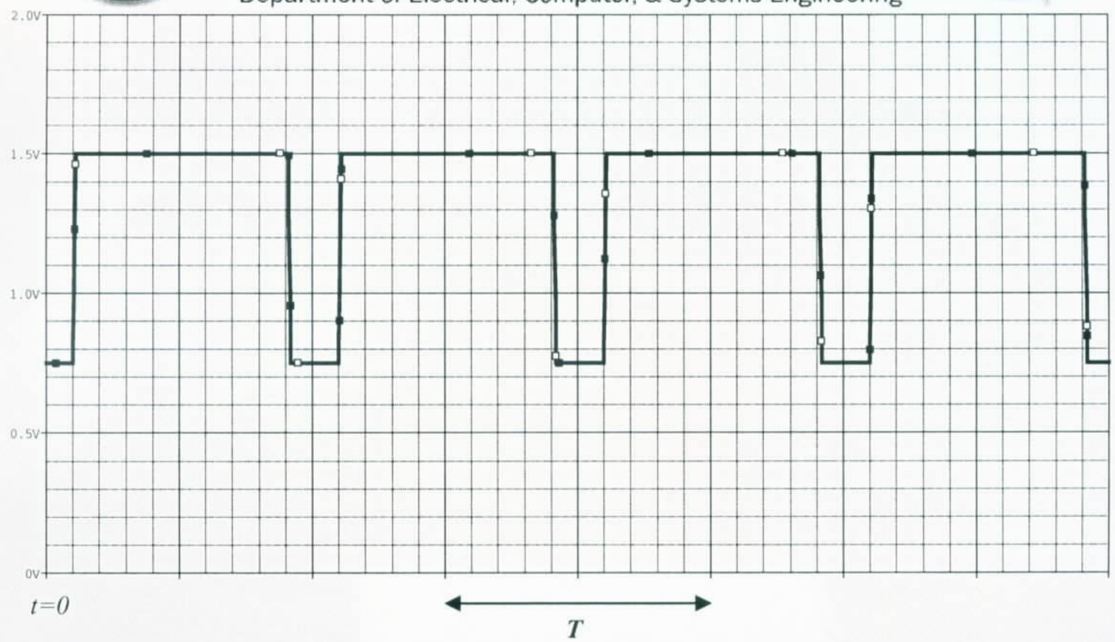
$t=0$

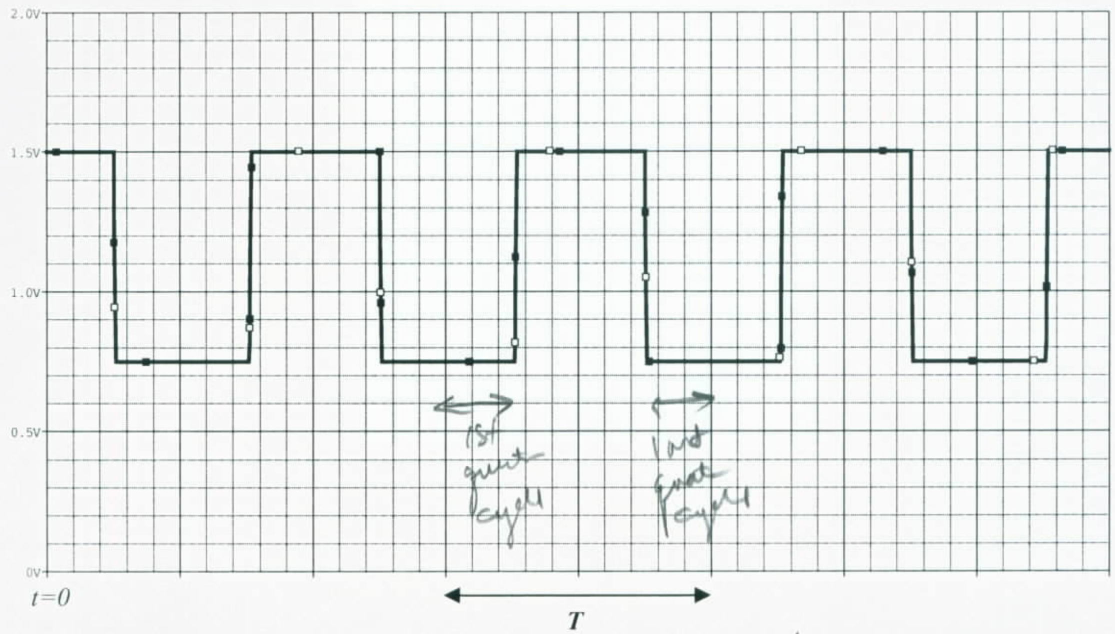
T





If not connected at t=0



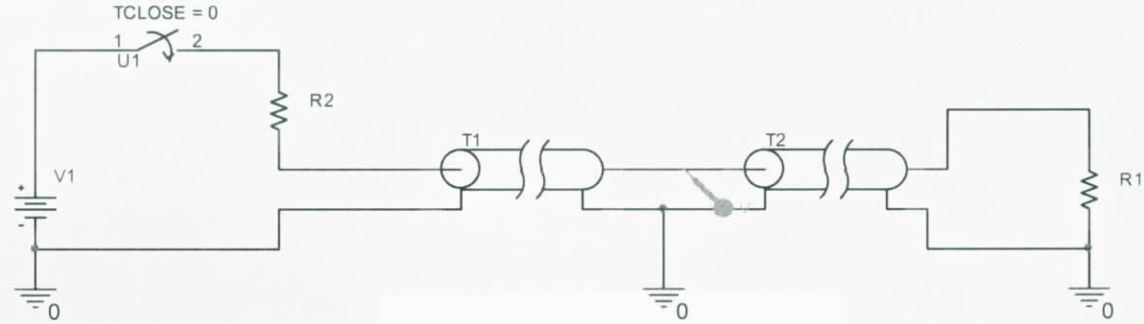


If connected at t = 0

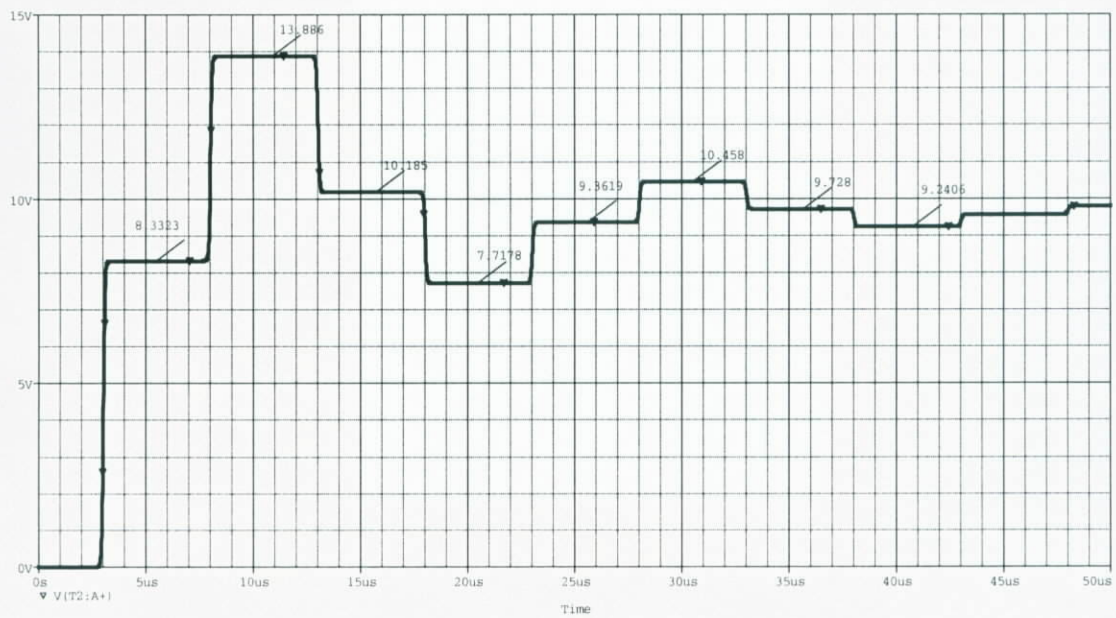
only one answer for this assumption

3. **Transients on Transmission Lines (4 Pts)**

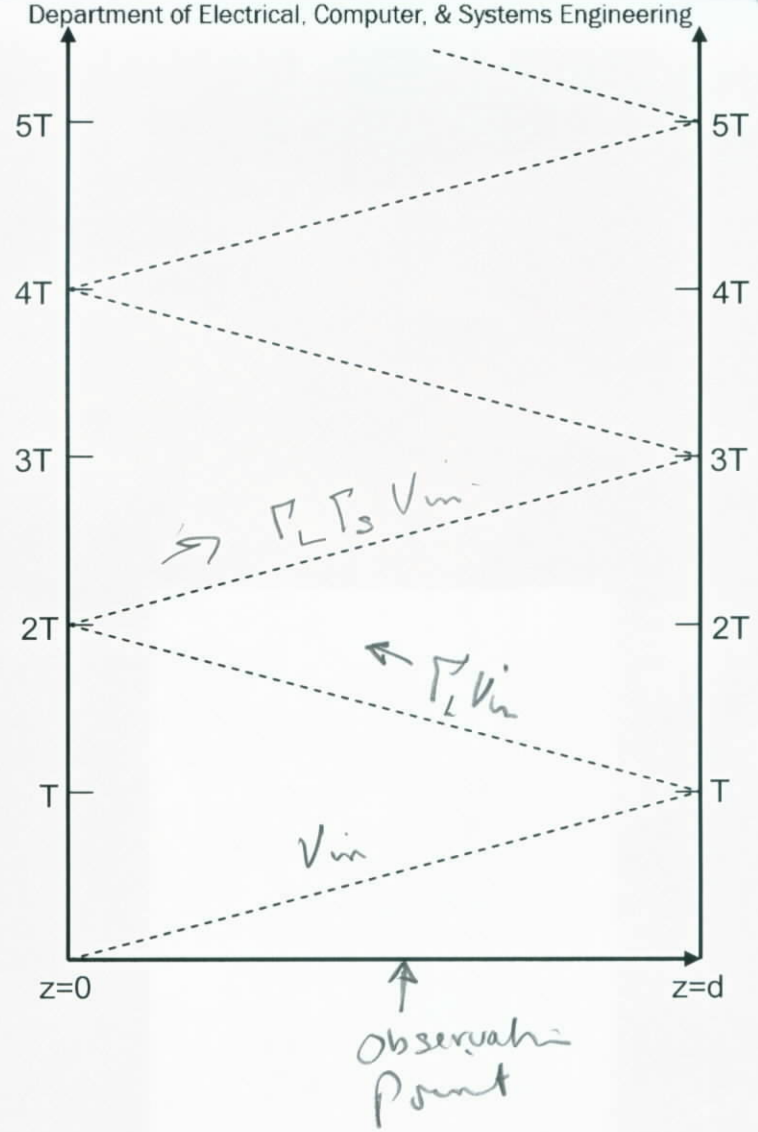
Two identical transmission lines in series are connected to a DC voltage source at time $t=0$. The source voltage, the source resistance and the load resistance are all unknown. The characteristic impedances of the two lines are both 50 Ohms and their propagation velocities are both $u = 2x10^8 m/s$. The lengths of the lines are also unknown except that they are also identical.



The voltage is observed at the point between the two lines is shown below.



The voltages observed are: 8.33V, 13.87V, 10.19V, 7.72V, 9.36V, 10.46V, 9.73V, 9.24V, … Generate a generic lattice diagram for this combination of two lines (not each line separately) and use the information in your diagram along with the measured voltages to determine the four unknowns: the source voltage, the source resistance, the load resistance and the length of each line.



