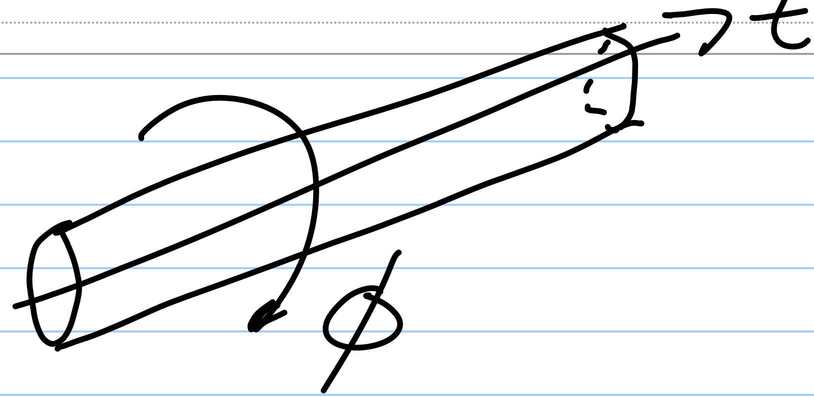
# Cylindrical Beam of Charge – Using Gauss' Law

Note Title 3/26/2006

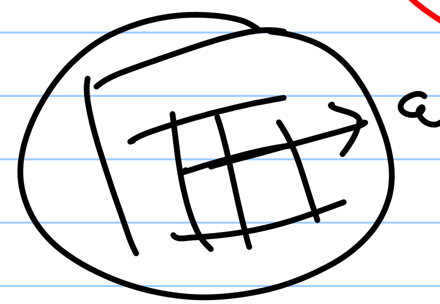



Beam of charge

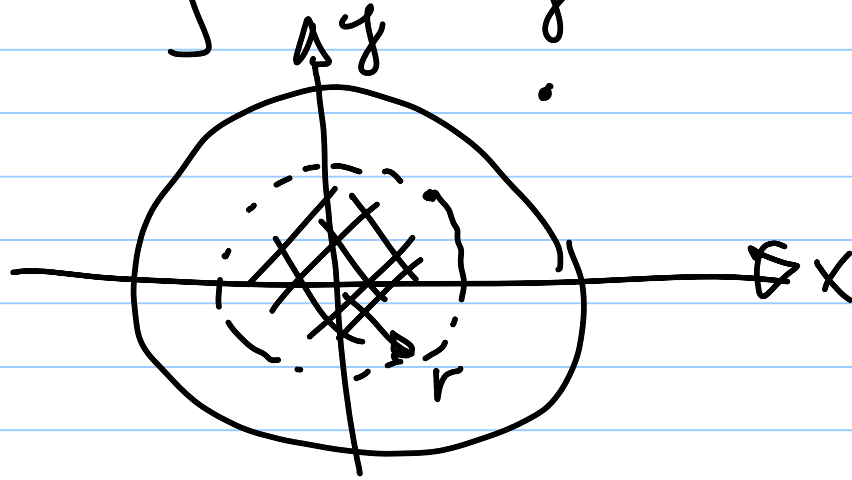
Cyl of charge

$\rho_v = \rho_{v0}$ C/m³

Uniform




Only charges




Cyl. Symmetry

$\rho_v = \rho_v(r)$

Gaussian Surface $\quad \oint \bar{D} \cdot d\bar{S} = Q_{encl}$


$$Q_{encl} = \int \rho_v \, dV$$

$$= \int \rho_v(r) \, r \, dr \, d\phi \, dz \cdot 2\pi \cdot 1$$

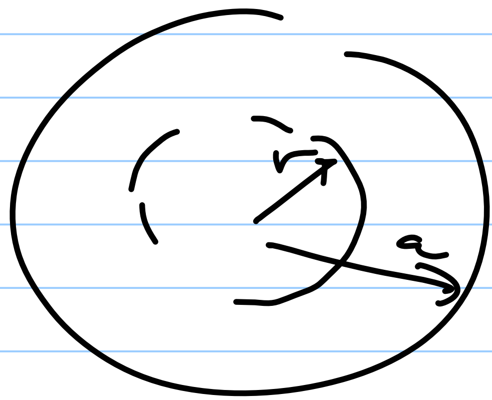
$$= 2\pi \int_0^r \rho_v(r) \, r \, dr = 2\pi \rho_{v0} \int_0^r r \, dr$$

$$= \cancel{2}\pi \rho_{v0} \frac{r^2}{\cancel{2}} = \pi r^2 \rho_{v0}$$

Writing the answer different ways:

Q per unit length $= \pi r^2 \rho_{v0}$

$= \pi r^2 \rho_{v0}$

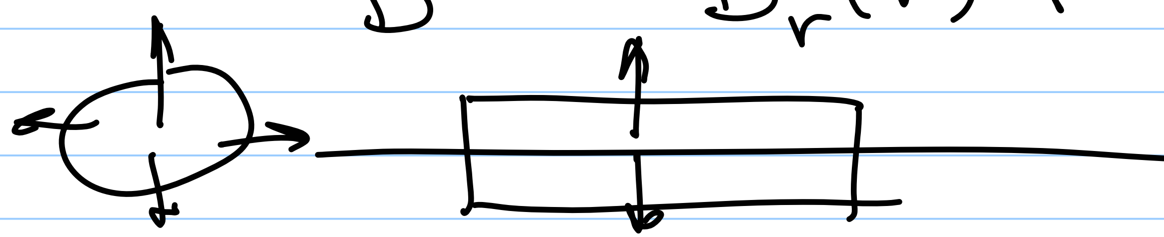



TOTAL Q $= \pi a^2 \rho_{v0}$

$$\frac{Q(r)}{Q_{TOTAL}} = \frac{r^2}{a^2}$$

$$\oint \bar{D} \cdot d\bar{S}$$

$$\bar{D} = D_r(r)\,\hat{r}$$



## General form of $\bar{D}$

$$= D_r(r)\,\hat{r}$$


From the ends → $\bar{dS}\cdot\bar{D} = 0$

$$\bar{dS} = \hat{r}\, r\, d\phi\, dz$$

$$\oint \bar{D}\cdot\bar{dS} = Q_{encl}$$

$$D_r(r)\cdot \text{area} = D_r(r)\, 2\pi r \cdot 1$$

per unit length

$$D\int dS$$

move D outside the integral

$$D_r(r)\, 2\pi r = \rho_{v0} \pi r^2$$

$$D_r(r) = \frac{\rho_{v0} \cancel{\pi} r \cancel{r}}{2 \cancel{\pi} \cancel{r}} = \frac{\rho_{v0}\, r}{2}$$

