Electronics and Instrumentation

Name ________________ ENGR-4220 Fall 1999 Section ____________

Modeling the Cantilever Beam

Supplemental Info for Project 1

The cantilever beam has a simple equation of motion. If we assume that the mass is located at the end of the beam (essentially that the beam itself is massless and is loaded by an additional mass at the end), then

$$Force = m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt}$$

where m is the mass, x is the vertical location of the mass (horizontal beam), k is the spring constant, b is the damping constant and *Force* is the external force applied to the beam. If we have our circuit set up correctly, the displacement of the beam x will be proportional to the output voltage of the strain gauge bridge

$$x \propto V_{SG}$$

Thus, we can write

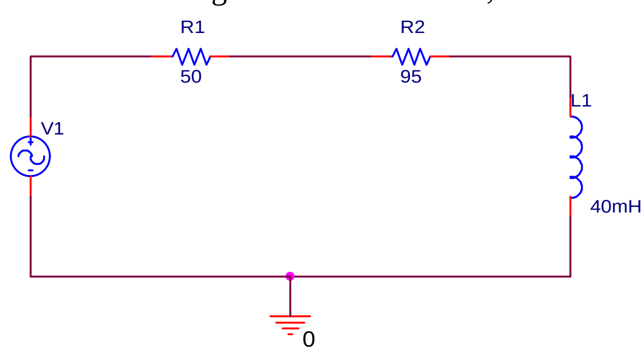
$$Force \propto m\frac{d^2V_{SG}}{dt^2} + kV_{SG} + b\frac{dV_{SG}}{dt}$$

The force is applied to the beam using the pickup coil. We connect the coil to the function generator, adjust the frequency to about 20 Hz (near resonance) and the amplitude to about 2 volts. We should then see the beam oscillating up and down at 20 Hz. The force is created by the magnetic field of the coil. By driving the coil with the function generator, we have created an electromagnet whose poles oscillate between north and south at the frequency we have chosen. It is the current in the coil that produces the magnetic field, so the force will be proportional to the current.

$$Force \propto I_{COIL}$$

The force on the beam is the force of the electromagnet on the permanent magnet that is attached to the end of the beam.

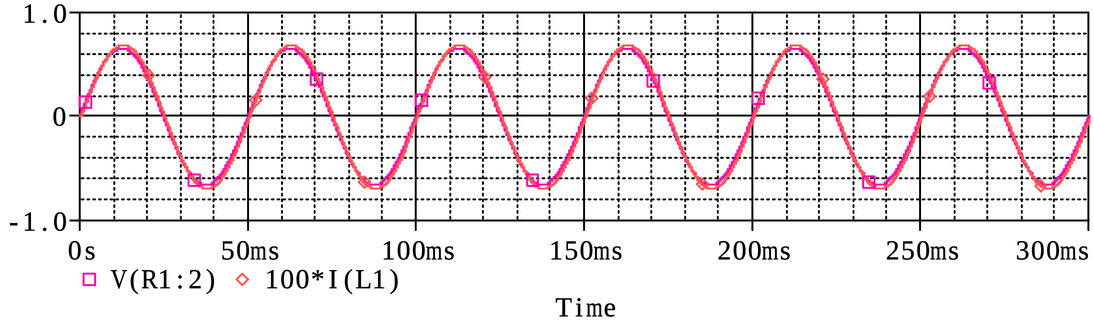
For the combination of the function generator and the coil, the circuit diagram looks like



where the coil is represented by the series combination of the 95 Ω resistor and the 40mH inductor. The 50 Ω resistor and the voltage source represent the function generator.

K. A. Connor
Rensselaer Polytechnic Institute

By plotting the voltage across the coil and the current through it, we can see that they look very similar. Note that the current has been multiplied by 100 to give it the same amplitude as the voltage.



Since the voltage and the current look the same, it is correct to say that the force is also proportional to the voltage or

$$Force \propto V_{COIL}$$

Combining the two expressions for the force and the motion of the beam, we have

$$V_{COIL} \propto m\frac{d^2V_{SG}}{dt^2} + kV_{SG} + b\frac{dV_{SG}}{dt}$$

Since we will be driving the coil with a sinusoidal voltage, these expressions can be simplified for particular frequencies. When we choose to drive the coil at the resonant frequency of the beam, $w_o = \sqrt{k/m}$, the first two terms on the right hand side cancel and we are left with