2×10^{8} m/s

$= 12.5$ V

$V_S = 8.33 \frac{75}{50}$

V source $\quad V_{in} = V_S \frac{50}{50+R_S} = 8.33$

R source $\quad \Gamma_L = \frac{R_L-50}{R_L+50} = \frac{13.866-8.333}{8.33} = 2/3 \qquad R_L = 200$

R load $\quad \Gamma_S = \frac{R_S-50}{R_S+50} = -\frac{13.86-10.165}{13.86-8.333} = -2/3$

Length of line

$R_S = 25$

Each line $= (2\times10^{8})(3\times10^{-6}) = 600$ m

4. **Standing Wave Patterns for Lossless Transmission Lines (3 Pts)**

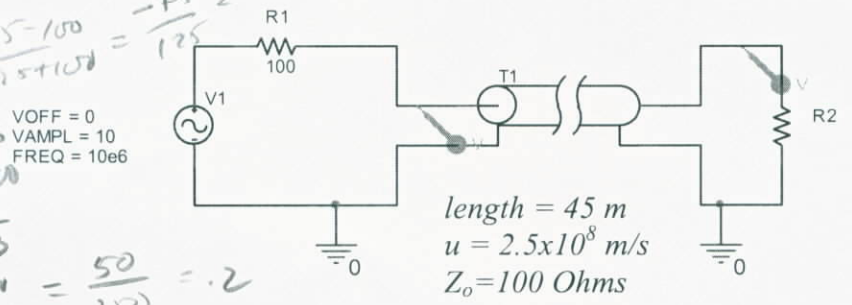
A sinusoidal voltage source is connected to a lossless transmission line as shown below. All parameters except for the load resistance are labeled including the source voltage, source resistance, characteristic impedance, velocity of propagation, and the length of the line.

$Z_L = 25 \quad \Gamma = \frac{25-100}{25+100} = \frac{-75}{125} = -.6$

$Z_L = 50 \quad \Gamma = \frac{50-100}{50+100} = -\frac{1}{3}$

$Z_L = 0 \quad \Gamma = -1$

$Z_L \to \infty \quad \Gamma = +1$



$Z_L = 150 \quad \Gamma = \frac{150-100}{150+100} = \frac{50}{250} = .2$

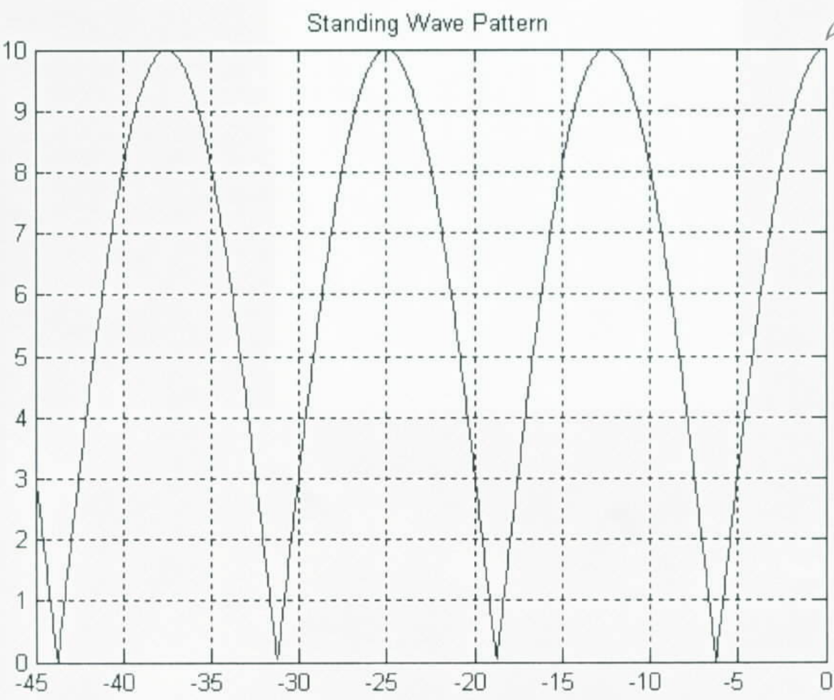
Six different loads are connected to the line resulting in the six different standing wave patterns shown below. The loads are $Z_L = 0\Omega$, $Z_L = 25\Omega$, $Z_L = 50\Omega$, $Z_L = 100\Omega$, $Z_L = 150\Omega$, $Z_L = 300\Omega$, and $Z_L = \infty$. Identify which standing wave pattern goes with which load. Be sure to explain your answer.

$Z_L = 300$

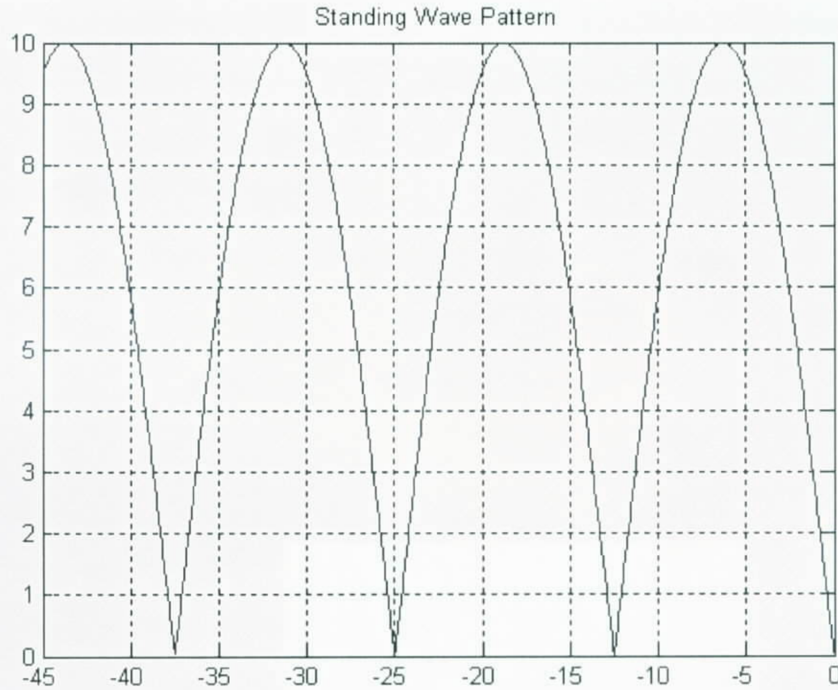
$\Gamma = \frac{300-100}{300+100} = \frac{1}{2}$