Note depends on r

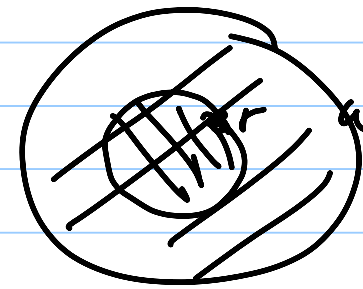
$\rightarrow \rho_L$ = Charge per unit length

$$= \rho_{v0} \pi a^2$$

$$D_r(r) = \frac{\rho_L}{\pi a^2} \frac{r}{2}$$

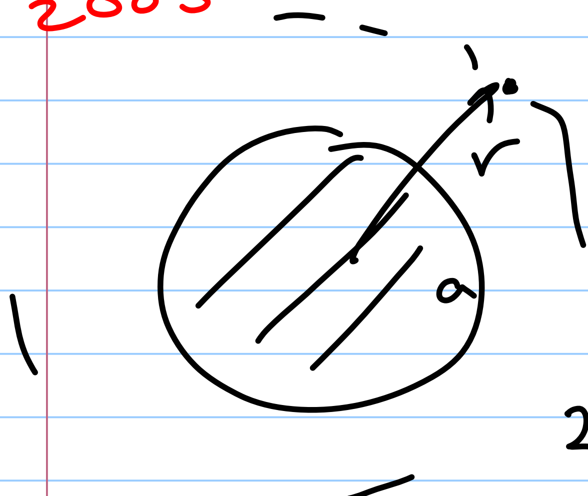
$$\bar{D} = \varepsilon \bar{E}$$

$$E_r(r) = \frac{\rho_L}{\pi a^2}\frac{r}{2\varepsilon_0}$$




Using notation from Quiz 2 Spring 2005

$$\rho_L = Q_0$$



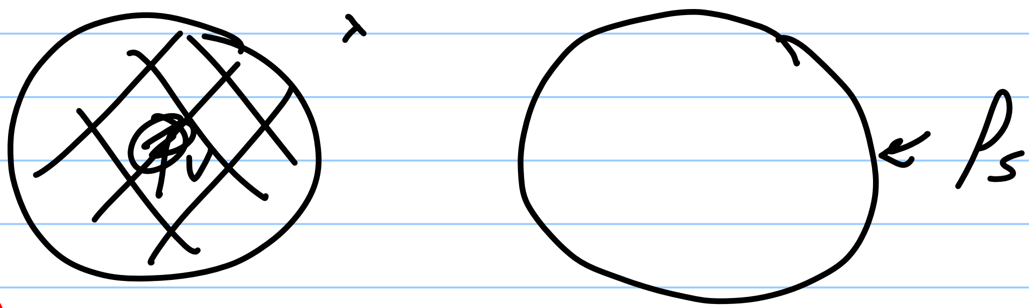

$$Q_{encl} = \text{all } Q$$
$$= \rho_{v0}\pi a^2$$

$$2\pi r D_r(r) =$$

$$D_r(r) = \frac{\rho_{v0}\cancel{\pi} a^2}{2\cancel{\pi} r} = \frac{\rho_{v0}}{2}\,\frac{a^2}{r}$$

$$Q_0 = \rho_{v0}\pi a^2 \Rightarrow$$

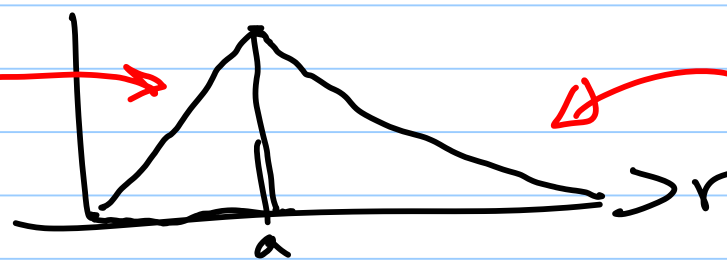
$$D_r(r) = \frac{Q_0}{2\pi r}$$




$$Q_0$$

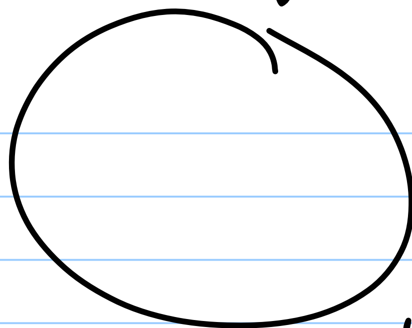
$$\rho_s = \frac{Q_0}{2\pi a}$$

We found the field here




Shows dependence outside of charge

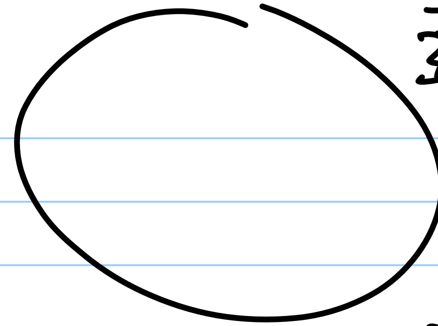
vol $\pi r^2 l$



area cyl

$2\pi r l$

$\frac{4}{3}\pi r^3$



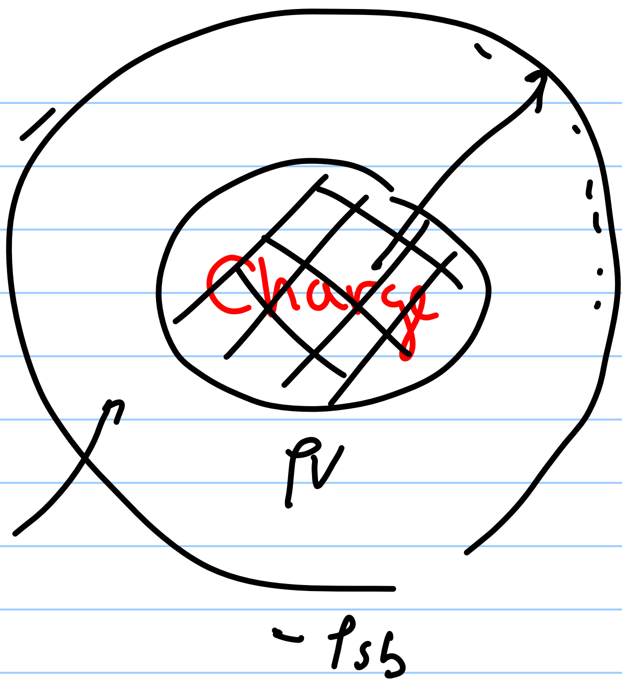
sph.

