## Spring 2004

1) Resistive Circuits (16 points)


In the circuit above, $\mathrm{V} 1=5$ volts. $\mathrm{R} 1=50 \Omega, \mathrm{R} 2=1000 \Omega, \mathrm{R} 3=2000 \Omega, \mathrm{R} 4=3000 \Omega$
a) Find the voltage across R1. (8 points)
b) Find the current through R4. (8 points)

## Spring 2004 solution

1) Resistive Circuits (16 points)


In the circuit above, $\mathrm{V} 1=5$ volts. $\mathrm{R} 1=50 \Omega, \mathrm{R} 2=1000 \Omega, \mathrm{R} 3=2000 \Omega, \mathrm{R} 4=3000 \Omega$
a) Find the voltage across R1. (8 points)
$1 /(R 234)=1 / R 2+1 / R 3+1 / R 4=1 / 1 K+1 / 2 K+1 / 3 K$ R234 $=545.5$ ohms
$V R 1=(R 1 /(R 1+R 234)) V 1=(50 /(50+545.4)) 5=0.42 V \quad$ VRI $=\mathbf{0 . 4 2 V}$
b) Find the current through R4. (8 points)
$V R 4=V 1-V R 1=5-0.42=4.58 \mathrm{~V}$
$I R 4=V R 4 / R 4=4.58 / 3000=1.53 E E-3 \mathrm{amps} \quad \operatorname{IR} 4=1.53 \mathrm{~mA}$

Spring 2003
4. Inductance and capacitance at very high and very low frequencies ( 20 points)


Consider the above circuit and apply your knowledge about the behavior of capacitors and inductors (i.e., open or short circuits at very high or very low frequencies).
a) (6 points) Redraw this circuit when V1 is very low frequency.
b) ( 2 points) According to your redrawn circuit, what would be the value of the voltage at Vout with respect to ground at very low frequencies? (Circle the best answer.)

$$
\text { Vout }=0 \quad \text { Vout }=\text { V1 } \quad 0<\text { Vout }<\text { V1/R1 } \quad \text { V1/R1<Vout } \ll \text { V1 }
$$

c) (6 points) Redraw this circuit when V1 is very high frequency.
d) (2 points) According to your redrawn circuit, what would be the value of the voltage at Vout with respect to ground at very high frequency? (Circle the best answer.)

Vout $=0 \quad$ Vout $=$ V1 $\quad 0<$ Vout $<$ V1/R1 $\quad$ V1/R1 $<$ Vout $<$ V1
e) (4 points) Using your knowledge of filters, what kind of filter would you say this is? (Circle the best answer).
high pass filter low pass filter band pass filter band reject filter

Spring 2003 solution
4. Inductance and capacitance at very high and very low frequencies ( 20 points)


Consider the above circuit and apply your knowledge about the behavior of capacitors and inductors (i.e., open or short circuits at very high or very low frequencies).
f) (6 points) Redraw this circuit when V1 is very low frequency.

g) (2 points) According to your redrawn circuit, what would be the value of the voltage at Vout with respect to ground at very low frequencies? (Circle the best answer.)

Vout=0 Vout $=$ V1 $\quad 0<$ Vout $<$ V1/R1 $\quad$ V1/R1 $<$ Vout $\ll$ V1
h) (6 points) Redraw this circuit when V1 is very high frequency.

i) (2 points) According to your redrawn circuit, what would be the value of the voltage at Vout with respect to ground at very high frequency? (Circle the best answer.)
$\underline{\text { Vout }=\mathbf{0}} \quad$ Vout $=$ V1 $\quad 0<$ Vout $<$ V1/R1 $\quad$ V1/R1<Vout $<$ V1
j) (4 points) Using your knowledge of filters, what kind of filter would you say this is? (Circle the best answer).
high pass filter low pass filter band pass filter band reject filter

Fall 2003
Question 1 -- RLC, RL and RC Circuits (30 points)
Shown below are 5 circuits. Assume the input voltage $\left(\mathrm{V}_{\text {in }}\right)$ is applied across the leftmost terminals and the output voltage ( $\mathrm{V}_{\text {out }}$ ) is measures across the rightmost terminals.