max at load

$\Rightarrow Z_L \to \infty$

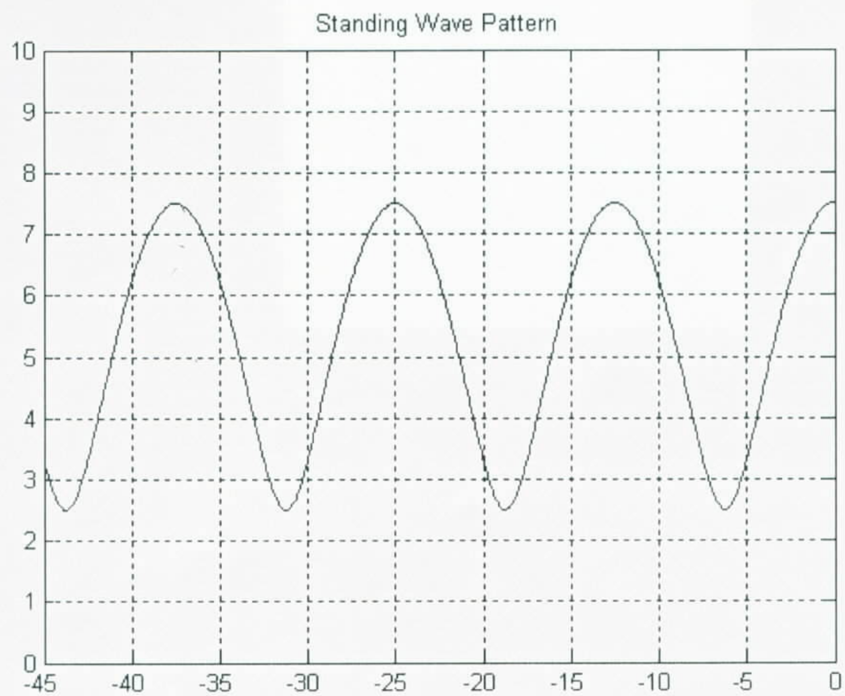


$SWR \to \infty \Rightarrow |\Gamma| = 1$



Min at load $\Rightarrow Z_L = 0$

$SWR = \infty = |\Gamma| = 1$



max at load

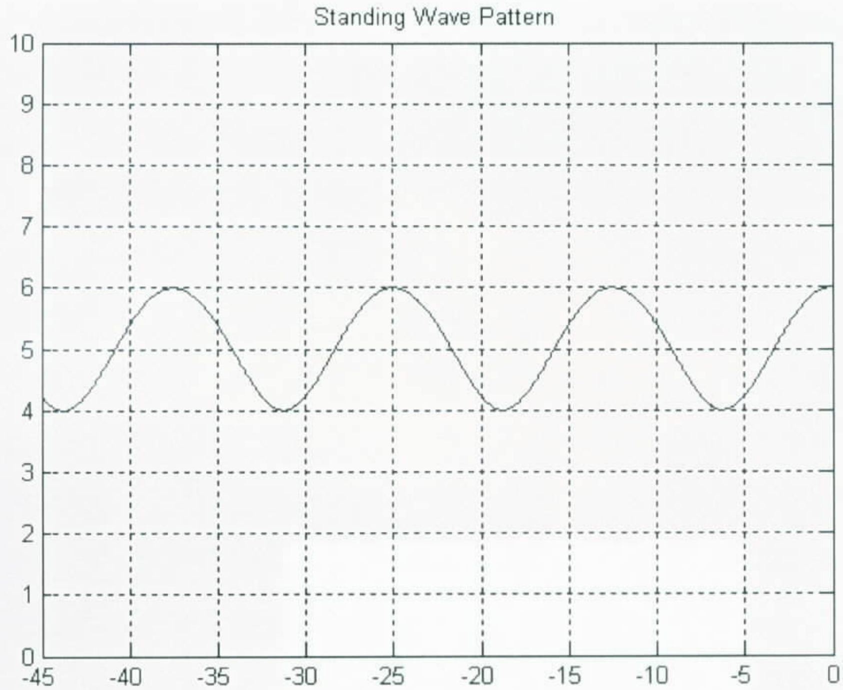
$\Rightarrow Z_L > Z_0$

SWR finite

$\Rightarrow Z_L$ finite

$\Gamma = \frac{2.5}{5} = .5$

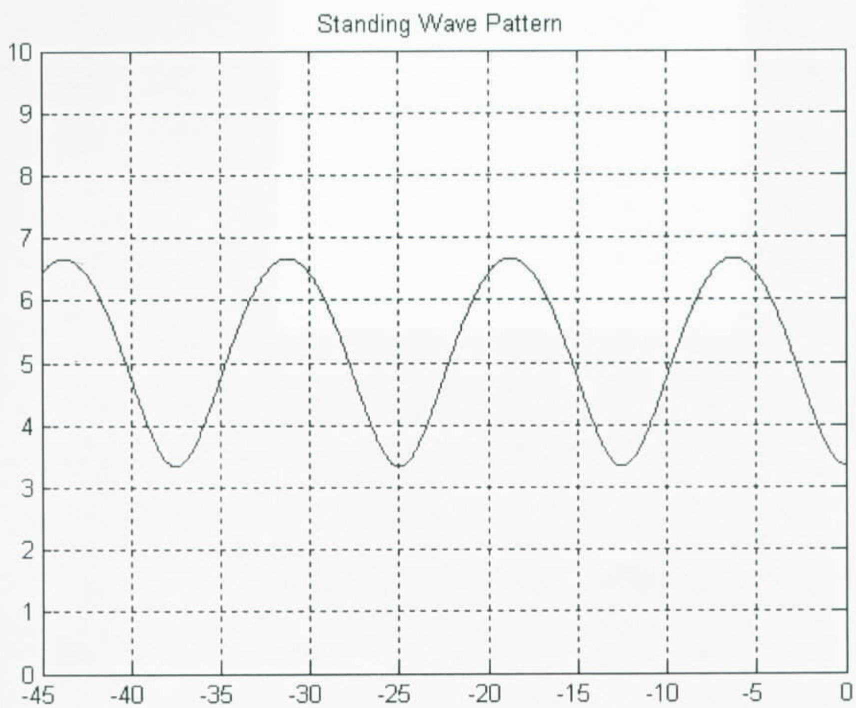
$\Rightarrow Z_L = 300\Omega$



Max at load $\Rightarrow Z_L > Z_0$

$\Gamma = .2$

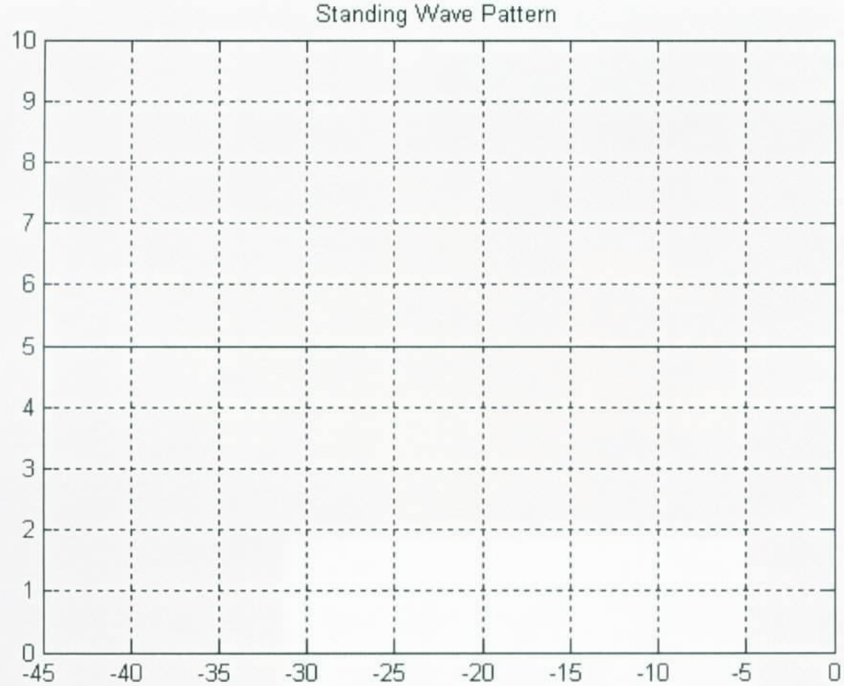
$\Rightarrow Z_L = 150\,\Omega$



Min at load $\Rightarrow Z_L < Z_0$

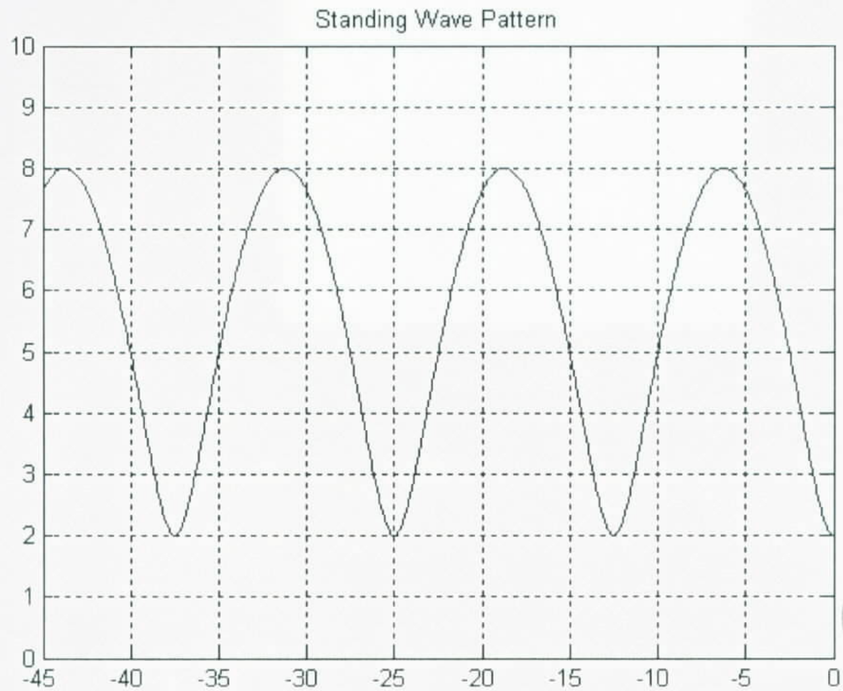