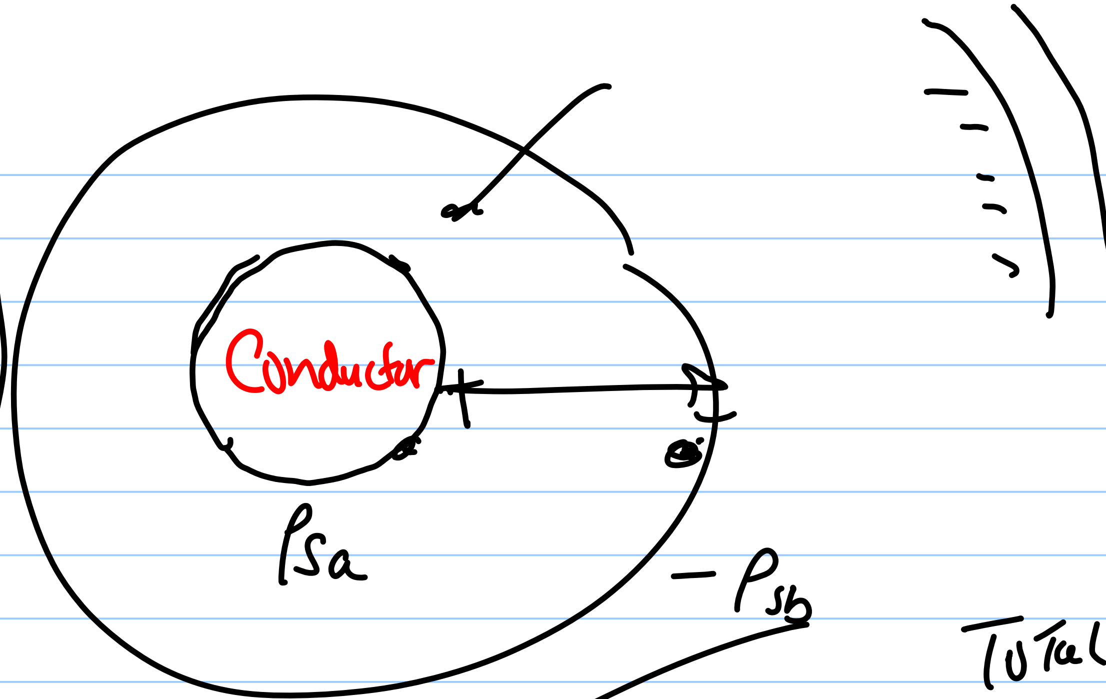
$4\pi r^2$

Cylinder

Sphere

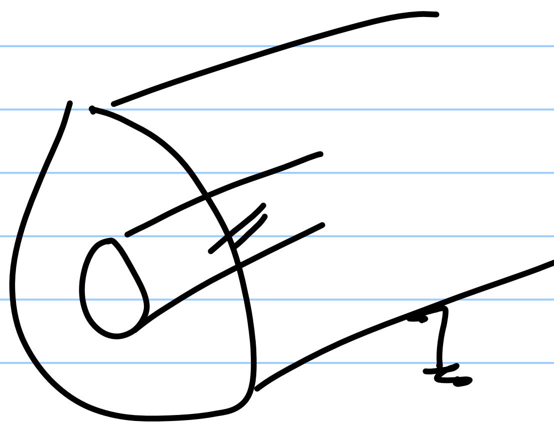
Everything we just did for the cylinder we can do for the sphere.

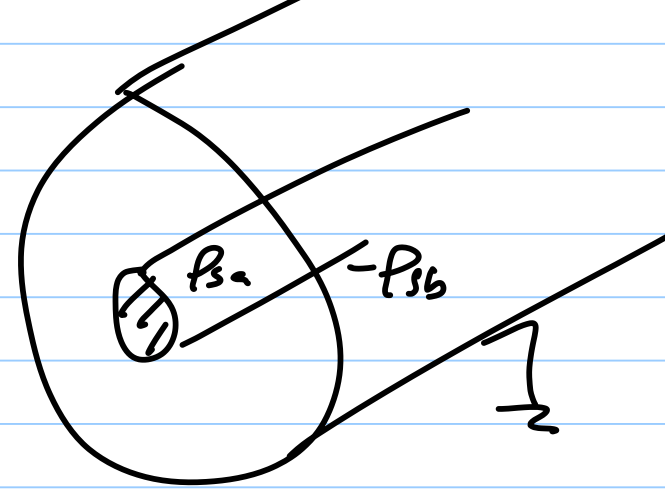


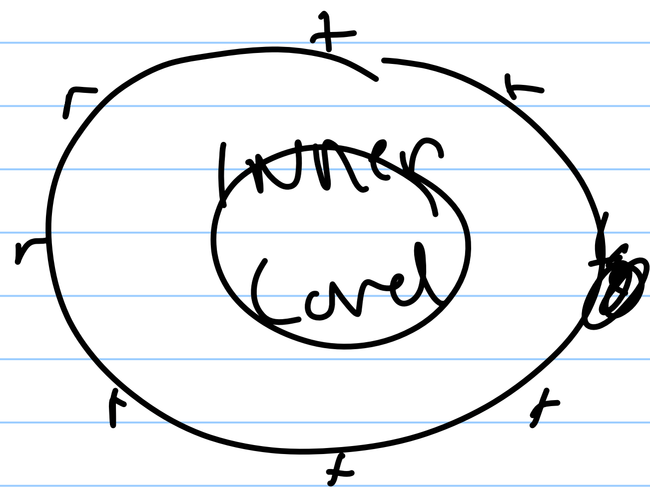




Total

$Q = 0$








$$\vec{E} = 0$$

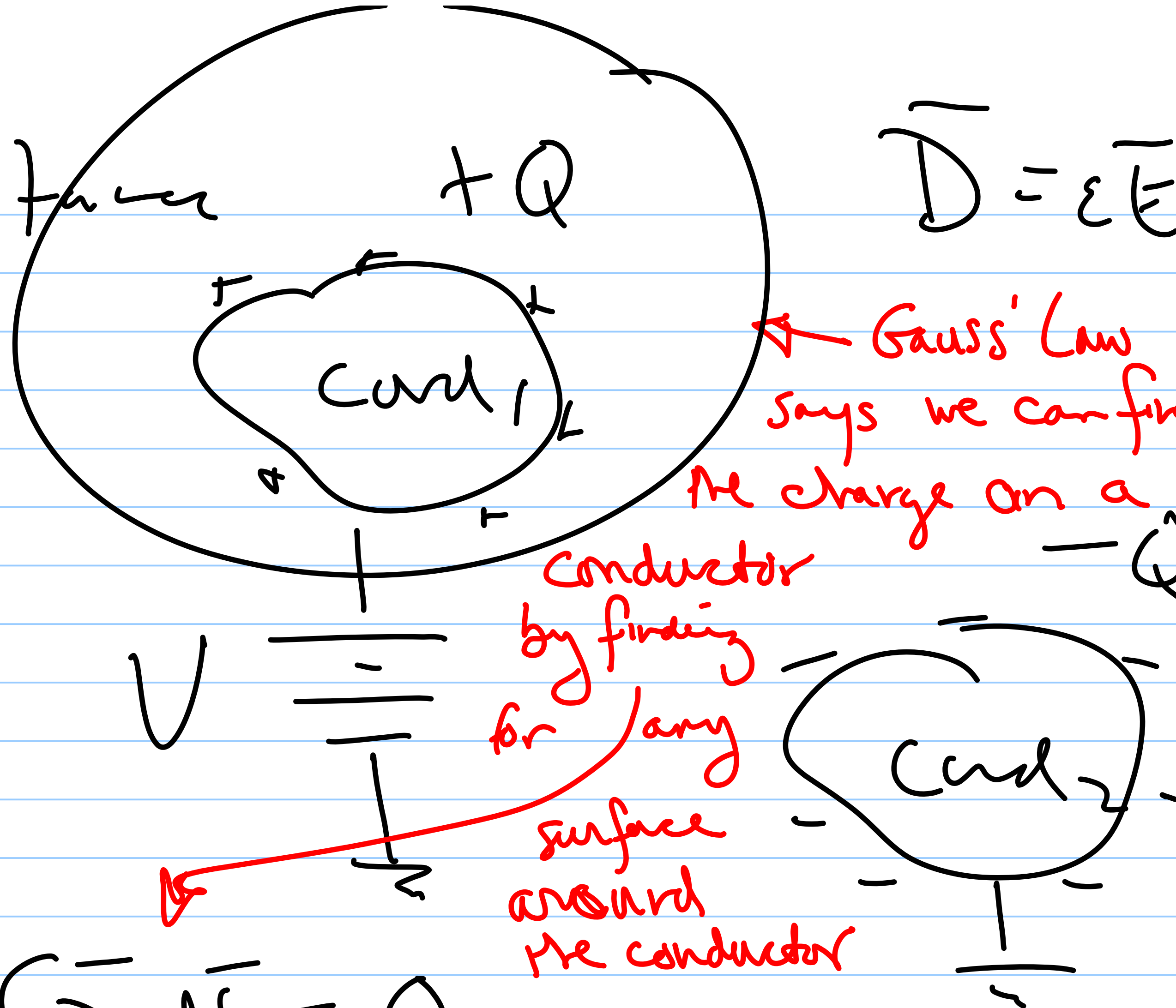
Electric field is zero inside the conductor