A


E

$\square$


D


E

Given below are several possible expressions for generic transfer functions for such circuits. Indicate which circuit goes with which function. (1 point each)
$\frac{j \omega L}{R+j \omega L+\frac{1}{j \omega C}}$


$$
\frac{j \omega L+\frac{1}{j \omega C}}{R+j \omega L+\frac{1}{j \omega C}}
$$

$$
\frac{\frac{j \omega L R}{R+j \omega L}}{\frac{1}{j \omega C}+\frac{j \omega L R}{R+j \omega L}}
$$

Find the approximate resonant frequency $\omega_{0}$ for the RLC circuits and the corner frequency $\omega_{\mathrm{c}}$ for the other circuits. That is, write the general expression for each frequency. (1 point each)
A.
B.
C.
D.
E.

Determine the complex transfer function for two of the five circuits (A and B) at the resonant frequency or corner frequency. Be sure your answer is given in terms of $\mathrm{R}, \mathrm{L}$, and/or C and does not contain $\omega$. This may seem like an obvious comment, but we want to make sure you have the simplest possible expression. Identify the magnitude and the phase of the transfer function at this frequency.
A. (4 points) $\mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}=\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{c}}\right)=$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{c}}\right)\right|=\quad \angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{c}}\right)=
$$

B. (4 points) $\mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}=\mathrm{H}\left(\mathrm{j} \omega_{0}\right)=$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{0}\right)\right|=
$$

$$
\angle \mathrm{H}\left(\mathrm{j} \omega_{0}\right)=
$$

Determine the transfer function, magnitude of the transfer function, and phase of the transfer function at low frequencies for C and D . These should be simplified and expressed these in terms of $\omega$ where appropriate.
C. (4 points) $\mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}=\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)\right|=
$$

$$
\angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=
$$

D. (4 points) $\mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}=\mathrm{H}\left(\mathrm{j} \omega_{\text {lo }}\right)=$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{10}\right)\right|=
$$

$$
\angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=
$$

Finally, find the transfer function, magnitude of the transfer function, and phase of the transfer function as $\omega$ approaches infinity for E . These should be simplified and expressed in terms of $\omega$ where appropriate.
E. (4 points) $\mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}=\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)=$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)\right|=
$$

$$
\angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)=
$$

Fall 2003 Solution

## Question 1 -- RLC, RL and RC Circuits (30 points)

Shown below are 5 circuits. Assume the input voltage $\left(\mathrm{V}_{\text {in }}\right)$ is applied across the leftmost terminals and the output voltage $\left(\mathrm{V}_{\text {out }}\right)$ is measures across the rightmost terminals.


A


E

$\square$


D


E

Given below are several possible expressions for generic transfer functions for such circuits. Indicate which circuit goes with which function. (1 point each)

$$
\begin{aligned}
& \frac{j \omega L}{R+j \omega L+\frac{1}{j \omega C}} \\
& \frac{R+j \omega L}{R+j \omega L+\frac{1}{j \omega C}} \\
& \frac{R}{R+j \omega L+\frac{1}{j \omega C}} \\
& \frac{j \omega L+\frac{1}{j \omega C}}{R+j \omega L+\frac{1}{j \omega C}} \\
& \frac{\frac{j \omega L R}{R+j \omega L} \quad E}{\frac{1}{j \omega C}+\frac{j \omega L R}{R+j \omega L}}
\end{aligned}
$$

Find the approximate resonant frequency $\omega_{0}$ for the RLC circuits and the corner frequency $\omega_{\mathrm{c}}$ for the other circuits. That is, write the general expression for each frequency. (1 point each)
A. $\omega_{c}=1 / R C$
B. $\omega_{0}=1 /(L C)^{1 / 2}$
C. $\omega_{c}=R / L$
D. $\omega_{0}=1 /(L C)^{1 / 2}$
E. $\omega_{0}=1 /(L C)^{1 / 2}$

