Fields and Waves I

Final

Spring 2002

1. Short Questions (25) ______

2. Multiple Choice (20) ______

Do Two of the Following Three Problems

3. Finite Difference (20) ______

4. Laplace/Poisson (20) ______

5. Faraday's Law (20) ______

----

6. Electrostatics (25) ______

7. Magnetostatics (25) ______

8. Trans Lines - Pulses (20) ______

Problem 9 has two options, do only one

9. Lossy/Lowloss Waves (20) ______

Option 1:

Option 2:

----

10. Oblique Incident (25) ______

Total ____________________

Name ___________________

Notes:

1) In the Multiple Choices section, each question may have more than one correct answer. Circle all of them.
2) For the Multiple Choice questions, you may add some comments to justify your answer.
3) Make sure your calculator is set to perform trigonometric functions in radians, not degrees.
4) All solutions should include the units. Points will be deducted if they are missing.
5) If you are stuck on part of a problem, use general expressions for the following sections. To earn the maximum credit, include all information that you know ($d\vec{l}$, $d\vec{S}$, limits, etc.)

Short Answers (25)

What is the surface current density, $\vec{J}_s$, of a perfectly conducting wire with radius *a* and total current *I*?

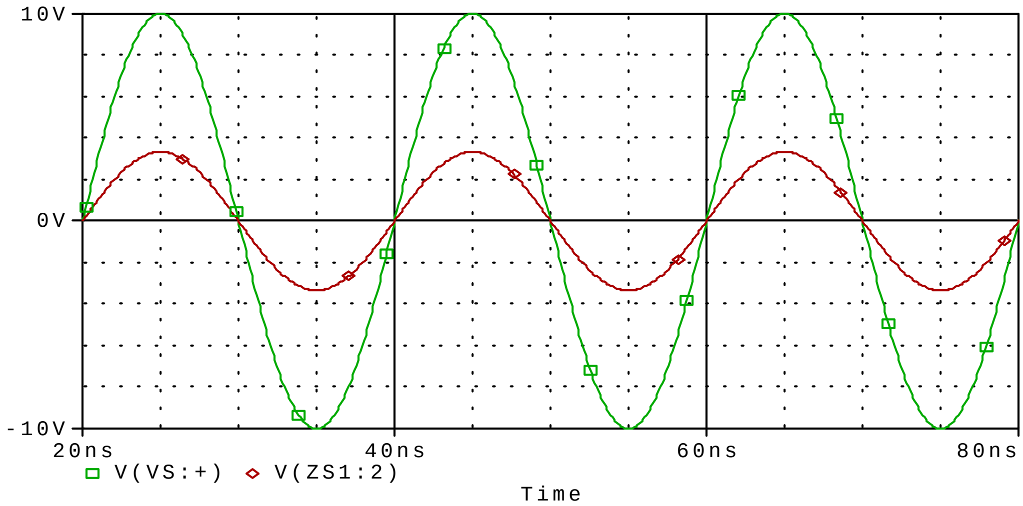
What is the divergence of the electric field, $\nabla \bullet \vec{E}$, in the dielectric material of a parallel plate capacitor that has voltage V_o?

For an extremely long ($l > 1000\ \lambda$) lossy transmission line with characteristic impedance $Z_o = 75\ \Omega$ and a short circuit load, what is the input impedance?

What is the maximum EMF induced in a fixed loop of area, *A*, moving in a uniform magnetic field, B_o, with constant velocity, v_o?

Where (in wavelengths) is the first <u>current</u> standing wave maximum for a short circuit load.?

Multiple Choices (30)



i) The above figure is a plot of the source voltage and the input voltage into a lossless transmission line with a resistive load:

a. The length of the line could be $\frac{\lambda}{4}$.

b. The length of the line could be $\frac{\lambda}{2}$.

c. The magnitude of the input impedance is greater than the source impedance, $Z_{in} > Z_s$.

d. The magnitude of the input impedance is less than the source impedance, $Z_{in} < Z_s$.

ii) Which of the following are possible fields in a source free region:

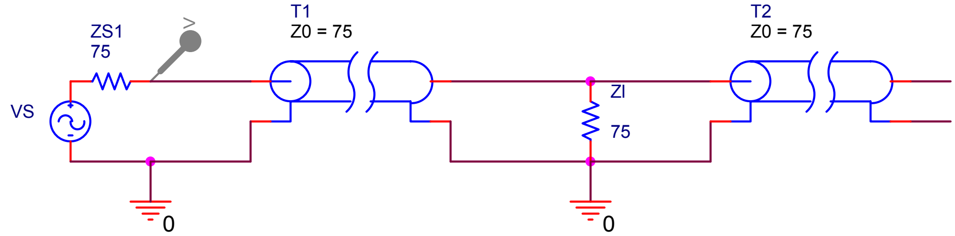
a. $\vec{E} = \frac{E_o}{r}\hat{r}$ (electric field, cylindrical coordinates)

b. $\vec{E} = \frac{E_o}{r}\hat{r}$ (electric field, spherical coordinates)

c. $\vec{B} = \frac{B_o}{r}\hat{r}$ (magnetic field, cylindrical coordinates)

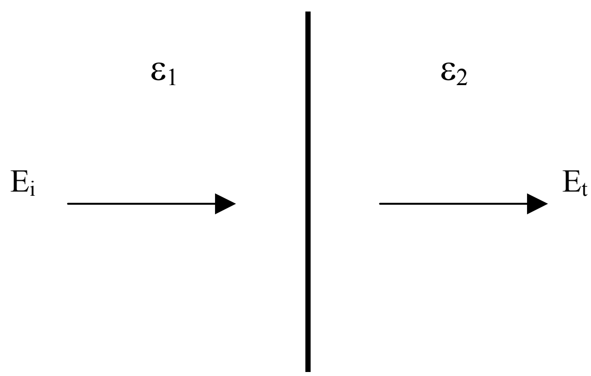
d. $\vec{B} = \frac{B_o}{r}\hat{\theta}$ (magnetic field, cylindrical coordinates) *iii)* For the electric field $\vec{E} = (10\hat{x} + A\hat{y})\exp(-j\omega z)$ [V/m] with unknown *A*, which of the following are true:

a. A = 10, the wave is linearly polarized.
b. A = 20, the wave is linearly polarized.
c. A = *j*10, the wave is circularly polarized.
d. A = *j*20, the wave is circularly polarized.



iv) In the above figure, the transmission lines have v_p = 2E8 [m/s]. T2 has a length of 0.5 [m]. Which of the following frequencies will be blocked:

a. 50 MHz.
b. 100 MHz.
c. 200 MHz.
d. 400 MHz.

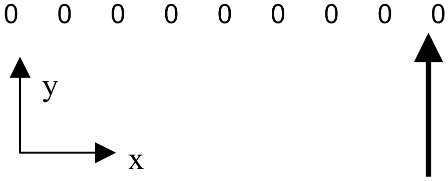


v) In the above figure, a plane wave is normally incident on a dielectric/dielectric boundary. (In the following choices, E_i is incident field magnitude, E_t is transmitted field magnitude, P_i is incident power magnitude, and P_t is transmitted power magnitude).

a. If $\varepsilon_1 > \varepsilon_2$, then $E_i > E_t$.
b. If $\varepsilon_1 > \varepsilon_2$, then $E_i < E_t$.
c. If $\varepsilon_1 > \varepsilon_2$, then $P_i > P_t$.
d. If $\varepsilon_1 > \varepsilon_2$, then $P_i < P_t$.

Finite Difference-Option 1 of 3 (20)

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2.2	4.3	6.4	8.4	10	12	13	14	14	14	13	12	10	8.4	6.4	4.3	2.2	0
0	4.3	8.7	13	17	21	24	26	28	28	28	26	24	21	17	13	8.7	4.3	0
0	6.6	13	20	26	32	37	41	43	44	43	41	37	32	26	20	13	6.6	0
0	8.8	18	26	35	44	51	57	60	61	60	57	51	44	35	26	18	8.8	0
0	11	22	33	45	56	67	76	79	80	79	76	67	56	45	33	22	11	0
0	13	26	40	55	70	84	100	100	100	100	100	84	70	55	40	26	13	0
0	15	30	46	64	83	100	100	100	100	100	100	100	83	64	46	30	15	0
0	16	33	51	72	100	100	-1	100	-1	100	-1	100	100	72	51	33	16	0
0	16	34	53	74	100	-1	-1	-1	-1	-1	-1	-1	100	74	53	34	16	0
0	16	33	51	73	100	100	-1	100	-1	100	-1	100	100	73	51	33	16	0
0	15	30	47	65	84	100	100	100	100	100	100	100	84	65	47	30	15	0
0	13	27	41	56	71	85	100	100	100	100	100	85	71	56	41	27	13	0
0	11	23	35	47	58	68	77	80	81	80	77	68	58	47	35	23	11	0
0	9.5	19	28	38	46	54	60	63	64	63	60	54	46	38	28	19	9.5	0
0	7.5	15	22	29	36	41	46	48	49	48	46	41	36	29	22	15	7.5	0
0	5.5	11	16	21	26	30	33	35	35	35	33	30	26	21	16	11	5.5	0
0	3.6	7.2	11	14	17	19	21	22	23	22	21	19	17	14	11	7.2	3.6	0
0	1.8	3.6	5.3	6.9	8.3	9.6	10	11	11	11	10	9.6	8.3	6.9	5.3	3.6	1.8	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0