# Capacitance

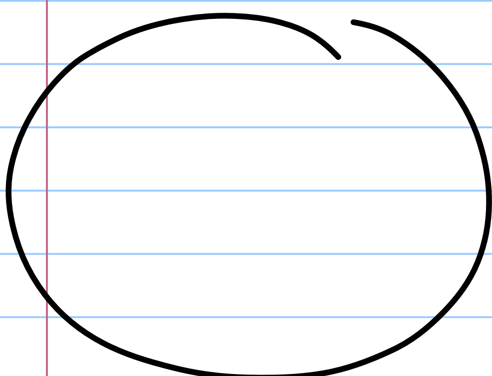
$\rho_s = D_n$

B.C.

$\bar{D} = \varepsilon \bar{E}$




Gauss' Law says we can find the charge on a conductor by finding for any surface around the conductor

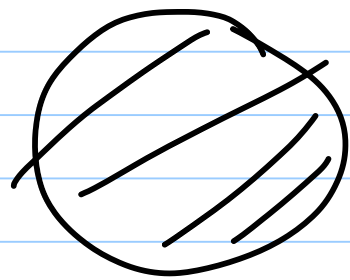


$$\oint \bar{D} \cdot d\bar{S} = Q_{enc}$$

Only with symmetry can we use Gauss' laws to

Sym $\qquad \oint \bar{D} \cdot d\bar{s} = Q_{encl}$

Solve for $\bar{D}$, $\bar{E}$



$\rho_s$ = const $\qquad Q = \rho_s \cdot \text{area}$

$\Rightarrow$ symmetry works & can find $D$ fn $Q$

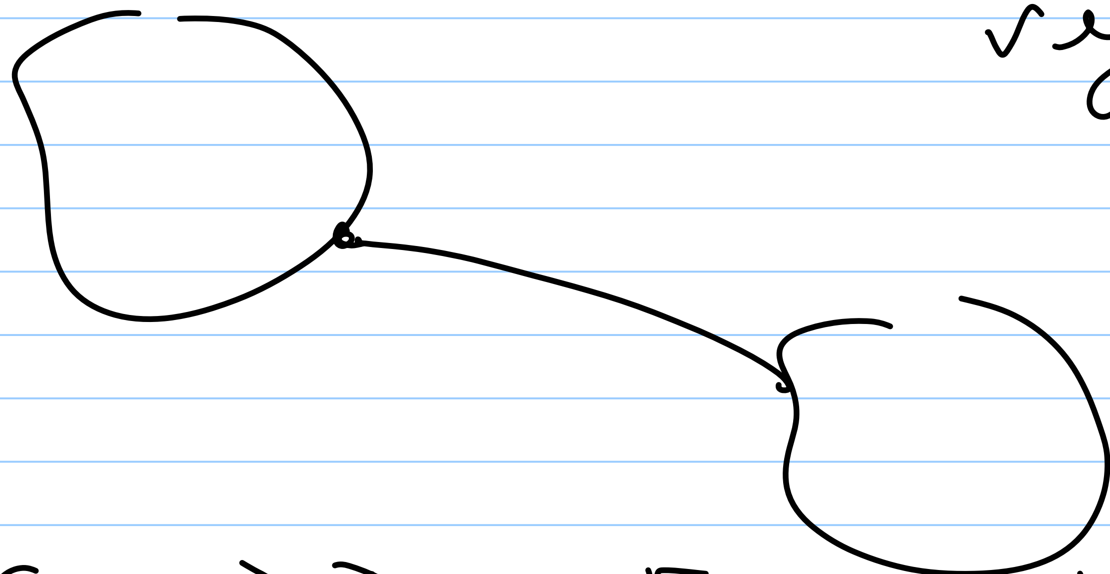
General Method for finding $C$

$Q \to D \to E \to V$

## Obtaining V from E

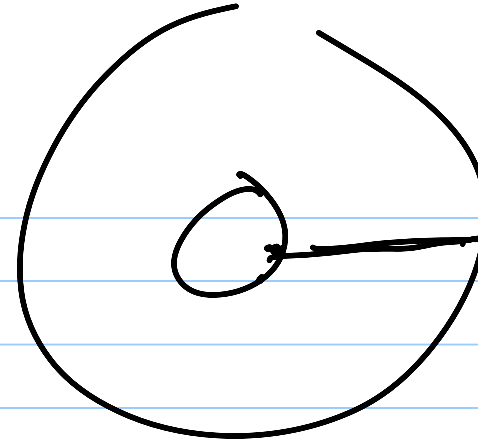
$$V = -\int_{P_1}^{P_2} \bar{E} \cdot d\bar{l}$$

know V here
any pt



$Q_0 \to D \to E \to V$ $\quad Q = CV$

$$D_r(r) = \frac{Q_0}{2\pi r}$$



$$\cancel{D_r}\; E_r(r) = \frac{Q_0}{2\pi \varepsilon_0 r}$$

Elec field in region between the two cyl.

$$V = -\int_{r=b}^{r=a} E_r(r)\,dr$$

$$V(a) = V(b) = \int_b^a \frac{Q_0}{2\pi\varepsilon_0 r}\,dr$$

$$\begin{array}{c} \| \\ 0 \end{array}$$

Reference Location

$$V(a) = -\frac{Q_0}{2\pi\varepsilon_0}\int_b^a \frac{dr}{r}$$

$$= \frac{Q_0}{2\pi\varepsilon_0} \ln b/a$$

Can find C from this

$$V(a) = \frac{Q_0}{2\pi\varepsilon_0} \ln b/a$$

$Q_0$ charge per unit length

$V(a) - V(b) =$ volt diff.