Determine the complex transfer function for two of the five circuits (A and B) at the resonant frequency or corner frequency. Be sure your answer is given in terms of $\mathrm{R}, \mathrm{L}$, and/or C and does not contain $\omega$. This may seem like an obvious comment, but we want to make sure you have the simplest possible expression. Identify the magnitude and the phase of the transfer function at this frequency.
A. (4 points) $\mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}=\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{c}}\right)=j \omega_{\mathrm{C}} R C /\left(j \omega_{\mathrm{C}} R C+1\right)=j(R C / R C) /(j(R C / R C)+1)=$ $j /(j+1)$

$$
H\left(j \omega_{c}\right)=j /(j+1)
$$

$$
\begin{array}{ll}
\left|H\left(j \omega_{c}\right)\right|=1 /(1+1)^{1 / 2} & \angle H\left(j \omega_{c}\right)=\pi / 2-\pi / 4 \\
\left|H\left(j \omega_{c}\right)\right|=1 /(2)^{1 / 2} & \angle H\left(j \omega_{c}\right)=\pi / 4
\end{array}
$$

B. (4 points) $\mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}=\mathrm{H}\left(\mathrm{j} \omega_{0}\right)=1 /\left(\mathrm{j} \omega_{0} \mathrm{RC}+1-\omega_{0}^{2} \mathrm{LC}\right)=1 /\left(\mathrm{j}\left(\mathrm{RC} /(\mathrm{LC})^{1 / 2}\right)=-j(\mathrm{LC})^{1 / 2} / R C\right.$

$$
H\left(j \omega_{0}\right)=-j(L C)^{1 / 2} / R C
$$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{0}\right)\right|=(\boldsymbol{L} \boldsymbol{C})^{1 / 2} / \boldsymbol{R} \boldsymbol{C} \quad \angle \mathrm{H}\left(\mathrm{j} \omega_{0}\right)=-\pi / 2
$$

Determine the transfer function, magnitude of the transfer function, and phase of the transfer function at low frequencies for C and D . These should be simplified and expressed these in terms of $\omega$ where appropriate.
C. (4 points) $\mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}=\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=\boldsymbol{j} \omega_{\mathrm{lo}} \boldsymbol{L} / \boldsymbol{R}$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)\right|=0 \quad \angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=\pi / 2
$$

D. (4 points)

$$
\text { Test } A: \mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}=\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=\left(\mathrm{j} \omega_{\mathrm{lo}} \mathrm{RC}+1\right) /\left(\mathrm{j} \omega_{\mathrm{lo}} \mathrm{RC}+1-\omega_{\mathrm{lo}}^{2} \mathrm{LC}\right)=1 / 1=1
$$

$$
H\left(j \omega_{l o}\right)=1
$$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)\right|=1 \quad \angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=0
$$

Test B: $\mathrm{V}_{\text {out }} / \mathrm{V}_{\mathrm{in}}=\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=\left(-\omega_{\mathrm{lo}}{ }^{2} \mathrm{LRC}\right) /\left[\mathrm{R}\left(1-\omega_{\mathrm{lo}}{ }^{2} \mathrm{LC}\right)+\mathrm{j} \omega_{\mathrm{lo}} \mathrm{L}\right]=-\omega_{\mathrm{lo}}{ }^{2} \mathrm{LRC} / \mathrm{R}=-\omega_{\mathrm{lo}}{ }^{2} \mathrm{LC}$

$$
H\left(j \omega_{l o}\right)=-\omega_{l o}{ }^{2} L C
$$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)\right|=\mathbf{0}
$$

$$
\angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=\pi \boldsymbol{o r}-\pi
$$