$\Gamma = -\frac{1\frac{2}{3}}{5} = -\frac{5/3}{5} = -1/3$

$\Rightarrow Z_L = 50$



Const ⇒ matched

⇒ $Z_L = 100$



Min at load

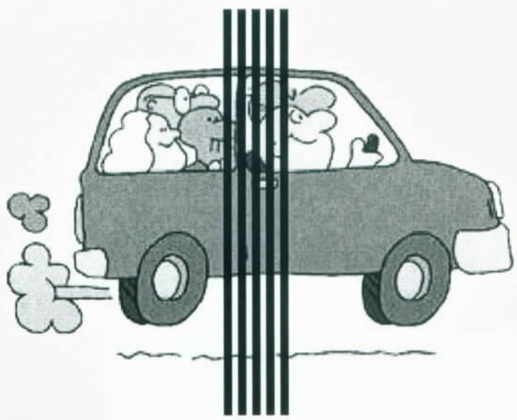
⇒ $Z_L < Z_0$

$\Gamma = -.6$

⇒ $Z_L = 25$

5. **Inductance Sensor (0.5 Pt)**

In an attempt to build a simple device to sense whether or not an automobile is passing through the entrance to a parking garage, a many turn coil is wrapped around the inside of the entrance door. When a car passes through the door, will the inductance of the coil increase, decrease or stay the same? Circle the correct answer and explain your choice.