$\quad = 0$

per unit length $\quad C = \frac{Q_0}{V} = \frac{2\pi\varepsilon_0}{\ln b/a}$

$\varepsilon$ (F/m)

# Recall Coaxial Cables

$$C = \underbrace{\frac{\varepsilon A}{d}}_{\text{for parallel plates}}$$

for parallel plates we can use this to see how thing change in general

RG59

$Z_0 = 75\,\Omega$

RG58

$Z_0 = 50\,\Omega$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\underbrace{Q \xrightarrow{\text{Sym}} D \rightarrow E \rightarrow V}_{C}$$

# A second method to find C

$$Q \rightarrow \bar{D} \rightarrow \bar{E} \rightarrow V$$

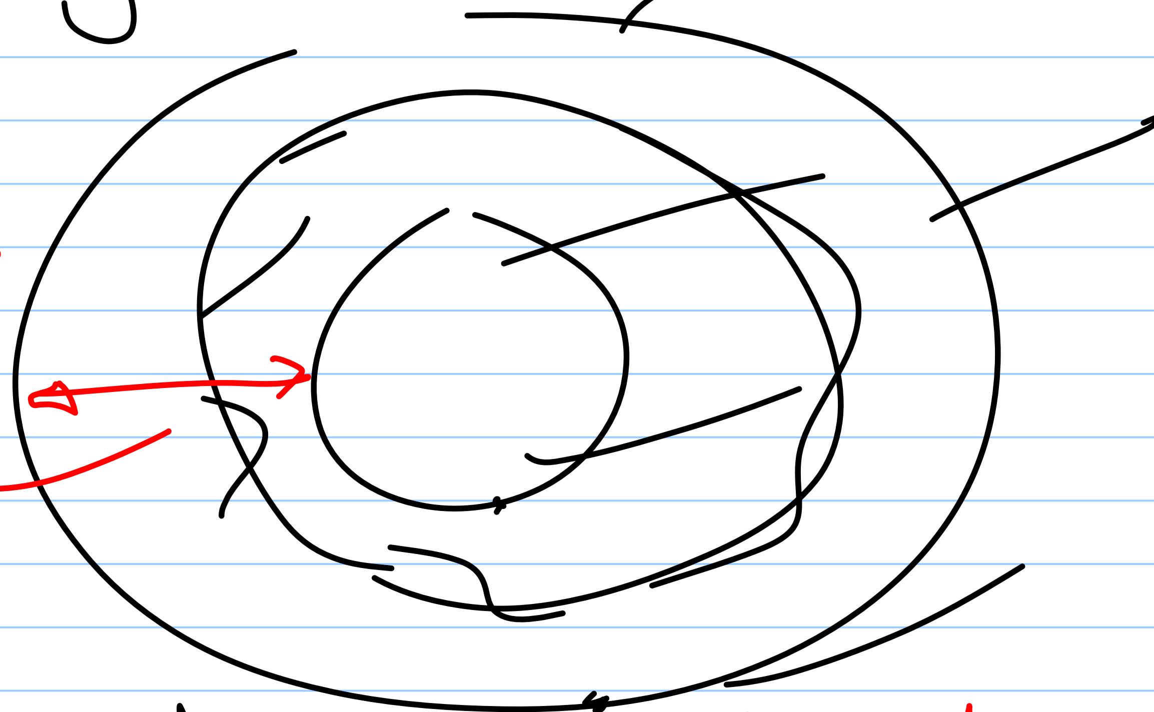
$$W_e = \underline{\underline{\frac{1}{2} C V^2}} = \underline{\underline{\frac{1}{2} \frac{Q^2}{C}}}$$

$$W_e = \int \bar{D} \cdot \bar{E} \, dV$$

$$Q \rightarrow \bar{D} \rightarrow \bar{E} \rightarrow W_e$$

$$C$$

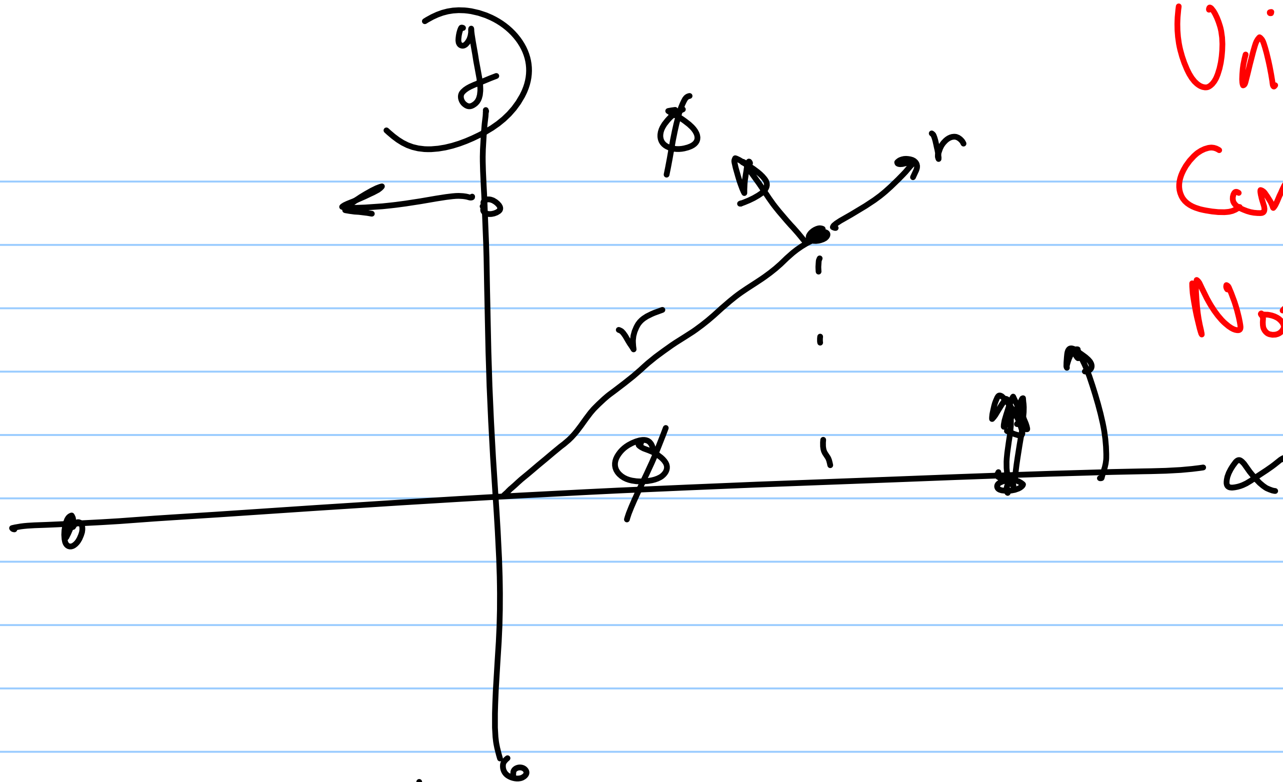
$$W_e = \int \bar{D} \cdot \bar{E} \, dV = \frac{1}{2} \frac{Q^2}{C}$$