The voltage in a coaxial capacitor is shown above. The outer conductor is grounded and the inner conductor is maintained at 100 [V]. The insulated material has permittivity, $\varepsilon_r = 4$. The –1 [V] points are part of my artwork and have no affect on the finite difference grid points. None of the grid points have been changed and for space reasons the order of accuracy is limited to two digits. Assume the grid spacing is 1 [mm] square.

Has the finite difference scheme converged reasonably close to a solution, or are more iterations required? (4)

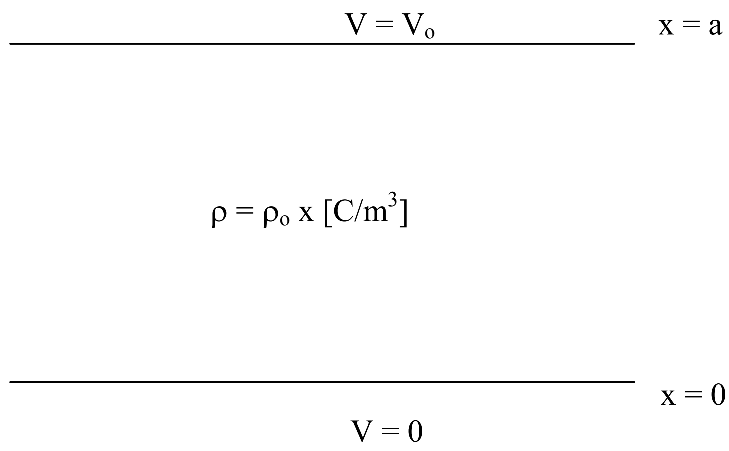
Using the values in the grid, circle the possible *x*-component(s) of the electric field at the cell with V = 9.6 [V] (just above the arrow)? (8)

i) $\vec{E}_x = 9.6$ [kV/m] *ii)* $\vec{E}_x = 0.4$ [kV/m] *iii)* $\vec{E}_x = 9.4$ [kV/m] *iv)* $\vec{E}_x = 1.3$ [kV/m]

What is the charge density, ρ_s, at the location of the arrow? (4)

Indicate the most likely area(s) that dielectric breakdown will occur. (4)
Extra Credit: What does the center conductor look like?

Laplace/Poisson's Equation-Option 2 of 3 (20)

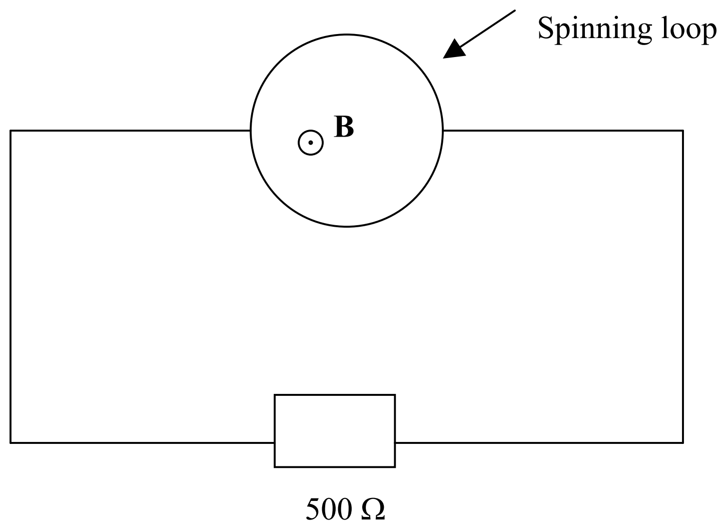


A parallel plate geometry with a charge density between the plates is shown above. You may consider that the geometry extends indefinitely in the *y* and *z* directions.