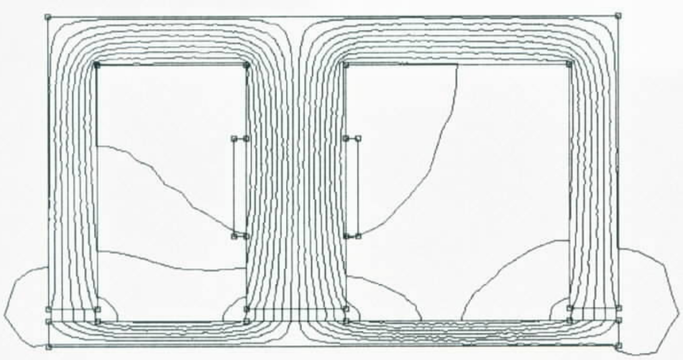
Car is part magnetic

$\mu > \mu_0$

$\Rightarrow L$ goes up

6. **Reluctance (0.5 Pt)**

The figure below shows a relatively accurate magnetic field configuration for a problem we have solved using the method of magnetic circuits. Using the calculated field, will the inductance of the configuration be smaller or larger than calculated using an ideal magnetic circuit? *Hint: consider the reluctance.* Explain your answer.



$R = \frac{l}{\mu A}$

because of spreading A goes up

$\Rightarrow R$ goes down

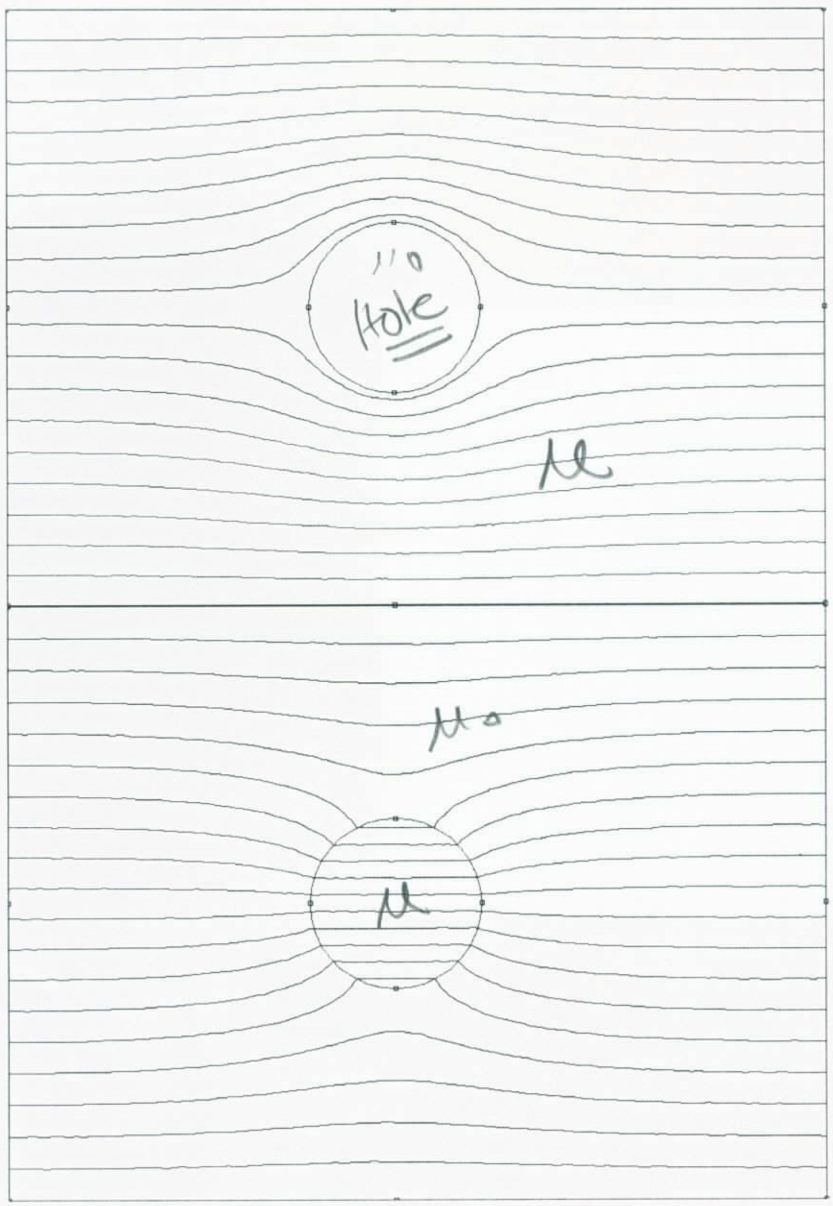
Note also that flux follows the path of least reluctance

$\Lambda = N\Phi = \frac{N^2 I}{R}$

$L = \frac{\Lambda}{I} = \frac{N^2}{R}$ R goes down $\Rightarrow$ L goes up

7. **Boundary Conditions (1 Pt)**

In the two figures shown below, a uniform magnetic field is modified by a circular region where the magnetic properties are different from the bulk material. For one case, the outer region has a very high permeability while the circular region is air and for the other case the outer region is air and the circular region has a high permeability.



$B_{n1} = B_{n2}$

$H_{t1} = H_{t2}$

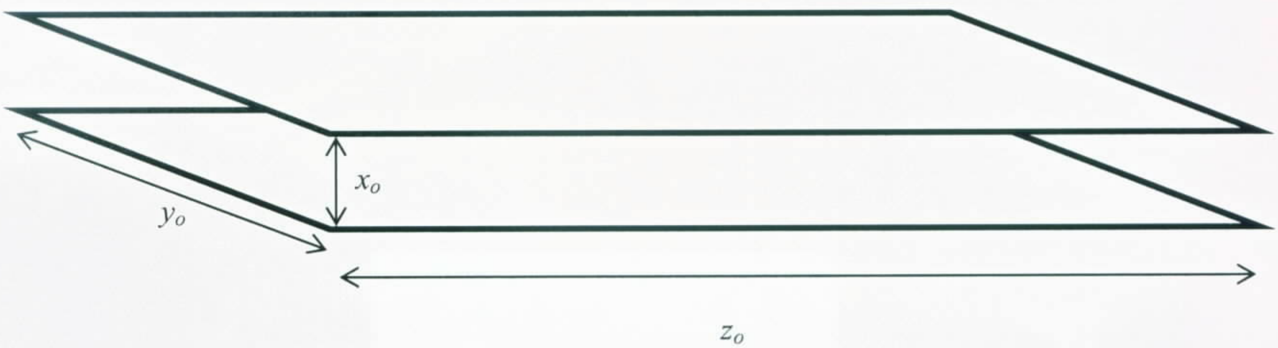
Flux follows the high permeability path.