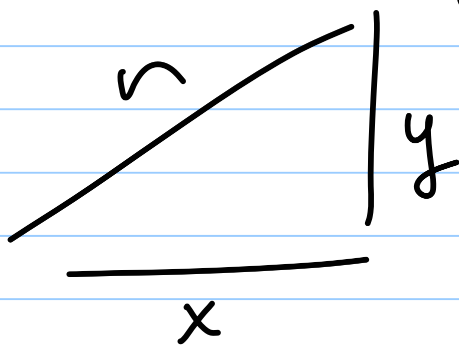
Energy stored in region between conductors.

$$dV = \underline{\underline{r \, dr \, d\phi \, dz}}$$

Volume element




**Unit Vector Conversions Not on Quiz**




$$r = \sqrt{x^2 + y^2}$$

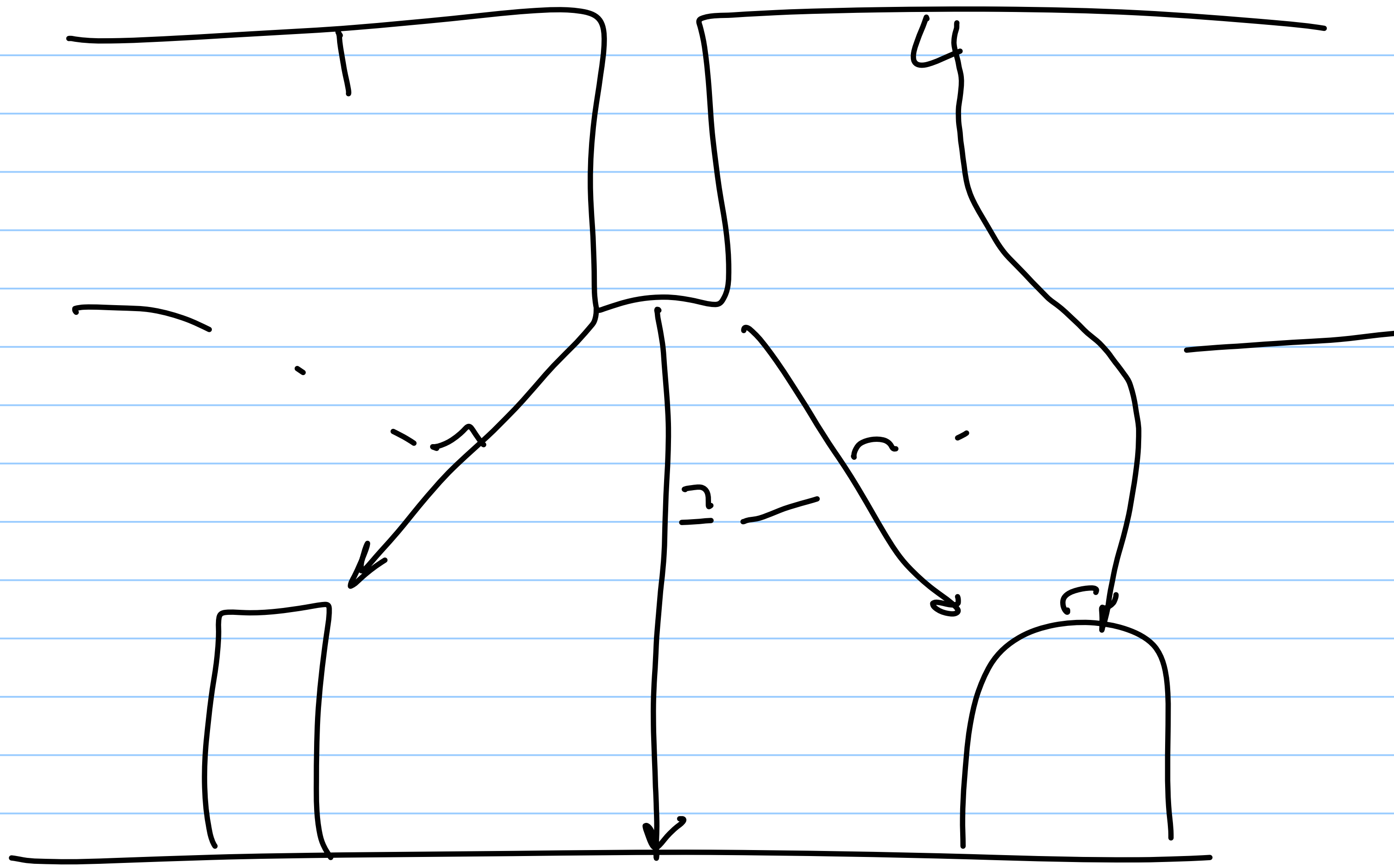
# Spreadsheet Solutions for Laplace's Eqn


ave $\Delta V$

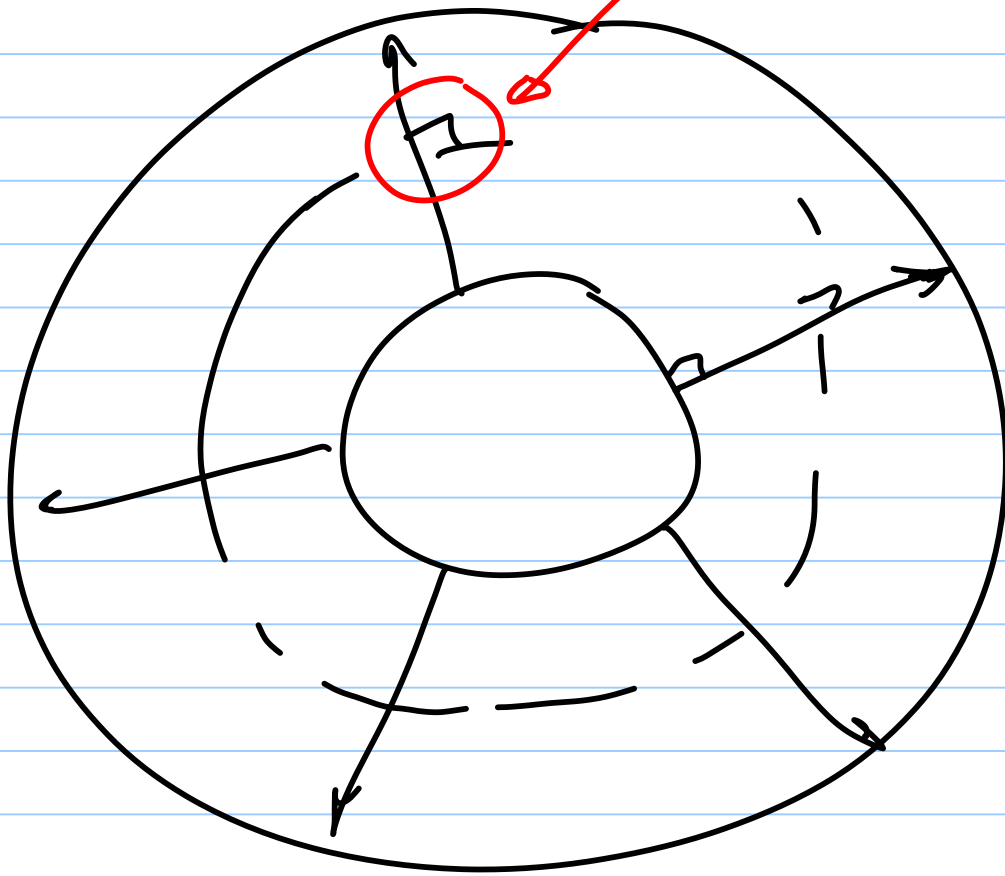
$$\text{Ave } E = \frac{\Delta V}{\Delta x}$$