Determine the voltage as a function of position between the plates. (15)

Determine the electric field, $\vec{E}$, as a function of position. (5)

Faraday's Law-Option 3 of 3 (20)



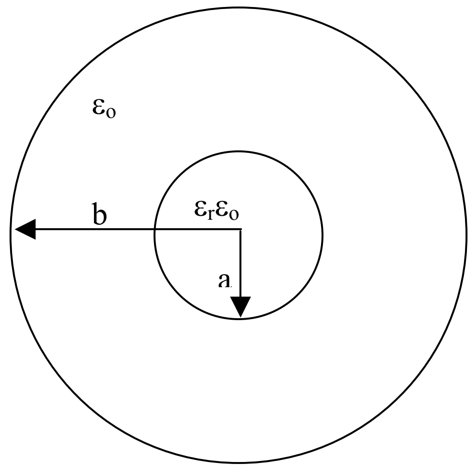
A simple representation of Beakman's motor is shown above. A 1 [cm] radius circular loop is rotating at 60 Hz in a uniform magnetic field, $\vec{B}$ = 1E-6 [Wb/m]. The magnetic field is coming out of the page. The loop is rotating such that you may consider the figure to represent *t* = *0*, and the top edge is starting to go into the paper and the bottom edge is coming out. (In the figure, only a single $\vec{B}$ is shown. As mentioned above, it is everywhere and uniform.)

Plot the EMF across the resistor as a function of time. Label your axis. (15)

At t = 0^+, is the direction of the current clockwise or counterclockwise? (3)

What is the maximum current through the resistor? (2)

Gauss's Law (25)



In the above figure a <u>spherical</u> volume charge distribution is inside a surface charge distribution. The volume charge is $\rho_v = \rho_o r$ [C/m^3] for $r < a$. The total charge on the surface is equal and opposite to the total charge in the volume. Additionally, the volume charge is deposited on a dielectric material with relative permittivity, ε_r. Free space exists for the region $r > a$.

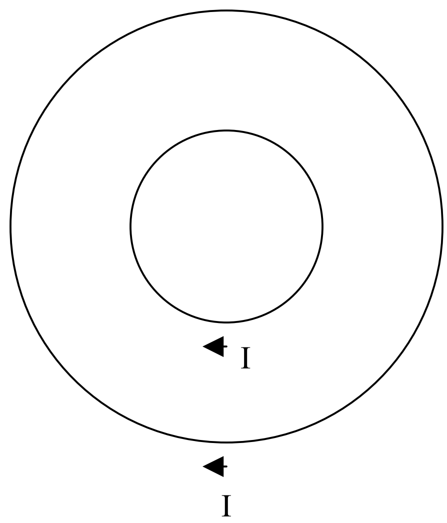
What is the surface charge density in terms of the volume charge and geometry? (5)

Determine the displacement field, $\vec{D}$, everywhere. (10)

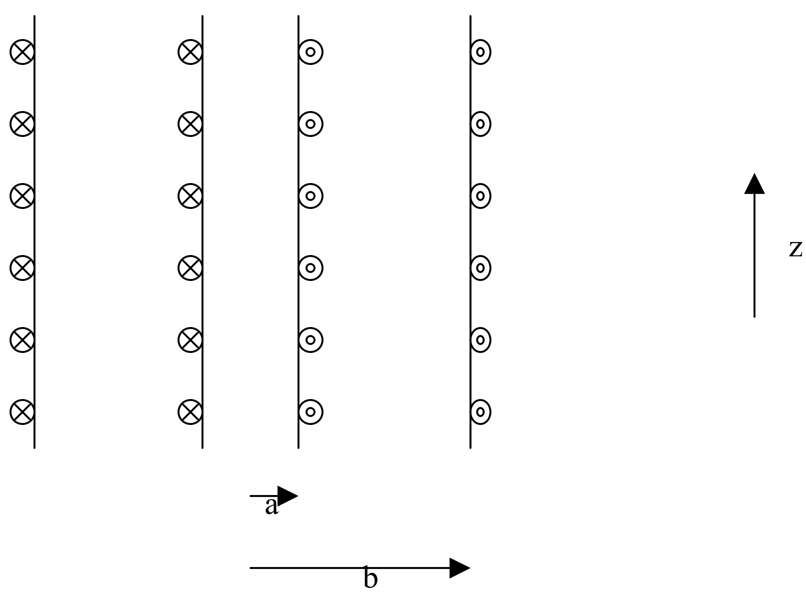
What is the voltage as a function of position for the region $a < r < b$? The voltage at infinity is zero. (5)