Finally, find the transfer function, magnitude of the transfer function, and phase of the transfer function as $\omega$ approaches infinity for E . These should be simplified and expressed in terms of $\omega$ where appropriate.
E. (4 points)

$$
\begin{aligned}
\text { Test } A: & \mathrm{V}_{\mathrm{out}} / \mathrm{V}_{\mathrm{in}}=\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)=\left(-\omega_{\mathrm{hi}}^{2} \mathrm{LRC}\right) /\left[\mathrm{R}\left(1-\omega_{\mathrm{hi}}^{2} \mathrm{LC}\right)+\mathrm{j} \omega_{\mathrm{hi}} \mathrm{~L}\right]= \\
& -\omega_{\mathrm{hi}}^{2} \mathrm{LRC} /-\omega_{\mathrm{hi}}^{2} \mathrm{LRC}=1 \\
& \boldsymbol{H}\left(\boldsymbol{j} \omega_{h i}\right)=\boldsymbol{1}
\end{aligned}
$$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)\right|=1 \quad \angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)=0
$$

Test B: $\mathrm{V}_{\mathrm{out}} / \mathrm{V}_{\mathrm{in}}=\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)=\left(\mathrm{j} \omega_{\mathrm{hi}} \mathrm{RC}+1\right) /\left(\mathrm{j} \omega_{\mathrm{hi}} \mathrm{RC}+1-\omega_{\mathrm{hi}}{ }^{2} \mathrm{LC}\right)=\mathrm{j} \omega_{\mathrm{hi}} \mathrm{RC} /-\omega_{\mathrm{hi}}{ }^{2} \mathrm{LC}$

$$
=-\mathrm{j} \mathrm{R} / \omega_{\mathrm{hi}} \mathrm{~L}
$$

$$
H\left(j \omega_{h i}\right)=-j R / \omega_{h i} L
$$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)\right|=\mathbf{0}
$$

$$
\angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)=-\pi / 2
$$

## Spring 2003

## 4. Transformer (20 pts)


a) In the circuit above, the transformer is ideal. If $\mathrm{R} 1=1 \mathrm{~K} \Omega$, find the equivalent impedance, $\mathrm{Z}_{\mathrm{AB}}$, seen from points A and B . ( 6 pts )
b) We have connected the above circuit to an AC source with a resistor of $\mathrm{R} 2=$ $1 \mathrm{~K} \Omega$.


If the input voltage has an amplitude of 10 V , find the voltage at point A . ( 8 pts )
c) What is the value of the voltage across R1? (6 pts)

## Spring 2003 solution

## 4. Transformer ( 20 pts )


a) In the circuit above, the transformer is ideal. If $\mathrm{R} 1=1 \mathrm{~K} \Omega$, find the equivalent impedance, $\mathrm{Z}_{\mathrm{AB}}$, seen from points A and B . ( 6 pts )
$Z_{A B}=R 1 / a^{2} \quad a=N 2 / N 1=4 \quad Z_{A B}=1 K / 16 \quad Z_{A B}=62.5 o h m s$
b) We have connected the above circuit to an AC source with a resistor of $\mathrm{R} 2=$ $1 \mathrm{~K} \Omega$.


If the input voltage has an amplitude of 10 V , find the voltage at point A . ( 8 pts )

$$
V_{A}=[(62.5) /(1062.5)] 10 V=.588 V \quad V_{A}=\mathbf{5 8 8 m} \boldsymbol{V}
$$

c) What is the value of the voltage across R1? (6 pts)
$V 2 / V 1=a \quad V_{A}=V 1 \quad V 2=.588(4)=2.35 V \quad V_{R 1}=2.35 V$

## Spring 2003

## 3. Instrumentation and Sine Waves (20 points)

You follow this procedure in the studio:

1) Turn on the 'scope and the function generator.
2) Connect a BNC cable from the function generator output to channel 1 of the 'scope.
3) Push the Freq button on the function generator.
4) Turn the dial until the frequency reads 2 K Hz .
5) Push the Ampl button on the function generator.
6) Turn the dial until the peak-to-peak voltage on the function generator display reads 100 mV .
7) Push the Offset button on the function generator.
8) Turn the dial until the DC offset on the function generator display reads 50 mV
9) Adjust the volts/div for channel 1 to $1: 50 \mathrm{mV}$.
10) Adjust the time/div for the horizontal trigger to $200 \mu \mathrm{~s} /$
11) Turn the position knob for channel 1 until the arrow lines up with the zero mark.
a) (2 points) When the function generator is connected to the scope you should notice a discrepancy between the reading on the display panel of the function generator, and the signal displayed on the scope. What discrepancy do you see and which device is correct?
b) (8 points) Find the following for the signal you have created (specify all units).
i) the frequency (f)
ii) the angular frequency ( $\omega$ )
iii) the period (T)
iv) the peak-to-peak voltage $\left(\mathrm{V}_{\mathrm{p}-\mathrm{p}}\right)$
v) the amplitude (A)
vi) the DC offset voltage $\left(\mathrm{V}_{\mathrm{dc}}\right)$
vii) the rms voltage ( $\mathrm{V}_{\mathrm{rms}}$ )
vii) the phase ( $\varphi$ )
c) (2 points) Write down the mathematical expression for the trace in the form: $\mathrm{v}(\mathrm{t})=\mathrm{V}_{\mathrm{dc}}+\mathrm{A} \sin (\omega \mathrm{t}+\varphi)$.
d) (8 points) Sketch what the output should look like on the display below.


## Spring 2003 solution

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1) Turn on the 'scope and the function generator.
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3) Push the Freq button on the function generator.
4) Turn the dial until the frequency reads 2 K Hz .
5) Push the Ampl button on the function generator.
6) Turn the dial until the peak-to-peak voltage on the function generator display reads 100 mV .
7) Push the Offset button on the function generator.
8) Turn the dial until the DC offset on the function generator display reads 50 mV
9) Adjust the volts/div for channel 1 to $1: 50 \mathrm{mV}$.
10) Adjust the time/div for the horizontal trigger to $200 \mu \mathrm{~s} /$
11) Turn the position knob for channel 1 until the arrow lines up with the zero mark.
a) (2 points) When the function generator is connected to the scope you should notice a discrepancy between the reading on the display panel of the function generator, and the signal displayed on the scope. What discrepancy do you see and which device is correct?

The function generator reads half of the scope. The scope is correct.
b) (8 points) Find the following for the signal you have created (specify all units).
i) the frequency (f) $\boldsymbol{f = 2 K \boldsymbol { H z }}$
ii) the angular frequency $(\omega) \quad \omega=2 \pi f=2 \pi x 2 k=4 k \pi$
$\omega=4 \mathrm{k} \pi \mathrm{rad} / \mathrm{sec}=12566 \mathrm{rad} / \mathrm{sec}$
iii) the period (T) $T=1 / f=1 / 2 \mathrm{k}=0.5 \mathrm{~ms} \underline{\boldsymbol{T}=0.5 \mathrm{~ms}}$
iv) the peak-to-peak voltage $\left(\mathrm{V}_{\mathrm{p}-\mathrm{p}}\right) \underline{V}_{p-p}=\mathbf{2 0 0 m V}$
v) the amplitude (A) $\boldsymbol{A}=\mathbf{1 0 0 m V}$
vi) the DC offset voltage $\left(\mathrm{V}_{\mathrm{dc}}\right) \underline{V_{d c}}=\mathbf{1 0 0 m V}$
vii) the rms voltage $\left(\mathrm{V}_{\mathrm{rms}}\right) \quad V_{r m s}=100 / \sqrt{2}=70.7 \mathrm{mV}$

$$
\underline{V}_{r m s}=70.7 \mathrm{mV}
$$

vii) the phase $(\varphi) \underline{\varphi=0}$ (scope automatically triggers at 0 )
c) (2 points) Write down the mathematical expression for the trace in the form: $\mathrm{v}(\mathrm{t})=\mathrm{V}_{\mathrm{dc}}+\mathrm{A} \sin (\omega \mathrm{t}+\varphi)$.

$$
v(t)=100 m V+100 m V \sin (4 \pi K t)
$$

d) (8 points) Sketch what the output should look like on the display below.