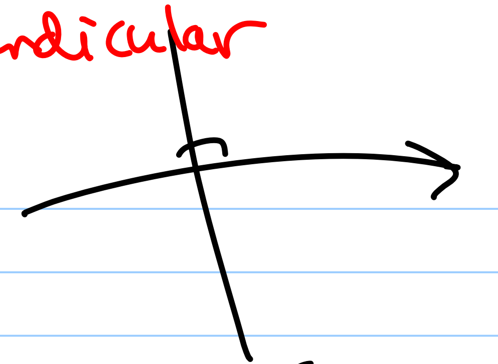
$$D = \varepsilon E \rightarrow \rho$$

# Sketching $\vec{E}$ lines & Equipotentials

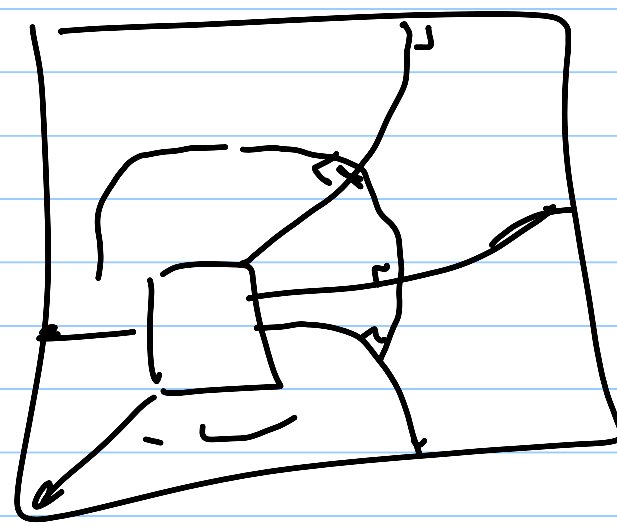


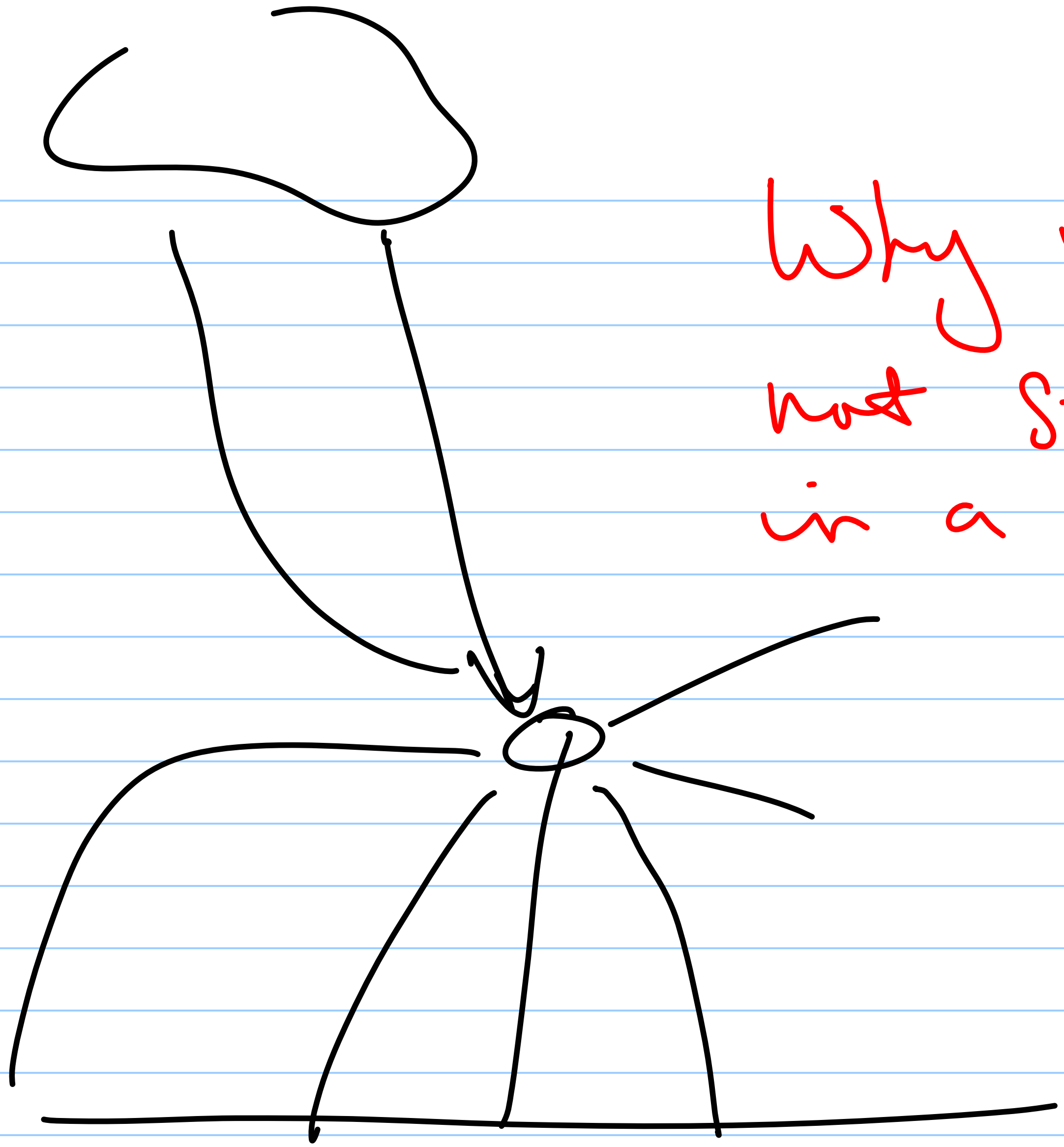
Note perpendicular



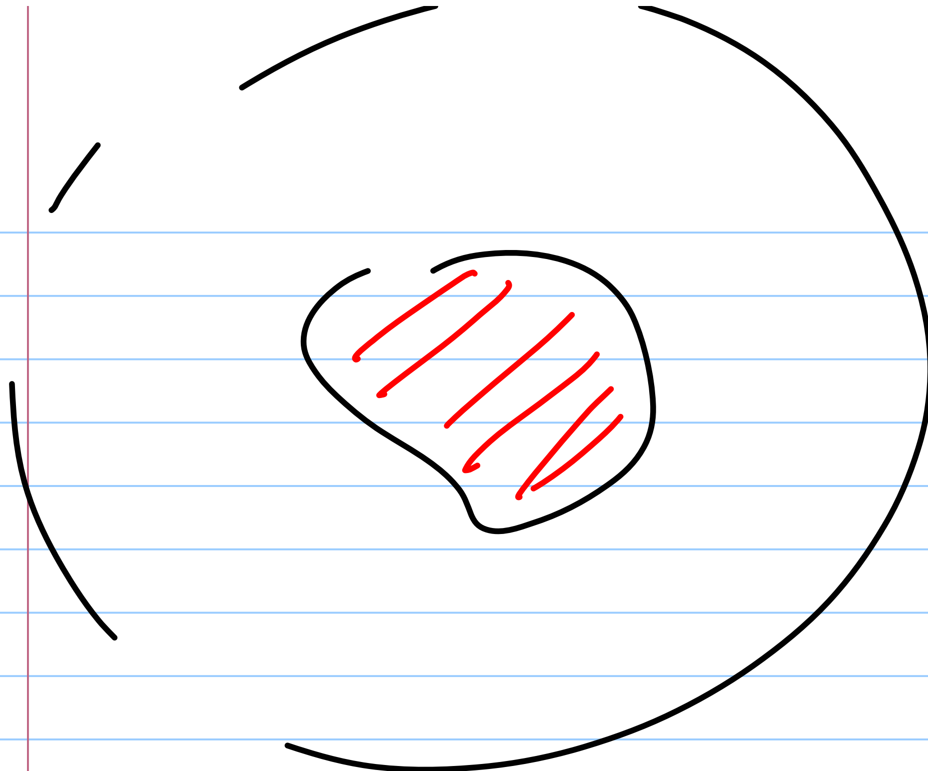


r = constant



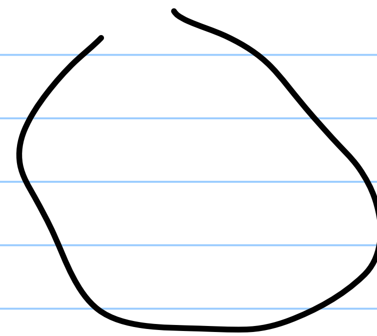


Why you should not stand out in a thunderstorm



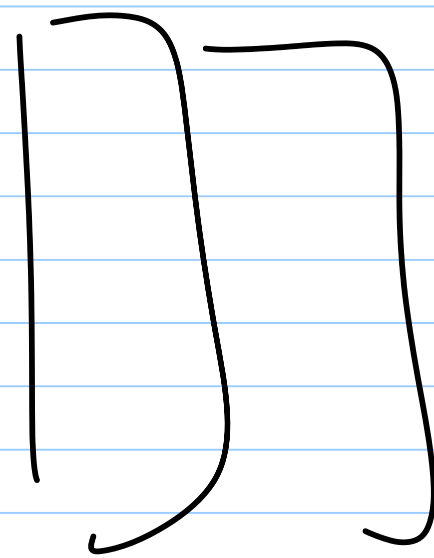
$$\oint \bar{D} \cdot d\bar{S} = Q_{encl}$$

Gives the $Q$ in the conductor



# Solving Poisson's Eqn Directly

$$\nabla^2 V = -\rho/\varepsilon$$

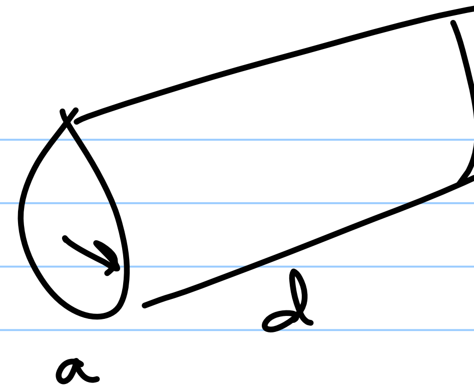


Not on Quiz

This is of value for microelectronics

# Resistance

$$R = \frac{\rho d}{\pi a^2} = \frac{d}{\sigma \pi a^2}$$