What is the total energy in the region, $r < a$? (5)

Ampere's Law (25)



Overhead cross-sectional view of two concentric wire wrapped solenoids
(positive *z*-direction out of the paper)



Lengthwise cross-sectional view of two concentric wire wrapped solenoids

⊗ - into the paper ⊙ - out of the paper

On the previous page, the two figures represent different viewpoints of a two-solenoid configuration. The geometries are concentric (have the same origin) and the current in the outer solenoid is in the same direction as the current of the inner solenoid. They both have a wire density of $n = \frac{N}{l}$ [wires/m] and the current in the wire is I [A].

Note: This problem is similar, but not identical to one you have seen before. Study the figures carefully.

On the previous page, draw the figure you would use to apply Ampere's Law (4).

Determine the magnetic field, $\vec{H}$, in all regions. (6)

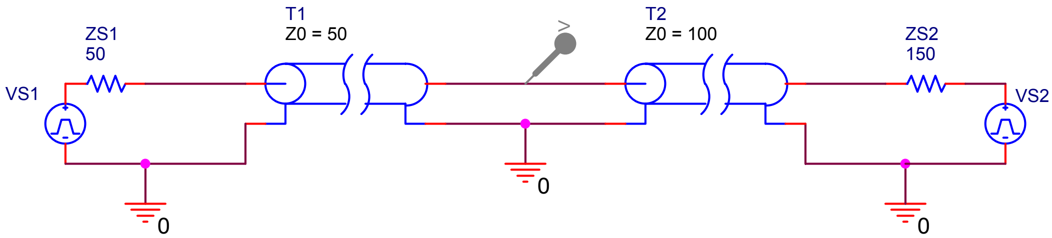
Verify that this result is consistent with both forms of the differential expressions for Maxwell's equations that relate to Magnetostatics. (5)

Determine the total flux, Ψ, that exists in this geometry. (4)

What is the inductance, L, of this geometry? (4)

What is the total energy, W_m, per unit length? (2)

Pulse Propagation (20)

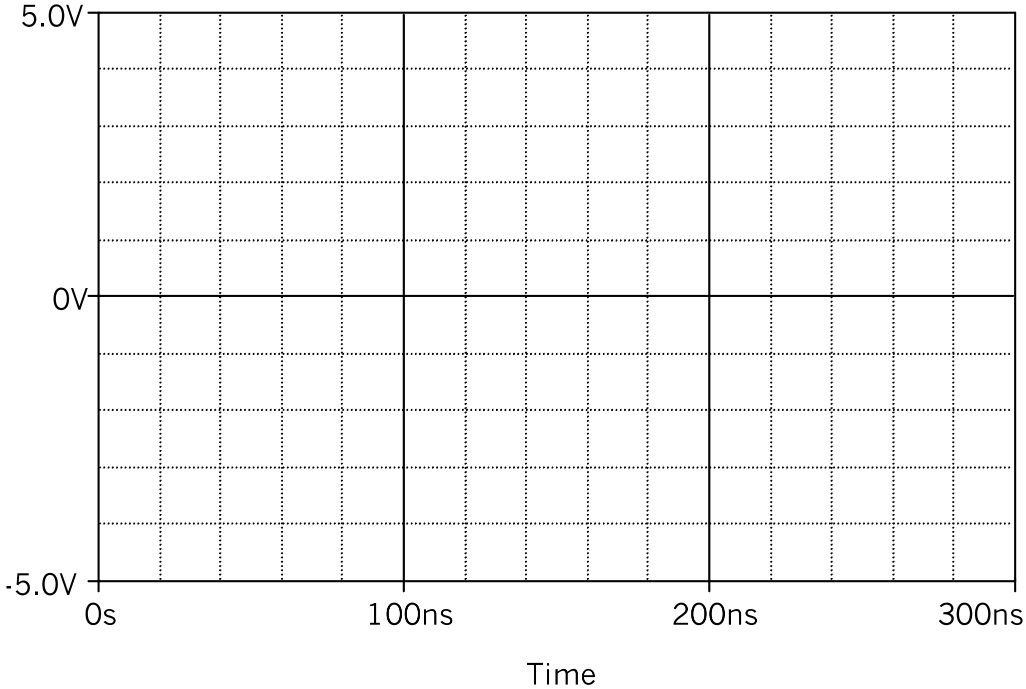


The above transmission line circuit connects two voltage sources together using two different impedance lines. This is a common application to make students lives miserable on a final. The circuit characteristics are as follows:

VS1: 5V, a single 10ns pulse at $t = 0$, 50 Ω internal impedance.
VS2: 5V, a single 10ns pulse at $t = 0$, 150 Ω internal impedance.
T1: 50 Ω characteristic impedance, 2E8 [m/s] velocity of propagation, 10 [m] length.
T2: 100 Ω characteristic impedance, 2E8 [m/s] velocity of propagation, 10 [m] length.