$$V_{COIL} \propto b\frac{dV_{SG}}{dt}$$

For high frequencies, the first term dominates and we have

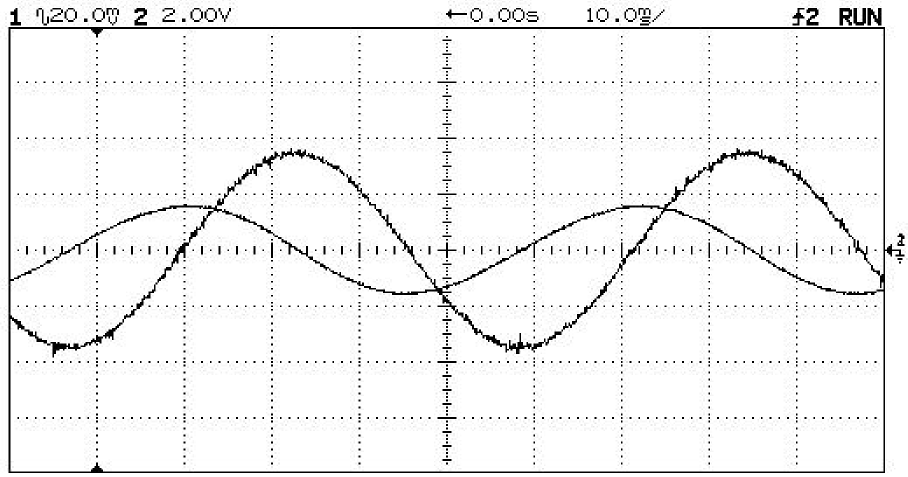
$$V_{COIL} \propto \frac{d^2V_{SG}}{dt^2}$$

For low frequencies, the middle term dominates and we have

$$V_{COIL} \propto V_{SG}$$

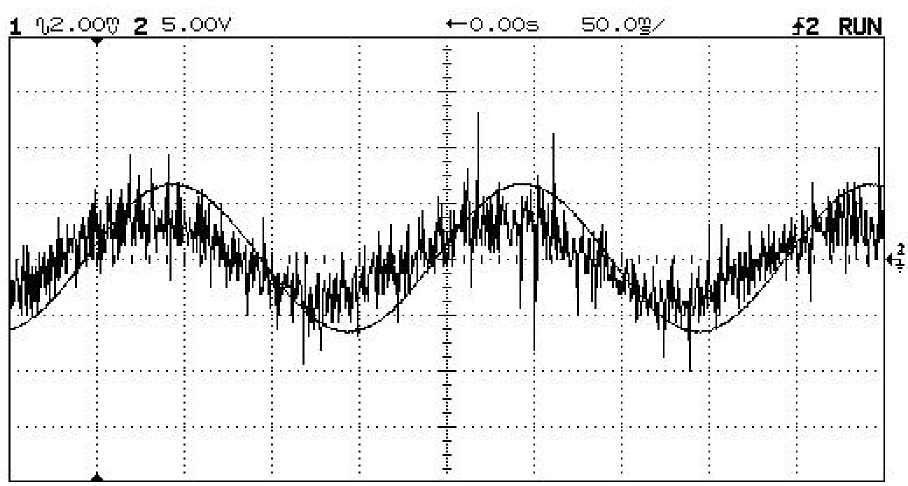
I took some data at two different stations (using the same beam) at three different frequencies. The resonant frequency of this beam was about 19.3 Hz. I drove the beam at this frequency, at 5 Hz and at 30 Hz. These last two frequencies were not chosen for any special reasons, other than that they were well away from resonance. The data I obtained are shown on the next page. The coil voltage and the strain gauge voltages are shown on each plot. I did not use any amplification for the strain gauge, so its voltage is smaller in magnitude and a lot noisier, especially away from resonance.

The first plot shows these voltages at resonance



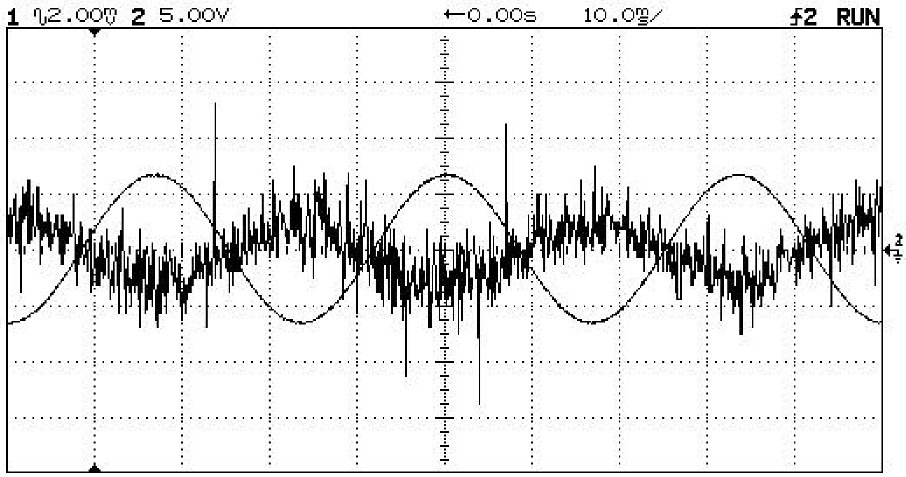
Note that the time derivative of the strain gauge voltage does indeed look like the coil voltage. If you cannot see this easily, assume that t = 0 occurs at end of the second time division. Then, the strain gauge voltage looks like a sine function while the coil voltage looks like a cosine function of time. Thus, the coil voltage looks like the derivative of the strain gauge voltage.