That is, in one case we have a hole in a magnetic material and in the other we have a circular magnetic material surrounded by air. Identify which region is which by first writing down the boundary conditions for the magnetic field at the interface between a magnetic material and air and then using this information to answer the question.

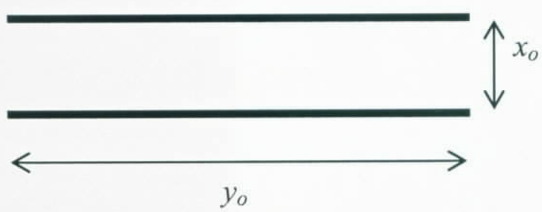
8. **Capacitance and Inductance (4 Pts)**

An *air insulated* parallel plate transmission line is configured as shown below. By connecting a resistor across the end of the line, a voltage source at the input will produce both a voltage difference between the plates and a current in the plates.

3D View

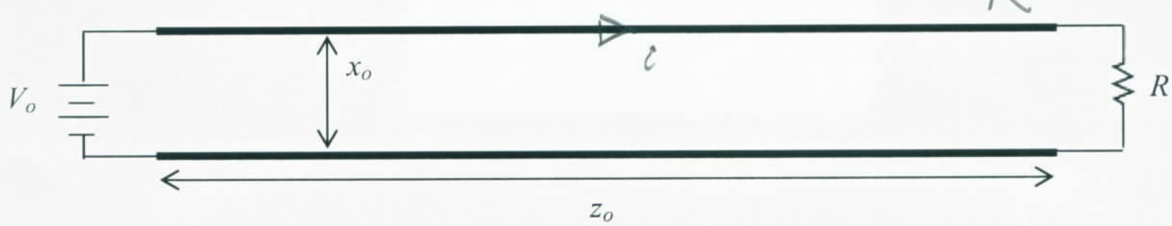


Cross Sectional View



Longitudinal View (as Transmission Line)

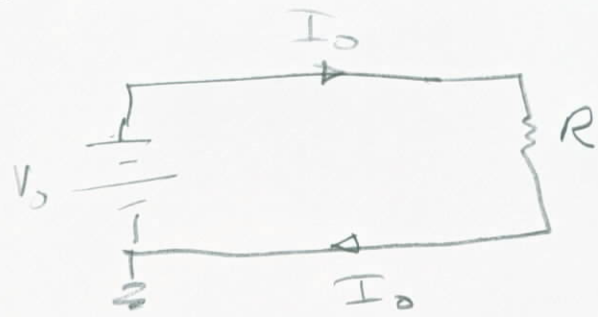
$i = \frac{V_o}{R}$



In this problem, you will be asked to find the electric and magnetic fields, charge and current distributions, total capacitance, total inductance, and energy stored for the parallel plate transmission line. Note that you are not to find the per-unit-length values but the totals for the entire line. You are given that the voltage on the top plate is V_o and the bottom plate is grounded. The voltages and currents are DC and do not change with time. The resistance connected across the output end of the line is R.

a. On the diagram above, indicate the direction of current flow. From the given information, determine the current.

$$I_o = \frac{V_o}{R}$$



b. Source of the Magnetic Field: Assuming that the plates are very thin (as they would be on a printed circuit board), the current can be represented as a surface current density. Determine the vector expression for the surface current in the top plate and the bottom plate. That is, find both the magnitude and direction of the surface current density.

$$J_s = \frac{I_o}{y_o}$$

$\vec{J}_s = \frac{I_o}{y_o}\hat{z}$ on the top

$\vec{J}_s = -\frac{I_o}{y_o}\hat{z}$ on the bottom

c. Source of the Electric Field: Assuming that the magnitude of the total charge on each of the plates is Q (you will be asked to determine the value of Q below), determine the sign and magnitude of the surface charge density on each plate.

$$\rho_s = \frac{Q}{\text{Area}} = \frac{Q}{y_o z_o}$$

d. Determine the electric field $\vec{E}(\vec{r})$ in the region between the plates using your answer to part c. Your answer should be expressed in terms of the charge distribution. Use your answer to find the total energy stored in the electric field of the transmission line.

$$E_x = -\frac{\rho_s}{\varepsilon_o} \qquad \vec{E} = -\hat{x}\frac{\rho_s}{\varepsilon_o}$$

$$W_E = \left(\frac{1}{2}\varepsilon_o E^2\right)(\text{Volume})$$

$$= \frac{1}{2}\varepsilon_o\left(\frac{\rho_s^2}{\varepsilon_o^2}\right)(x_o y_o z_o)$$

$$= \frac{1}{2}\frac{\rho_s^2}{\varepsilon_o}x_o y_o z_o = \frac{1}{2}\frac{Q^2}{(y_o z_o)^2}x_o y_o z_o$$

$$= \frac{1}{2}\frac{Q^2}{\varepsilon_o y_o z_o}x_o \qquad C = \frac{\varepsilon\,\text{Area}}{\text{separat.}} = \frac{\varepsilon_o y_o z_o}{x_o}$$

$$= \frac{1}{2}\frac{Q^2}{C}$$ ✓

e. Determine the voltage as a function of position in the region between the plates. That is, find the electric scalar potential $V(\vec{r})$. Your answer should be expressed in terms of the charge distribution.

$$V(x) = -\int_0^x \vec{E}\cdot d\vec{l} = \frac{\rho_s}{\varepsilon_o} x$$

f. From your answer to either part d or part e, find the total capacitance of the transmission line. Then find the surface charge density in terms of the given voltage V_o.

from d (already completed)

$$C = \frac{\varepsilon_o y_o z_o}{x_o}$$

from e $\quad V(x_o) = \frac{\rho_s}{\varepsilon_o} x_o = \frac{Q}{\varepsilon_o y_o z_o} x_o = V_o$

$$C = \frac{\varepsilon_o y_o z_o}{x_o}$$

$$\rho_s = \frac{V_o}{x_o}\varepsilon_o$$

g. Determine the magnetic field $\vec{H}(\vec{r})$ and the magnetic flux density $\vec{B}(\vec{r})$ in the region between the plates using your answer to part b. Your answer should be expressed in terms of the current. Use your answer to find the total energy stored in the magnetic field of the transmission line.