Note that the sources' voltage characteristics are identical, but the source impedances are different. The transmission lines are the same length, but have different characteristic impedances and velocities of propagation.

On the following page, indicate the voltage as a function of time for t ∈ [0,300ns] at the location of the marker, the connection between the two transmission lines. I recommend using the boxes provided to make a lattice (bounce) diagram for the two lines. This will enable you to get partial credit should you make a mistake.


5.0V
0V
-5.0V
0s
100ns
200ns
300ns
Time

Plane Waves in Lossy/Lowloss Materials (20)

A 200 MHz plane wave is propagating in the *z*-direction through a nonmagnetic lossy material. The relative permittivity of both materials listed below is $\varepsilon_r = 8$. They have different conductivities. At $z = 0$, the electrical field is 0.5 [V/m] and is polarized in the *y*-direction.

Choose one of the following materials with their respective power losses (**circle it**). Be careful, your approach to this problem is dependent on the material you select. (1)

Option 1: Material 1: at 1.04 [m], -3dB loss

Option 2: Material 2: at 7.8E-6 [m], -3dB loss

(Reminder: -3dB corresponds to half power)

Is your material a good conductor or good insulator? (3)

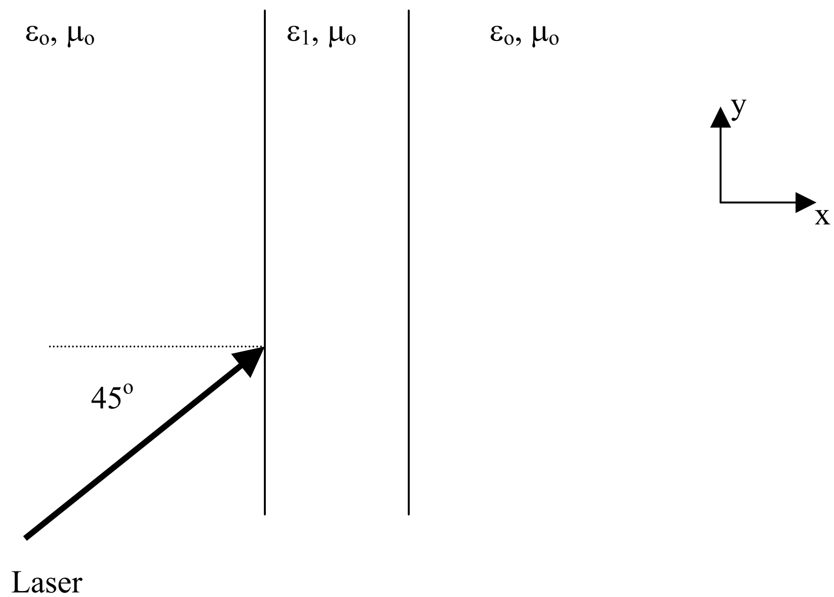
What is the attenuation constant, α? (5)

What is the spatial frequency, β? (5)

Determine the time domain form of the electric field, $\vec{E}$. (5)

If the region $z < 0$ is free space (the wave is incident on the material), what is the approximate reflection constant, Γ? (2)

Oblique Incidence and Polarization (25)



In the above figure, a green laser (527 nm) beam is incident on a dielectric layer. Free space is on either side of the dielectric. The dielectric has a relative permittivity, $\varepsilon r = 2.4$. The electric field is polarized in the *z*-direction.

(Note: Lasers are similar, but not the same as plane waves. This is not an input impedance problem.)

In the above figure, draw the path(s) of the laser beam. Indicate the all angles relative to the normal at the boundaries. Make your drawing complete enough to indicate that your understanding of the problem to the grader. (5)

For the incident beam, determine the phasor form of the electric field. (8)

Determine the phasor form of the transmitted field. (8)

What is the maximum power of a laser beam transmitted through the layer relative to the incident beam? (4)