$$= \frac{\text{length}}{\sigma \text{ area}}$$

$\rho$ charge ~~density~~

resistivity

Resistivity

$$\rho = \frac{1}{\sigma}$$

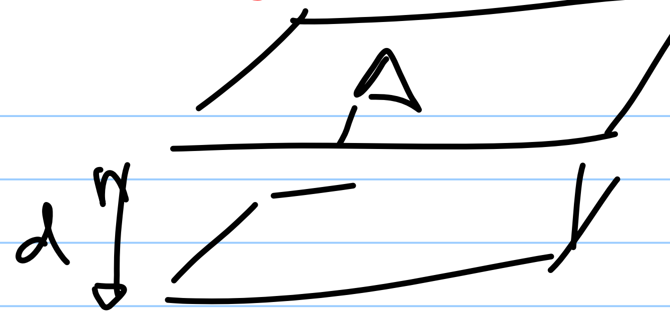
Conductivity

$$\sigma = \frac{1}{\rho}$$

cond.

## Relationship between conductance G & capacitance

$$R = \frac{d}{\sigma A}$$



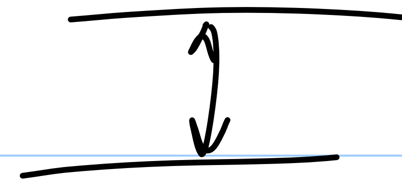

$$C = \varepsilon \frac{A}{d}$$

$$C = \frac{\varepsilon}{\sigma} G$$

$$G = \frac{\sigma A}{d}$$

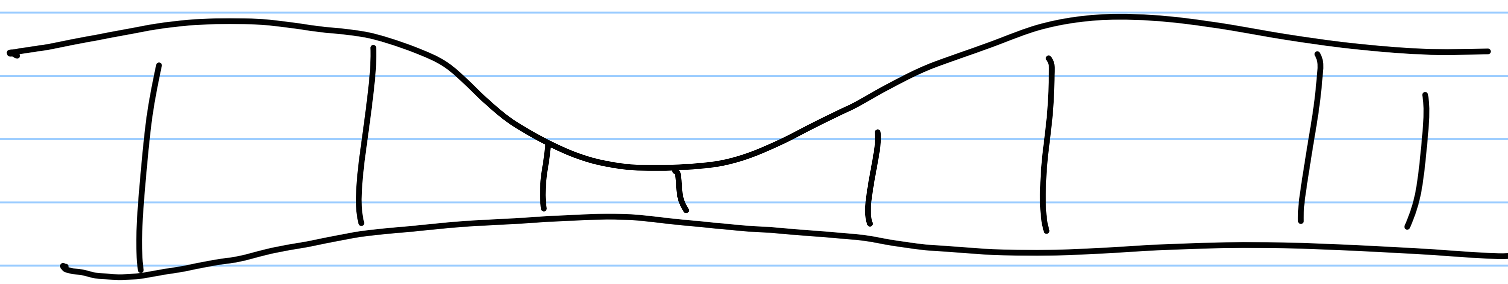
$$G = \frac{\sigma}{\varepsilon} C$$

$C, L, R, G$



Generalized Resistance

$$R = \int \frac{d\ell}{\sigma(\ell)\, A(\ell)}$$



# Fuse

lower

$\sigma$ in this region

Note:
changing
area