From the B.C. $\quad H_y = J_s = \frac{I_o}{y_o} = \frac{V_o}{R y_o}$

$$\vec{B} = \mu_o \vec{H} = \mu_o \frac{V_o}{R y_o}\hat{y}$$

$$W_M = \left(\frac{1}{2}\mu_o H^2\right)(\text{Volume}) = \frac{1}{2}\mu_o \frac{I_o^2}{y_o^2} x_o y_o z_o$$

$$= \frac{1}{2}\mu_o I_o^2 \frac{x_o z_o}{y_o}$$

h. From your answer to part g, find the total magnetic flux flowing through the region between the plates.

$$\text{Flux} = \Psi_m = \int \bar{B} \cdot d\bar{s} = \frac{\mu_0 I_0}{y_0} \underbrace{x_0 z_0}_{\text{area}}$$

i. From your answer to either part g or part h, determine the total inductance of the transmission line.

$$\text{From flux} = LI \Rightarrow L = \frac{\mu_0 x_0 z_0}{y_0}$$

$$\text{From energy} = \frac{1}{2} LI^2 \Rightarrow L = \frac{\mu_0 x_0 z_0}{y_0}$$

j. From your answers to part a, d and g, determine the conditions under which the energy stored in the electric field will equal the energy stored in the magnetic field.

$$W_E = \frac{1}{2} \frac{Q^2}{\varepsilon_0 y_0 z_0} x_0 = W_M = \frac{1}{2} \mu_0 I_0^2 \frac{x_0 z_0}{y_0}$$

$$Q^2 = \rho_s^2 y_0^2 z_0^2 = \frac{V_0^2}{x_0^2} \varepsilon_0^2 y_0^2 z_0^2 \qquad = \frac{1}{2} \mu_0 \frac{V_0^2}{R^2} \frac{x_0 z_0}{y_0}$$

$$\cancel{\frac{1}{2}} \frac{\cancel{V_0^2} \varepsilon_0 y_0 \cancel{z_0}}{x_0} = \cancel{\frac{1}{2}} \frac{\mu_0}{R^2} \frac{x_0 \cancel{z_0}}{y_0} \cancel{V_0^2}$$

$$R^2 = \frac{\mu_0}{\varepsilon_0} \frac{x_0^2}{y_0^2}$$

$$R = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{x_0}{y_0} = Z_0 \;!$$

9. Electromagnetic Plane Waves (2 Pts)

a. A uniform plane wave with a frequency of $f = 10GHz = 10^{10}Hz$ is propagating in a dielectric medium with a permittivity of $\varepsilon = 6\varepsilon_o$. If the magnitude of the magnetic field of the wave is *100A/m*, determine the vector phasor expressions for both the electric and magnetic fields in this material. Also assume that the wave propagates in the *z* direction and that the magnetic field is in the *x* direction.

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi 10^{10}}{3\times10^8}\sqrt{6} = 16.33\pi = 51.3$$

$$H_x = 100 e^{-j\beta z}$$

$$\eta = \frac{\eta_o}{\sqrt{6}} = 49\pi = 154$$

$$E_y = -\eta 100 e^{-j\beta z}$$

b. Determine the average power density carried by this wave.

$$S_{ave} = \frac{\eta 100^2}{2} = \frac{154\times10^4}{2} = \frac{1.5\times10^6}{2}\frac{W}{m^2} = 7.5\times10^5 \frac{W}{m^2}$$

c. Now assume that the medium has a small conductivity $\sigma = 0.1 S/m$. Determine the propagation constant β and the decay constant α for the wave.

$$\gamma = j\omega\sqrt{\mu\varepsilon} = j\omega\sqrt{\mu_o\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}} = j\omega\sqrt{\mu_o\varepsilon}\left(1 - j\frac{\sigma}{2\omega\varepsilon}\right)$$

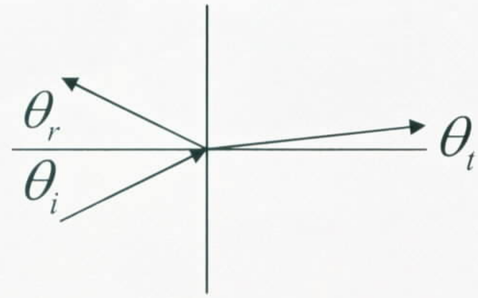
β = the same $= 51.3$

$$\alpha = \underbrace{j\omega\sqrt{\mu_o\varepsilon}}_{\beta}\left(-\frac{j}{2}\frac{\sigma}{\omega\varepsilon}\right) = \beta\frac{\sigma}{2\omega\varepsilon} = \beta\frac{(0.01)}{2(2\pi\times10^{10})(\frac{6}{36\pi}\times10^{-9})}$$

$$= \beta\frac{(0.01)3}{20} = \beta\frac{.03}{20}$$

$$= \frac{(51.3)(.03)}{20} = 0.077$$

10. Oblique Incidence (2 Pts)



$\eta_2 = \frac{\eta_0}{\sqrt{6}}$

a. A uniform plane wave propagating in air is incident obliquely on the surface of an insulating medium at an angle of 35 degrees. Determine the angle of reflection and the angle of transmission if the permittivity of the second medium is $\varepsilon = 6\varepsilon_o$.

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\sin\theta_2 = \frac{n_1 \sin 35^\circ}{n_2} = \frac{\sin 35^\circ}{\sqrt{6}} = 13.5^\circ$

1.4

b. Determine the ratio of the reflection coefficient for perpendicularly polarized waves to parallel polarized waves $\frac{\Gamma_\perp}{\Gamma_\parallel}$ for this angle of incidence.

$$\Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_2 \cos\theta_i} = -.35$$

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = -.49$$