At low frequencies like 5 Hz, we expect the two signals to look mostly alike, since the two voltages are supposed to be proportional to one another. The next plot shows that this is indeed the case.



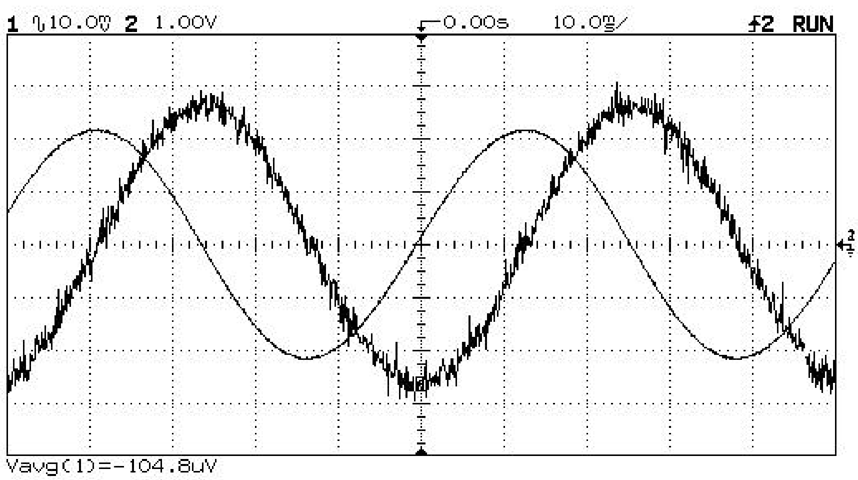
The agreement is not perfect, but quite good considering the amount of noise.

At high frequencies like 30 Hz, the two voltages should look like the negatives of one another, which again appears to be true.



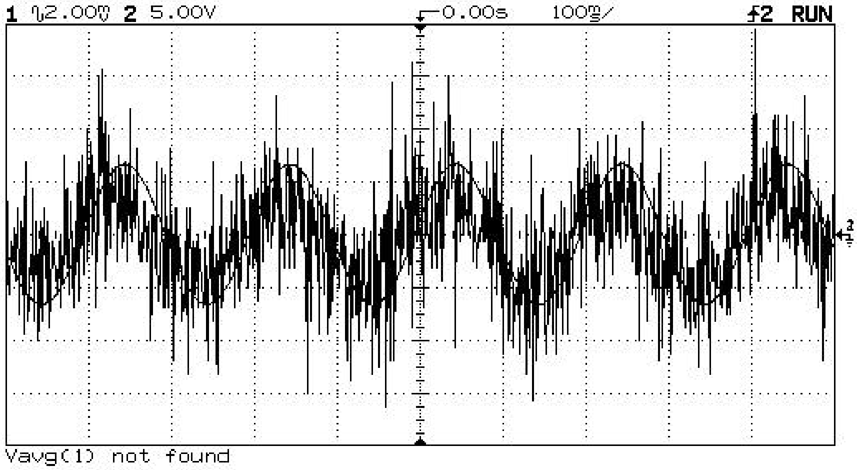
Thus, we can be confident that all the assumptions in our simple model are justified.

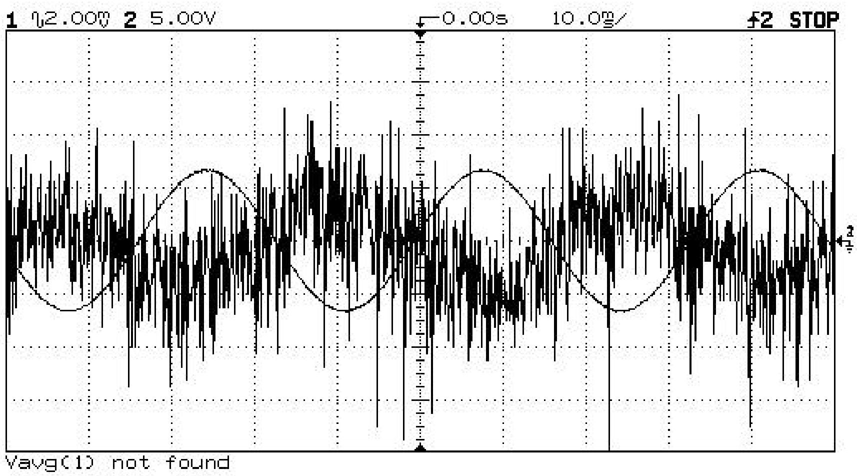
The data does not always look as good as this. At some stations, the noise level is a lot higher. Making no attempt to filter out the noise, the same three plots are.



Electronics and Instrumentation

Name _______________ ENGR-4220 Fall 1999 Section ____________





This modeling information has been provided to help you in preparing your Project 1 report. You should have some confidence in the signals if they look anything like these.